
MAGNANIMITER CRUCEM SUSTINE
GEORGE KENYON
of Peel Esq^r.

LOGARITHMOLOGIA:
OR THE WHOLE
DOCTRINE
OF
LOGARITHMS,
Common and *Logistical*,
IN
THEORY and PRACTICE.

IN THREE PARTS.

PART. I. The THEORY of LOGARITHMS; Shewing their Nature, Origin, Construction, and Properties, demonstrated in various Methods, *viz.* 1. By Plain Arithmetic. 2. By the Logarithmic Curve. 3. By Dr. HALLEY's Infinite Series. 4. By Fluxions. 5. By the Properties of the Hyperbola. 6. By the Equiangular Spiral. 7. By a Logarithmic inspectional Scale of twenty-two Inches length. With the Construction of the artificial Lines of Numbers, Sines, and Tangents. Also the Nature and Construction of Logistical Logarithms. The whole illustrated and made easy by many and suitable Examples.

PART II. The PRAXIS of LOGARITHMS; Wherein all the Rules and Operations of Logarithmical Arithmetic, both Common and Logistical, by Numbers and Instruments, are copiously exemplified. Together with the Application thereof to the several Branches of Mathematical Learning.

PART III. A Three-fold CANON of LOGARITHMS; In a new and more compendious Method than any extant;

Viz.
1. A Canon of Logarithms of NATURAL NUMBERS.
2. A Canon of Logarithms of SINES and TANGENTS.
3. A Table of LOGISTICAL LOGARITHMS.

The whole being a *Compleat System* of this most useful Art; and enrich'd with all the Improvements therein from its Original to the Present Time.

By *BENJAMIN MARTIN*,
Author of *the Philological Library of Literary Arts and Sciences*, &c.

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THE

PREFACE.

THO' Logarithms *may justly be esteem'd the* principal Invention *of* modern Ages, *on account of their* excellent *and most* extensive Use *in* Mathematical Literature, *yet it may with equal Truth be said, that little more is known of them, generally speaking, than their* practical Use *in some Rules of* common Arithmetic *and* trigonometrical Calculations; (*and how few are perfect in this!*) *For* (*saith the* great Improver *of this Art, the learned Dr.* Halley) " *I find very few of*
" *those, who make constant use of* Logarithms,
" *have attain'd an adequate Notion of them;*
" *to know how to make or examine them, or*
" *to understand the* Extent *of the* Use *of*
" *them: contenting themselves with the* Ta-
" bles *of them, as they find them; without*
" *daring to* question them, *or caring to know*
" *how to* rectify *them, should they be found*
" *amiss; being, I suppose, under the Appre-*
" *hension of some great Difficulty therein, &c.*"

For the ſake of ſuch Perſons the following Treatiſe *is principally intended; wherein they will find every thing neceſſary relating to the* Theory *and* Practice *of this* admirable Art.

For, as to the Theory, (*the* principal Part, *and ſo very rarely known) I have exhibited all the* Methods, *whereby it has been taught and explain'd by the* Inventor, *and ſeveral* Improvers *thereof, ſince his time; as by the* Extraction of Roots; *by the* Logarithmic Curve *from Dr.* Keil; *by an* Infinite Series, *from Dr.* Halley; *by the Method of* Fluxions, *from Mr.* Ditton; *by the* Hyperbola, *from Mr.* Domky; *by the* Equiangular Spiral, *from* Halley, Wallis, *&c. by the large* Logarithmic Scale, *of my own* conſtructing: *the like to which, for* Largeneſs, *was never before publiſhed; for a compleat Account of which ſee Chap.* X. *of the Theory.*

I ſay, by all theſe various Methods *I have endeavour'd to* explain, illuſtrate, *and* facilitate *the Knowledge of the* Nature, Properties, *and* Conſtruction *of thoſe* excellent Numbers, *called* Logarithms. *I have alſo exemplified the Manner of* making Logarithms *for the* prime Numbers, *by* many *and* different Examples, *and in ſeveral ways; and have taken all poſſible Care to render this moſt* abſtruſe *and* difficult *Part, as* eaſy *and* intelligible, *as the Nature of the Subject will admit.*

Having

Having thus explain'd the Nature of Logarithms, *I then ſhew how they are laid on* Inſtruments, *and thereby the* Conſtruction *of the* Artificial Lines *of* Numbers, Sines, *and* Tangents; *firſt contrived by the famous Geometer, Mr.* Gunter *of* Greſham-College; *and for that reaſon they are ſtill called* Gunter's Line, *and all together* Gunter's Scale.

Laſtly, I have largely explain'd the Nature, and ſhewn the Manner of making or conſtructing the Logiſtical Logarithms, *according to* Shakerley's *and* Street's *Form thereof; and which I have not ſeen done by any other Hand.*

Theſe things together will, I hope, be allow'd to make a regular, univerſal, *and* compleat *Theory of* Logarithms, common *and* logiſtical, *for Integers and Fractions*, numerical *and* inſtrumental; *and ſuch, as for* Brevity, *yet* Copiouſneſs *and* Variety, *has not been before extant.*

As touching the Praxis, *or* Uſe *of* Logarithms, *which makes the ſecond Part of this Work, I have made it as compleat and perfect as poſſible; having illuſtrated all the Rules of* Practice *with all the* Variety *of* Examples *I could deviſe, that were neceſſary. And that none may be unappriz'd of the moſt* extenſive Service *of* Logarithms *in the* Mathematical Diſciplines, *I have applied them to the* Arithmetic

Arithmetic *of all kinds of Numbers ; to* Trigonometry, *in the Solution of all Cases of* Plain *and* Spherical Triangles ; *to* Mercator's Sailing *particularly, showing how all its Cases may be resolv'd solely by the* Canon *of* Logarithmic Tangents ; *to the* Mensuration *of* Superficies *and* Solids, *&c. All which are* fundamental Operations, *and may each of them be extended or branched out into* particular Sciences ; *but that would have been too tedious a Task, and not absolutely necessary to my Design. I have therefore only applied the* Doctrine *of* Mensuration *to the Arts of* Gauging, Timber-Measure, *and* Surveying ; *because they are the most common and necessary Arts in Life ; and because the Use of* Gunter's Scale, *and* Sliding-Rule (*though before fully taught, and all along applied ; yet*) *in them is more* extensive *and* various *than in any other Arts: and therefore I have taken care not only to shew all the different ways of using those* Instruments, *but likewise the* Rationale *of every Operation ; a Matter of the greatest Importance, and too often neglected, in Books which treat thereof.*

Lastly, I have in the last Chapter given a Variety of Examples of the Use of Logistical Logarithms *in the* practical Parts *of* Astronomy ; *both with respect to* Time *and* Motion, *have made it appear that* Street's Logistical Logarithms *answer all the Ends of* Shakerley's ; *and how they are to be used along with the* Common Logarithms *of* Numbers, Sines, *and* Tangents.

Tangents. *And throughout this* second Part, *as well as the* first, *you will find a great Variety of new and useful Particulars, not here to be express'd.*

The Third Part *of this Work consists of a* three-fold *Canon of* Logarithms, viz. (1.) *Of common Numbers from* 1 *to* 10000; *and is sufficient for any Number under* 10000000 *by proper Rules.* (2). *Of* Sines *and* Tangents *to every* Degree *and* Minute *of the* Quadrant. (3.) *Of* Logistical Logarithms *of Mr.* Street's *Form. Concerning which Tables, I shall only observe* two things *in general,* viz.

First, that they are here contrived in a new *and most* compendious Form, *equally* easy *and* useful *as those of the common Form; tho' in this they take up but* one half *the room, as they do in that. An* Abbreviation *very commodious, and I hope will prove acceptable.*

Secondly; the Correctness of these is a matter of the last Concern, and the greatest Argument to recommend them. In order to prove this, I need only say, that those large Tables of Mr. Sherwin's *are granted to be the most correct of any extant, from the most* careful *and* exquisite Method *he took to make them so, which see in his Preface.*

From these large Tables thus correct, I have made mine, every Figure of the two first Tables with my own Hand; in doing which

which, I discover'd several Errors, here and there, as I went along, in them, as exact as they were, which accordingly correcting in mine, I can, I presume, justly pronounce my Tables the most certain *and* exact, *as well as the most compendious of any in being.*

Having thus largely declared the several Parts of this Work, and shewn the Reader what an useful Variety he may expect to meet with both in the Theory, Praxis, *and* Tables, *of this most useful and excellent Art; I must leave it to himself, to use or reject it, as he shall judge of the Merits thereof.*

THE

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LOGA-

LOGARITHMOLOGIA.

PART I.

The Theory of LOGARITHMS.

CHAP. I.

Of the Definition, Origin, and Nature of LOGARITHMS.

1. THE beſt *Definition* of thoſe Numbers we call *Logarithms*, is contain'd in the very *Name* or *Word* (*Logarithm*) itſelf; for it is compoſed of the two *Greek* Words λὸγων ἀριθμὸς, which properly or literally ſignify, *a Number of Ratio's:* and a *Logarithm* is no other than a Number, which denotes or ſhews what Number of *Ratio*'s is contain'd between *Unity*, and *that Number* of which it is ſaid to be the *Logarithm*.

2. Whence 'tis evident, that in order to have a clear Notion of *Logarithms*, 'tis abſolutely neceſſary to underſtand firſt, and that very well, what is meant by the Word *Ratio*, or *Ratio's*, as here uſed in the Definition of Logarithms, and making an eſſential part thereof.

3. *Ratio, then, is a certain mutual Habitude of Magnitudes of the ſame kind, according to Quantity.* This is *Euclid*'s Definition: in which four things muſt be obſerv'd; as (1.) he ſaith Ratio is *a certain mutual Habitude*; by which he means no more than

what we commonly call the *Proportion of any two things of a like ſort to each other*, when by us they are compar'd together. (2.) He uſeth the general Word *Magnitude* to denote, that all *Subjects of Quantity*, as *Numbers*, *Lines*, *Superficies*, and *Solids*, are capable of ſuch *Ratio*, *Habitude*, or *Proportion*, as aforeſaid. (3.) He adds this Reſtriction, *of the ſame kind*; thereby inſinuating there can be no *Ratio* or *Proportion of Quantity* between Magnitudes of a *different kind*; thus we cannot compare a *Line* to a *Superficies*; becauſe the Quantity of a Line is eſtimated in *Length* only, but the *Quantity* of a *Superficies* ariſeth from the joint Conſideration of Length and Breadth, or the Product of each, and ſo importeth *Space*; which is entirely different from a *Line*, and therefore theſe two things cannot be the Terms of Ratio or Compariſon. (4.) Laſtly, he ſays, this *Ratio* is according to *Quantity*; that is, we compare Magnitudes in this Caſe, only to obſerve and maintain the *Proportion of Greatneſs or Bulk* which is between them; or to find *how often*, or *how many times*, one *leſſer Magnitude* is contain'd in another *greater Magnitude*; neglecting all other Conſiderations and Affections of the ſaid Magnitudes.

4. Having thus conſider'd the general Nature of *Ratio's* or *Proportions*; I ſhall apply it to *Numbers*, as they are immediately the Subject of *Logarithms*. The *Ratio* therefore, or *Proportion* of a *Number to a Number* is two-fold; for firſt the Ratio of a *greater Number* to a *leſſer* may conſiſt in the *Addition* of ſome certain Number to that *leſſer Number*; thus the Ratio of 6 to 2 is made by adding 4 to 2. And if from Unity you begin the conſtant Addition of the ſame Number, ſuppoſe 2, you will then have a Series of Numbers, whoſe Differences will be the ſame, as 1, 2, 4, 6, 8, 10, 12, &c. and ſuch Numbers are ſaid to be in *Arithmetical Proportion* or *Pro-*

greſſion;

greſſion; and this *common Difference* of the Terms, as here 2 is called the *Ratio* of the Progreſſion.

5. Secondly, the Ratio of a *greater Number* to a *leſſer* may conſiſt in a *Multiplication* of the *Leſſer Number* by ſome other Number; thus the Ratio of 12 to 4, is made by multiplying 4 by 3; and if from *Unity* you begin *a conſtant Multiplication* by the *ſame Number*, ſuppoſe 2, you will then have a *Series* of Numbers, as 1, 2, 4, 8, 16, 32, 64, *&c.* which are ſaid to be in *Geometrical Proportion*, or *Progreſſion*; and the *common Multiplier*, as here 2, is called the *Geometrical Ratio* of this Progreſſion.

6. Wherefore in the two Series or Progreſſions of Numbers, *viz.*

Arith. 1, 2, 4, 6, 8, 10, 12, 14, *&c.*
Geom. 1, 2, 4, 8, 16, 32, 64, 128, *&c.*

'tis eaſy to obſerve, that as the ſecond Term exceeds the firſt by one Ratio, ſo the third Term exceeds the firſt by two Ratio's, the fourth by three Ratio's, the fifth by four Ratio's, *&c.* Thus in the Arithmetical Series, the Ratio of 4 to 1, is *double* of the Ratio of 2 to 1; the Ratio of 6 to 1, is *triple* the Ratio of 2 to 1; the Ratio of 8 to 1, is *quadruple* the Ratio of 2 to 1, *&c.* And in the Geometrical Series, the Ratio of 4 to 1 is the *duplicate* of the Ratio of 2 to 1; the Ratio of 8 to 1 is *triplicate* of the Ratio of 2 to 1, and the Ratio of 16 to 1 is *quadruplicate* of the Ratio of 2 to 1, and ſo on. Where 'tis to be obſerved, that the Words, *double*, *triple*, *quadruple*, &c. are proper to the Ratio's of the *Arithmetical Series*, and imply the *Addition* of them only; but the Terms *duplicate*, *triplicate*, *quadruplicate*, &c. are proper to the *Geometrical Series*, and imply the *Multiplication* of thoſe *Ratio's*.

7. What has been thus far related of the Doctrine of Ratio's, is ſufficient for our preſent purpoſe, *viz.* the underſtanding the *Nature* of *Logarithms*. For

ſuppoſe a Series of Numbers in *Arithmetical Progreſſion*, beginning from 0, and whoſe *Ratio*, or common Difference, is Unity or 1; and to them be adapted a Series in *Geometrical Progreſſion*, beginning from *Unity*; and whoſe common Ratio be any aſſign'd Number, ſuppoſe 2, as before; then will thoſe two Series ſtand as below:

viz. { Arith. 0. 1. 2. 3. 4. 5. 6. 7. 8. 9 *&c.*
Geom. 1. 2. 4. 8. 16. 32. 64. 128. 256. 512. *&c.*

8. 'Tis evident the Numbers in the firſt Series ſhew the Number of Ratio's between their correſpondent Numbers and Unity in the ſecond Series. For inſtance, the Figure 2 in the firſt Series, ſhews the Ratio's between its correſponding Number 4 and 1, in the lower Series, are 2; the Numbers 5, 7, 9, in the upper Series, ſhew the Number of Ratio's between their correſponding Numbers 32, 128, 512, and 1 or Unity, in the lower Series, to be reſpectively 5, 7, and 9; or that the Ratio is ſo often repeated from Unity to thoſe Numbers, and conſequently ſo often compounded in them; or farther, that the Ratio of 32 to 1 is compounded of five times the Ratio of 2 to 1; and the Ratio of 128 to 1, of ſeven times the Ratio of 2 to 1; and the Ratio of 512 to 1, of nines times the Ratio of 2 to 1.

9. Wherefore ſince the Numbers in the upper Series ſhew the Number of Ratio's contain'd between their correſponding Numbers and Unity in the lower Series; therefore thoſe Numbers in the upper Series in Arithmetical Progreſſion ſhall be the *Logarithms* of the Numbers in the lower Series of Geometrical Progreſſion, and that according to the Definition of Logarithms beforegoing.

10. From the ſaid Series, 'tis farther manifeſt, that the Product of any two Terms in the lower Series correſponds to the Sum of their reſpective Terms in the upper Series. See the following Examples.

Arith.

Arith. or Logar. $\left\{ 2+3=5;\ 2+4=6;\ 3+6=9. \right.$

Geom. or N.Num. $\left\{ 4\times 8=32;\ 4\times 16=64;\ 8\times 64=512. \right.$

Alſo if any two Numbers in the lower Series be divided the one by the other, the Quotient thence ariſing ſhall correſpond to the Difference of the reſpective Numbers in the upper Series. Examples in

Arith. $5-3=2;\ 6-2=4;\ 9-3=6.$
Geom. $32\div 8=4;\ 64\div 4=16;\ 512\div 8=64.$

11. And univerſally, if any four contiguous Numbers be taken in the ſecond Series, as the Product of the Extremes is equal to the Product of the Means; ſo in the firſt Series of the four correſponding Numbers, the Sum of the Extremes will be equal to the Sum of the Means; as in the Examples below.

Arith. $1+4=2+3=5;\ 3+6=4+5=9.$
Geom. $2\times 16=4\times 8=32;\ 8\times 64=16\times 32=512.$

and *vice verſa*.

12. Again the *Square*, *Cube*, *&c.* of any Number in the lower Series of Geometricals will be anſwer'd by *double*, *triple*, &c. the correſponding Number in the upper Series in Arithmetical Ratio; for Example;

Square $\left\{ \begin{array}{l} \text{Arith. } 2\times 2=4;\ 4\times 2=8;\ 0\times 2=0. \\ \text{Geom. } 4\times 4=16;\ 16\times 16=256;\ 1\times 1=1. \end{array} \right.$

The Cube $\left\{ \begin{array}{l} \text{Arithmet. } 1\times 3=3;\ 3\times 3=9. \\ \text{Geomet. } 2\times 2\times 2=8.\ 8\times 8\times 8=512. \end{array} \right.$

And the Converſe of this Article is alſo true; as is evident enough without example.

13. It appears then by theſe ſix laſt Articles, that *Logarithms are a Series of Numbers in Arithmetical Progreſſion, ſo fitted and adapted to another Series of Numbers in Geometrical Progreſſion, as that each*

Term of the firſt ſhall expound (or be the Exponent *of) the Ratio of its correſpondent Term to Unity in the ſecond Series.* And that on this very Principle: For every *Addition*, *Subſtraction*, *Multiplication*, or *Diviſion* of the *Logarithmic Numbers* there correſponds a *mutual Multiplication*, *Diviſion*, *Involution*, and *Extraction* of the *reſpective Terms* in the Geometrical Series.

14. Now 'tis a Matter entirely arbitrary or indifferent, what Number be made the *firſt Term* in either Series; for ſince the *firſt* are made by *equal Additions*, the *latter* by equal *Multiplication*, be the *Ratio* what it will, the *former* will ſtill be the *Logarithms* of the *latter*; as is evident in the Table adjoin'd.

		Series of Logarithms.					
6	1	1	3	6	7	0	0
18	2	2	5	7	12	1	10
54	4	3	7	8	17	2	20
162	8	4	9	9	22	3	30
486	16	5	11	10	27	4	40
1458	32	6	13	11	32	5	50
4374	64	7	15	12	37	6	60
13122	128	8	17	13	42	7	70
39366	256	9	19	14	47	8	80
118098	512	10	21	15	52	9	90
354294	1024	11	23	16	57	10	100

15. Wherefore to the *ſame Series* of *Proportionals* there may be an *infinite Number* of *Series* or *Scales* of *Logarithms* contrived; and *vice verſa*. Yet of *all* thoſe *infinite* kinds of Logarithms, only thoſe whoſe firſt Term is 0, and the *common Difference* 1, 10, 100, *&c.* are adapted for uſe. Becauſe if the firſt Term be a ſignificant Figure, we muſt neceſſarily have reſpect to it in uſe; and ſo, in this caſe, four

four Terms of the *Logarithmical Series* becomes unavoidable; whereas if 0 be the first Term, three other Terms only suffice in the *Multiplication* of any two *Proportionals* whatever: for the *Sum* of their Logarithms will point out the *Product*, and shew its *Place*, that is, its *Distance* from, or *Ratio* to, the first Term of *that Scale* of *Proportionals*. But if the *first Term* in the Scale of Logarithms were significant, it must be subducted from the *Sum* of the Mean, in order to find the Product of the two *Proportionals*, *&c.* as before: and so we should perpetually have *double Labour* in every Operation. All this is evident from the different Series of Logarithms in the foregoing Table.

16. This being the Nature, and such the wonderful Properties of those Numbers called Logarithms, 'tis natural to suppose that he who first discover'd them, would make such a *noble Discovery* as subservient as possible to the Uses of Life, for the general Benefit of Mankind, but more immediately of *Artists* or *Mathematicians*.

Proport.	Logarith.
1	0.0000000
10	1.0000000
100	2.0000000
1000	3.0000000
10000	4.0000000
100000	5.0000000
1000000	6.0000000

17. Now to do this, 'twas necessary to calculate and fit a *Scale* or *Canon* of *Logarithms* to all Numbers which Men commonly make use of in Business, that so the *Fatigue* and *Labour* of *Multiplying*, *Dividing*, &c. large *Sums* or *Numbers* might be avoided by only the Addition or Substraction, *&c.* of Logarithms. And of consequence nothing less than a *general Table of Logarithms* for all Numbers between Unity, or 1 and 1000000 or 10000000, could suffice; because no Number must here be wanting, as in the other intermitting Series 1, 2, 4, 8, 16, *&c.* since general Use requires them all, and therefore their Logarithms.

18.

18. Towards this 'twas eaſy to find Logarithms for a Series of Numbers proceeding in a *Decuple Proportion* or *Ratio* from Unity, as 1, 10, 100, 1000, *&c.* in the Table above. For putting 0 = Logarithm of 1; and 1 = Logarithm of 10; the Logarithm of 100 will be 2; and of 1000, 3; and ſo on from what I have already ſaid.

19. But the *Difficulty* all conſiſted in finding the Logarithms for the intermediate Numbers between 1 and 10, 10 and 100, 100 and 1000, *&c.* or thoſe of them which are called *Prime Numbers*, becauſe they being once found, the Logarithms of all others are eaſily obtain'd, as we ſhall ſee by and by.

20. Now tho' the Numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, are not in *Geometrical Proportion* (but indeed in an *Arithmetical one*) and ſo their Logarithms not to be obtain'd like the Logarithms of thoſe which are ſo, yet as the *Ratio* of 10 to 1, ſhews only what *Diſtance*, or *how far* 10 is from Unity in the Scale of Proportion, and that the Numbers 2, 3, 4, 5, *&c.* poſſeſs ſeverally a certain part of that Diſtance, if the ſaid Ratio or Diſtance of 10 to 1, be ſuppoſed to be divided into a vaſt number of *equal Parts*, ſuppoſe 10000000, *&c.* 'tis evident a certain number of thoſe equal Parts are to be allotted to the Numbers 2, 3, 4, 5, *&c.* which ſhall as truly expreſs the *Diſtances* from, or the *Ratio's* of thoſe ſeveral Numbers to Unity, as that of 10 to Unity is expreſs'd by the whole Number 10000000.

21. This indefinite Number (1000000, *&c.*) of *equal Parts*, into which the whole Ratio of 10 to 1, is divided, may be conceived as a Number of ſo many *ſmall Ratio*'s or *Ratiunculæ*, ſince they are all equal to each other. And ſo ſince the whole Number of *Ratiunculæ* between 10 and 1, is the Logarithm of 10 to 1; therefore that Number of thoſe *Ratiunculæ* which lie between, or expreſs the Diſtance of 2, 3, *&c.* to 1, ſhall alſo be as much the

Logarithms

Logarithms of thoſe Numbers 2, 3, *&c.* or, rather, of their Ratio's to Unity. And thus it appears that even the Logarithms of 2, 3, 4, 5, *&c.* (and for the ſame Reaſon) 11, 12, 13, *&c.* 21, 22, 23, *&c.* 101, 102, 103, *&c.* may be found to as great Exactneſs as is neceſſary. The Method of doing this by Numbers, ſhall be the Subject of the next Chapter.

CHAP. II.

Of the Method of making a Table of Logarithms by plain Arithmetic.

1. HAving explain'd the Nature and Properties of Logarithms, and ſhewn not only how they are adapted to Series of Geometrical Proportionals, but alſo by what means they are to be calculated for all Numbers from Unity to any large Number either above or below it:

2. The *Method* or *Proceſs* it ſelf, then, of doing this, is the next thing to be propoſed; an *arduous Taſk* this, to him that firſt aggreſs'd it! At ſuch an Enterprize, he indeed might truly have ſaid, *Hic Labor, hoc Opus eſt.* And this any but an *inhuman Reader* will be convinced of, by viewing the following *tedious* and *laborious* Proceſs for gaining the Logarithm of one ſingle Prime Number, and that only to ſeven Places of Figures.

3. For, as I ſaid before, 'tis ſufficient to produce the Logarithms of *Prime Numbers* only, becauſe, by the *Addition*, Subſtraction, *&c.* of theſe, we obtain the Logarithms of all Compoſit Numbers, with Eaſe; as will appear further on. Now, tho' there are but three Prime Numbers, *viz.* 3, 5, 7, between

1 and 10; yet, because the Logarithm of no Number between 1 and 10 can be found by the Logarithms of any of the Proportionals 1, 10, 100, 1000, *&c.* it follows, that the Logarithm of any one of the nine Digits is found with equal Difficulty for the first. Therefore I shall give an Example in finding the Logarithm for the Number 9.

4. The Number 10, then, being already supposed at such a Distance from Unity as contains 10000000 equal Parts or *Ratiunculæ*, which is the Logarithm of 10; therefore to find the Logarithm of 9, is to find *how many*, or *what Number* of those *Ratiunculæ* are contain'd between 1 and 9, which is done in the following Manner.

5. First, make A=1, whose Log. is 0.0000000; and B=10, which Logarithm 1.0000000; as in the Table below.

Secondly, between A and B, or 1 and 10, find a *Mean Geometrical Proportional* C=3.1622777; the Logarithm whereof will be half the Logarithm of 10, *viz.* 5000000.

Thirdly, because C is much less than 9, find another *Mean Proportional* D=5.623413 between B and C; whose Logarithm will be an *Arithmetical Mean* between the Logarithms of B and C, *viz.* 0.7500000.

Fourthly, because D is still much less than 9, find another *Geometrical Mean* between B and D, *viz.* E=7.4989421, whose Logarithm is an Arithmetical Mean between those of B and D, *viz.* 0.87500000.

Fifthly, since E is yet much less than 9, find yet another *Geometrical Mean* F=8.6596432, between B and E; the Logarithm of which will be an *Arithmetical Mean* 0.9375000, between those of B and E.

6. And thus continue finding *Mean Geometrical Proportionals* between the Numbers *next Greater* and *next Lesser* than 9, till you arrive to the Number 9 itself,

itſelf, which ſhall be clear of all other Figures, but *Cyphers*, to the Number of Places propoſed ; which will happen at the twenty-ſixth Trial, as you ſee in the following Table.

	Proportionals.	Logarithms.
A	1.0000000	0.00000000
C	3.1622777	0.50000000
B	10.0000000	1.00000000
B	10.0000000	1.00000000
D	5.6234132	0.75000000
C	3.1622777	0.50000000
B	10.0000000	1.00000000
E	7.4989421	0.87500000
D	5.6234132	0.75000000
B	10.0000000	1.00000000
F	8.6596432	0.93750000
E	7.4989421	0.87500000
B	10.0000000	1.00000000
G	9.3057204	0.96875000
F	8.6596432	0.93750000
G	9.3057204	0.96875000
H	8.9768713	0,95312500
F	8.6596432	0,93750000
G	9.3057204	0.96875000
I	9.1398170	0.96093750
H	8.9768713	0.95312500
I	9.1398170	0.96093750
K	9.0579777	0.95703125
H	8.9768713	0.95312500
K	9.0579777	0.95703125
L	9.0173333	0 95507812
H	8.9768713	0.95312500
L	9.0173333	0.95507812
M	8.9970796	0.95410156
H	8.9768713	0.95312500

	Proportionals.	Logarithms.
L	9.0173333	0.95507812
N	9.0072008	0.95458984
M	8.9970796	0.95410156
N	9.0072008	0.95458984
O	9.0021388	0.95434570
M	8.9970796	0.95410156
O	9.0021388	0.95434570
P	8.9996088	0.95422363
M	8.9970796	0.95410156
O	9.0021388	0 95434570
Q	9.0008737	0.95428467
P	8.9996088	0.95422363
Q	9.0008737	0.95428467
R	9.0002412	0.95425415
P	8.9996088	0.95422363
R	9.0002412	0.95425415
S	8.9999250	0.95423889
P	8.9996088	0.95422363
R	9.0002412	0.95425415
T	9.0000831	0.95424652
S	8.9999250	0.95423889
T	9.0000831	0.95424652
V	9.0000041	0.95424271
S	8.9999250	9.95423889
V	9.0000041	0.95424271
X	8.9999650	0.95424080
S	8.9999250	0.95423889
V	9.0000041	0.95424271
Y	8.0000845	0.95424217
X	8.9999650	0.95424080
V	9.0000041	0.95424271
Z	8.9999943	0.95424223
Y	8.0000845	0.95424217

	Proportionals.	Logarithms.
V	9.0000041	0.95424271
a	8.9999992	0.95424247
Z	8.9999943	0.95424223
V	9.0000041	0.95424271
b	9.0000016	0.95424259
a	8.9999992	0.95424247
b	9.0000016	0.95424259
C	9.0000004	0.95424253
a	8.9999992	0.95424247
c	9.0000004	0.95424253
d	8.9999998	0.95424250
a	8.9999992	0.95424247
c	9.0000004	0.95424253
e	9.0000000	0.95424251
d	8.9999998	0.95424250

7. Thus, after a long and laborious Calculation, (consisting of 26 Multiplications and Extractions of large Numbers) we at length have obtain'd the Logarithm of 9, *viz.* 0.95424251, which is exactly true to 7 Places or Figures: That is, if the *Logarithm*, or Distance of 10 from Unity be supposed to consist of 10000000 equal Parts, then the Distance of 9 from Unity shall consist of 9542425 of those equal Parts precisely; which therefore is the Logarithm of 9, because it expresses the Ratio of 9 to 1. If the said Logarithm, or Distance of 10 from Unity were supposed to consist of 100000.00000.00000.-00000. 00000. 00000. 00000. 00000 00000.00000.-00000.000000 equal Parts; then the Distance of 9 from 1, would be 95424.25094.30324.87459.00558.-06510. 23061. 84002. 57728. 38139. 17296. 597313 of the same equal Parts; and is its Logarithm to 62 Places of Figures. But the Reader is not to imagine that this prodigious Logarithm was produced by the

foregoing operoſe Method: No; for tho' 'tis not impoſſible, 'tis impracticable by that Method; but it is the Product of modern Invention; an Inſtance of which will be given in due Place.

8. Having thus found the Log. of 9=0.9542425 if it be divided by 2, the Quotient or Half, 0.4771212 will be the Logarithm of 3, by the Converſe of Article 12. Chap. 1. The Double of the Logarithm of 9 is the Logarithm of 81=1.9084850 by the ſame Article. And thus by a continual Multiplication of this one Logarithm by 2, 3, 4, 5, *&c.* you gain the Logarithms of all the Powers of Nine.

9. 'Tis farther evident, that if the Ratio of 10 to 1, 100 to 10, 1000 to 100, *&c.* be ſuppoſed only Unity or 1, as expreſſed in the Table of Art. 17. Chap. I. the Logarithm of Nine, *viz.* 0.95424225 will be no other than a Decimal Fraction, whoſe Denominator is the ſaid *Ratio*, or Unity, with Cyphers annex'd, thus $\frac{9542425}{10000000}$.

Therefore alſo the Log. of any Number, *viz.*		may be expreſs'd Fraction wiſe, as follows, *viz.*	
	3=0.4771212		$3=0.\frac{4771212}{10000000}$
	9=0.9542425		$9=0.\frac{9542425}{10000000}$
	10=1.0000000		10=1.0000000
	81=1.9084850		$81=1.\frac{9084850}{10000000}$
	100=2.0000000		100=2.0000000
	729=2.8627275		$729=2.\frac{8627275}{10000000}$
	1000=3.0000000		1000=3.0000000

10. Hence 'tis obvious that the Logarithms are only the Decimal Parts of ſeveral *Ratio's*, 1, 2, 3, *&c.* of 10, 100, 1000, *&c.* to Unity; and that thoſe *Ratio's* themſelves make the *integral Part* of the *mixt Decimal* Logarithm, (as I may call it.) Theſe *integral Parts* of thoſe *mixt Decimal Logarithms*, then are what we call the *Indexes*, or rather *Indices*, of thoſe Logarithms, *or fractional Parts*; wherefore, the *Ratio's* of 10, 100, 1000, 10000, 100000, *&c. to Unity, are the Indices of or belonging to the Logarithms of all intermediate Numbers immediately*

mediately above those Proportionals. So the Ratio of 10 to 1, *viz.* 1, is the *Index* of the Logarithms of all Numbers between 10 and 100. And 2 (the Ratio of 100 to 1) is the *Index* of all Logarithms of Numbers, from 100 to 1000, and ſo on.

11. From whence 'tis eaſy to obſerve, *That the Index of any Logarithm contains a Number of Units leſs by one than is the Number of Figures in that Number which belongs to the ſaid Logarithm.*

$$\text{Therefore the Number} \left\{ \begin{array}{r} 9 \\ 81 \\ 729 \\ 1000 \end{array} \right\} \text{the Numb. of whose Fig}^{s}\text{. are} \left\{ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\} \text{hath the } \textit{Index} \text{ of its Log.} \left\{ \begin{array}{l} 0. \\ 1. \\ 2. \\ 3. \end{array} \right\}$$

12. Now as this *Index* is thus called, from its *indicating* or ſhewing how many Places of Figures there are in the Number of the Logarithm; ſo, on the contrary, the Number of the Logarithm, as plainly ſhews what the Index of the Logarithm muſt be. And therefore, ſince in the Tables of Logarithms, all the Numbers of thoſe Logarithms are appoſitely placed in proper Columns by them, 'tis entirely needleſs to print the *Indices*, as being well enough known by the Numbers; and thus they are (for this Reaſon) omitted in ſeveral of the ſaid Tables; and conſequently in thoſe we have here, *in ſo conciſe a Form*, now firſt of all made publick.

13. What has been ſaid hitherto, relates altogether to the Logarithms of *Whole Numbers*; but the *ſame Doctrine* is equally applicable both in *Theory and Practice*, to *fractional Numbers*; for in Effect, the Properties of *Decimal* and *Whole* Numbers are the ſame. Thus the Series of *decuple Proportionals*, 1, 10, 100, 1000, 10000, *&c.* may be continued as well *below* Unity, as *above* it; for the following Rank of Numbers, *viz.* $\frac{1}{10000}$, $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, 1, 10, 100, 1000, 10000, *&c.* which may be thus expreſs'd, 0,0001, 0,001, 0,01, 0,1, 1, 10, 100, 1000, 10000,

10000, *&c.* are all in a *Geometrical Decuple Ratio or Proportion*; and confequently equally diftant from Unity on each Side, and fo their Logarithms equidifferent, and the fame. The Ratio's then, in the Scale above Unity, may be called *Pofitive Ratio's*; and thofe below Unity, *Negative Ratio's*; and thus the *Index* of a Logarithm of a *Whole Number*, or *Integer*, fhall be *pofitive*, but the Index of the Logarithm of *Fraction* fhall be *negative.*

14. And fince *Vulgar Fractions*, *Duodecimal*, *Sexagefimal*, &c. Fractions, are all reducible to *Decimals*; 'tis evident that all *Numerical Arithmetic* whatfoever is fubject to, and manageable by this Art of *Logarithmical Arithmetic.* The only Difficulty being in adapting or readily finding the proper *Index* to the Logarithm of a Decimal Number. For, fince the *Index* in the Logarithm of *Integers*, fhews only the *Number of Places* or *Figures* in the faid Integers, 'tis plain thofe *Indices decreafe*, as the Places of Figures in the Integer decreafe, and intirely vanifh when thofe Places of Figures become Unity or one, that is, the *Index* is then 0; and confequently cannot ferve for *Decimal Numbers.*

15. Therefore fome new kind of *Indices* or *Characteriftics* muft be invented, which fhall be proper only to *Decimal Numbers*, as the other are to *Integers*; and fuch as fhall as readily difcover the *Number of Cyphers* to be *prefix'd* to the *fignificant Figures* in the *Decimal*, as the other determine the *Number of Figures* in the *Integer*, are in the fame degree ufeful for *Decimals*, as they for *Integers.* For as to the fignificant Figures of the *Decimal*, the *Logarithm it felf* difcovers them; all therefore that is farther neceffary, is to procure *fuch Indices* as fhall at all times denote *how far*, or *how many Places* from *Unity*, the *firft fignificant* Figure of the *Decimal* muft ftand; or, which amounts to the fame, *how many Cyphers muft be prefix'd* to compleat the true Value of the Decimal.

16.

16. The *Indices* for this Purpoſe are therefore judg'd beſt, *which being ſubſtracted from* 9, *ſhall have a Remainder expreſſing the Number of Cyphers to be prefix'd*; thus the *Index* of a Decimal whoſe firſt Figure (to the Left,) is ſignificant,

Integers	47500=4.6766936
	4750=3.6766936
	475=2.6766936
Decimals	47,5= 1.6766936
	4,75= 0.6766936
	,475=.9.6766936
	,0475=.8.6766936
	,00475=.7.6766936
	,000475=.6.6766936
	,0000475=.5.6766936

muſt be 9, becauſe 9—9=0, that is *no Cypher* is to be prefix'd. And thus, a *Decimal* that has 1, 2, 3, 4, *&c. Cyphers*, muſt have the *Indices*, accordingly 8, 7, 6, 5, *&c.* becauſe 9—8=1, 9—7=2, 9—6=3, 9—5=4, *&c.* denoting the *Cyphers* to be prefix'd to the *Decimal Numbers*. This is evident by the Examples in the Table above; but it is proper to dott theſe new *Indices* on *each Side*, to diſtinguiſh 'em from thoſe which belong to the Logarithms of *Integers*.

17. From all that has been ſaid hitherto, 'tis evident, that while the *ſignificant Figures* in *any Number* whatever, *Integral* or Decimal, are the ſame, the *Logarithm* of thoſe Figures will be the *ſame* alſo; the Variation occaſion'd by the differing Nature of the Number, being only in the *Indices*, as denoted in the ſaid Table. But ſomething farther will be ſaid of the *Nature* and *Variety of Indices*, when we come to treat of the *Practical Rules* of *Addition* and *Subſtraction* of Logarithms; I ſhall now proceed to the next Chapter, wherein the *myſterious Nature*, or *Doctrine of Logarithms* will be farther illuſtrated, and render'd more obvious to the Senſes by *Geometrical Schemes* and *Demonſtrations*.

CHAP. III.

The Doctrine of the Nature and Properties of Logarithms farther explained and illuſtrated, by means of the LOGARITHMIC CURVE.

1. IN explaining the Nature of *Logarithms* from the *Logarithmic Curve*, 'twill be expedient to repreſent *Proportional Quantities* by *Letters or Species*; as being ſuited to a more *univerſal Theory* than *Numbers*, and better applicable to *Right Lines*, by which alone both *Numbers* and their *Logarithms* are repreſented in the *Geometrical* Method now before us.

2. In a Series of *Proportionals*, then, increaſing from Unity, let the firſt be $=a$; then will the Series be $1, a^1, a^2, a^3, a^4, a^5$, &c. where 'tis evident the *Indices* or *Exponents* of the ſeveral *Powers* of a, are a Series of Numbers in *Arithmetical Progreſſion*, each whereof ſhews the *Place* or *Diſtance* of its *Term* from Unity; thus the *Term* a^4 is ſhewn by its *Index* 4 to be in the *fourth Place* from Unity; and a^5 is in the *fifth Place*; or a^4 and a^5 are *four* and *five times* more diſtant from Unity than the *firſt Term* a; which is here the *common Ratio* alſo.

3. From hence you obſerve, that the *Exponents* of the *Powers* of the Terms in the Series $1, a^1, a^2, a^3, a^4, a^5, a^6, a^7$, &c. are the *Logarithms* of thoſe Terms reſpectively. Let the *Exponent* be $=e$; then

$$\begin{cases} \text{the Terms, } 1, a, aa, aaa, aaaa, aaaaa, aaaaaa, \text{ \&c.} \\ \text{the Expon. } 0, e, 2e, 3e, 4e, 5e, 6e, \text{ \&c.} \end{cases}$$

Therefore

$$\begin{cases} \text{As } aa \times aaa = a^5;\ a \times a^5 = a^6;\ \text{and } aa \times aaa\sqrt{a} = a^5\sqrt{a}, \\ \text{So } 2e + 3e = 5e;\ e + 5e = 6e;\ \text{and } 2e + 3\tfrac{1}{2}e = 5\tfrac{1}{2}e. \end{cases}$$

and ſo every where, agreeable to the Nature of *Logarithms*

garithms before deſcrib'd, in the two preceding Chapters.

4. If between 1 and a there be put one *mean Proportional*, viz. $\sqrt{a}$, its *Index* or *Exponent* muſt be wrote $\frac{1}{2}$; thus $\sqrt{a} = a^{\frac{1}{2}}$; for 0, $\frac{1}{2}$, 1, have an *Arithmetical Ratio*; and ſo the Terms 1, $a^{\frac{1}{2}}$, a, are *Geometrical Proportionals*. Thus, a *mean Proportional* between a and aa, is $a\sqrt{a} = a^{1\frac{1}{2}} = a^{\frac{3}{2}}$. Alſo if you conceive two mean Proportionals between 1 and a, they ſhall be $a^{\frac{1}{3}}$, $a^{\frac{2}{3}}$; for $1 : a^{\frac{1}{3}} :: a^{\frac{2}{3}} : a$; and 0, $\frac{1}{3}$, $\frac{2}{3}$, 1, have an *equal Difference*.

5. Moreover the ſame Series of *Geometrical Proportionals* may be continued both ways, and be made *decreaſing* as well as *increaſing*; that is, it may as well *deſcend below Unity* to the *left Hand*, as *aſcend* above it to the *Right*. Thus the Terms $a^{\frac{1}{5}}$, $a^{\frac{1}{4}}$, $a^{\frac{1}{3}}$, $a^{\frac{1}{2}}$, $\frac{1}{a}$, 1, a, a^2, a^3, a^4, a^5, &c. are all in the ſame *Geometrical Progreſſion*. And ſince the Diſtance of a^2 from Unity towards the *right Hand* is *poſitive*, and denoted by the *poſitive Exponent* $+2$; ſo $a^{\frac{1}{2}}$ being equally diſtant on the *contrary Side*, or *below Unity*, and this Diſtance being *negative*, therefore the Exponent thereof may be thus *negatively* expreſs'd, $a^{-2} = a^{\frac{1}{2}}$ 1, and ſo $a^{\frac{1}{3}}$ is the ſame as a^{-3}; thus the Series a^{-5}, a^{-4}, a^{-3}, a^{-2}, a^{-1}, 1, a, a^2, a^3, a^4, a^5, &c. is the ſame as the laſt foregoing.

6. Therefore, in thoſe Series, if a repreſent any Right Line (ſee Fig. 1.) aa, or the Square thereof, is not to be taken as a Quantity of *two Dimenſions*, or *Surface*, viz. a *Geometrical Square*; but only as a *Line* that is a *third Proportional* to ſome *Line* taken as *Unity*, and the *Line a*. So likewiſe $aaxa$, or aaa, is not a *Quantity* of *three Dimenſions*, or a *Geometrical Cube*; but a Line that the *fourth Term* or *Proportional* in a *Geometrical Series*, whoſe *firſt Term* is 1,

and *second a.* And thus you are to conceive of all the rest; *viz.* simply as *Proportionals of Length* only.

7. This being premised then, if on any Line, as AN (Fig. 2.) both ways indefinitely extended from A, be taken the equal Distances AC=CE=EG=GI=IL=LN, to the *right Hand*; and Ac=ce=eg, *&c.* on the left; and if on the Points g, e, c, A, C, E, G, I, L, N, be erected to the Right Line gN, the Perpendiculars gh, ef, cd, AB, CD, EF, GH, IK, LM, NO, which let be in a continual *Geometrical Proportion* to each other, and represent *Numbers*, whereof AB=1, or Unity. The Tops of these Lines being duely join'd, from what is call'd the *Logarithmetic Curve*, viz. hBHO; by which we are farther to explain and illustrate the *Nature* and *Properties* of *Logarithms*.

8. But a more compleat Idea of the *Logarithmical Curve*, may be conceiv'd by a twofold Motion of the Line AB; the one *equable*, the other *accelerated* or *retarded* in any given *Geometrical Ratio*: For Example, if the Right Line AB moves uniformly along the AN, so that the End A describes *equal Spaces* AC=CE=EG, *&c.* in *equal Times*; while in the same Time the said Line AB so *increases*, that the *Increments* thereof generated in *equal Times*, be *proportional* to the *whole* increasing Line; that is, if AB in moving forward to *ab*, be *increased* by the *Increment ob*, and in an *equal Time*, when it comes to CD, the *Increment* thereof is D*p*; and D*p* : *ab* :: *bo* : AB; then the End B of the said Line AB thus *continually increasing* or *decreasing* in the *same Ratio*, will describe the *Logarithmic Curve*. For since AB : *bo* :: *ab* : D*p* :: DC : *dq*, &c. it shall be, by Composition of Ratio, as AB : *ab* :: *ab* : DC :: DC : *dc*; and so on.

9. Since (from what has been said) the Line AB=1, and the other Lines CD, EF, GH, *&c.* proceed from thence to *increase* in a *Geometrical Ratio*, and

and their Diſtances AC, CE, EG, *&c.* are all *equal* to each other (by *Art.* 7.) Therefore it follows that the Diſtances or Lines AC, AE, AG, *&c.* are the *Logarithms* of thoſe Numbers repreſented by the Right Lines CD, EF, GH, *&c.* according to the *Definition of Logarithms*, Chap. I. Art. 1, and 13. For if AC=1, then AE=2, AG=3, *&c.* and ſo in the Series of *proportional Numbers* AB, CD, EF, GH, IK, LM, NO, *&c.* we have the *Logarithms*

0, AC, AE, AG, AI, AL, AN, } *&c.*
0, 1AC, 2AC, 3AC, 4AC, 5AC, 6AC, }

10. As *Logarithms*, then, are the *Exponents* of the *Ratio's* of *proportional Numbers* to *Unity* in any Series; or ſhew the *Place*, *Power*, or *Order* of the *Numbers* with reſpect thereto; 'tis plain that the Logarithm of AB or Unity muſt be =0, becauſe Unity is not diſtant from it ſelf. Alſo if the *Ratio* of CD to AB be $=x$, then ſhall the *Ratio* of the Number EF to AB$=x\times x=xx$; or *Duplicate* of the former; and the Ratio of GH to AB$=x\times x\times x=x^3$, or *Triplicate* of the firſt Ratio x. Thus the Ratio of NO to AB$=x^6$. And conſequently the *Numbers*, their *Ratio's*, and *Logarithms*, will ſtand in the following Order, *viz.*

Prop. N°.	AB,	CD,	EF,	GH,	IK,	LM,	NO,	*&c.*
Ratio's . . .	1,	x,	x^2,	x^3,	x^4,	x^5,	x^6,	*&c.*
Logarithms	0,	AC,	2AC,	3AC,	4AC,	5AC,	6AC,	*&c.*

11. If four Numbers be ſuch, that the Diſtance between the *firſt* and *ſecond*, be equal to the Diſtance between the *third* and *fourth*, (be the Diſtance between the *ſecond* and *third* what it will) then theſe Numbers will be proportional; for let the four Numbers be AB, CD, LM, and NO; then becauſe AC=LN, it will be AB : D*p* :: LM : OT, by Art. 8. Therefore (by Compoſition of Ratio) AB : DC :: LM : NO. And thus of any other four Numbers in the Geometric Series. The *Converſe* alſo of this Propoſition is as manifeſtly true.

12. Since AB=1, and its Logarithm =0; and since *Unity* is to the *Multiplier* as the *Multiplicand* is to the *Product*, in every Multiplication; therefore to every *Addition* of *Logarithms*, there corresponds a *Multiplication* of their *Numbers*. Thus,

Numbers, CD×EF=GH; and EF×IK=NO.
Logarithms, AC+AE=AG; and AE+AI=AN.

And in *Division*, since *Unity* is to the *Divisor* as the *Quotient* to the *Dividend*; then (by *Art.* 11.) for every *Subtraction* or *Difference* of *Logarithms*, there corresponds a *Division* of their *Numbers*. Thus,

Numbers, GH÷EF=CD; and NO÷EF=IK.
Logarithms, AG—AE=AC; so AN—AE=AI.

agreeably to what was shewn in *Chap.* I. *Art.* 10.

13. Again, since *Unity*, any *assum'd Number*, the *Square* thereof, the *Cube*, the *Biquadrate*, &c. are all continual *Proportionals*, their *Distances* from each other will be *equal* to one another; and are therefore as the Numbers AB, CD, EF, GH, *&c.* in the Series. Consequently if the Distance or Logarithm of the Number CD be *multiplied* by 2, 3, 4, *&c.* there will answer an *Involution* of the said Number to the *Square*, *Cube*, *Biquadrate*, &c. Power.

Thus for the Square
Numb. CD×CD=EF; and EF×EF=IK.
Log. AC×2=AE; so AE×2=AI.

For the Cube
Numb. CD×CD×CD=GH; and EF×EF×EF=NO.
Log. A×3=AG; and so AE×3=AN.

For the Biquadrate
Numb. CD×CD×CD×CD (=EF×EF)=IK.
Log. AC×4= (AE×2=) AI.

Which is the same as was observed in Chap. I. Art. 12.

14. If the equal Distances AC, CE, EG, GI, IL, be bisected, and in the Points of Bisection *a*, *c*, *e*, *g*, *i*, there

there be erected the Perpendiculars *ab*, *cd*, *ef*, *gh*, *ik*; these, by means of the Curve, will be all *Proportionals*; and the Number LM will be in the *tenth Place* from *Unity* or AB: if then we put LM=10, and suppose its *Ratio* to *Unity* be 10000000, such *Ratiunculæ*, as that of *no* to AB, or Unity is 1. Then will the *Numbers* and their *Logarithms* be as express'd in the following Table.

Numbers.	Logarithms.
AB = 1	0.
ab = 1.259, &c.	A*a* = 1000000
CD = 1.585, &c.	AC = 2000000
cd = 1.996, &c.	A*c* = 3000000
EF = 2.512, &c.	AE = 4000000
ef = 3.163, &c.	A*e* = 5000000
GH = 3.982, &c.	AG = 6000000
gh = 5.072, &c.	A*g* = 7000000
IK = 6.310, &c.	AI = 8000000
ik = 7.944, &c.	A*i* = 9000000
LM = 10.	AL = 10000000.

Numbers.	Logarithms.
AB = 1	0
CD = 2	AC = 3010300
EF = 3	AE = 4771212
GH = 4	AG = 6020600
IK = 5	AI = 6989700
LM = 6	AL = 7781512
NO = 7	AN = 8450980
PQ = 8	AP = 9030900
RS = 9	AR = 9542425
TV = 10	AT = 10000000

15. If BKV (Fig. III.) be the *Logarithmetic Curve*, and AB=1, TV=10, or 10AB; and all the intermediate Lines CD, EF, GH, &c. be as *Digits* 2, 3, 4, &c, to 10; these Lines will be situated

ated at *unequal Diſtances* from each other; and ſo their *Logarithms* AC, AE, AG, *&c.* not proceeding to increaſe by equal Differences, ſhew the Numbers AB, CD, EF, *&c.* are not *ordinately* (or all of them) in a *Geometrical Ratio* or *Proportion.* If the Ratio of 10 to 1, or TV to AB be ſuppoſed to conſiſt of 10000000 *Ratiunculæ*, and conſequently the whole Logarithm AT of 10000000 *equal Parts*, then the Diſtances of each of the *nine Digits* from Unity, *viz.* AC, AE, AG, *&c.* ſhall conſiſt of ſuch Numbers of thoſe ſmall *equal Parts*, as are expreſſed in the Table, oppoſite to the ſaid Digits.

16. Yet ſince ſome of the Digits are in a *Geometric Ratio*, as 1, 2, 4, 8, their Logarithms will be *equidifferent*; ſo AC=CG=GP; and becauſe 1 : 3 :: 3 : 9, there is AE=ER; and thus it appears that tho' the nine Digits are not all in continual *Geometrical Proportion*, yet ſome of them are ſo; and the reſt are ſome of *thoſe Proportionals*, of which there be 10000000 between AB=1, and TV=10. If the *firſt Term* from Unity be called x, the ſecond will be x^2, the third x^3, &c. and ſince the Number 10 is the 10000000th Term of the Series, it will be $TV = x^{10000000} = 10$. Alſo $CD = x^{3010300} = 2$; and $EF = x^{4771212} = 3$; and ſo on. Whence *every Digit* is *ſome Power* of *that Number* which is the *firſt Proportional* from Unity: The *Exponents* or *Indices* of the Powers being the *Logarithms* of the *Numbers*, agreeable to Art. 10. of this.

CHAP.

CHAP. IV.

The Nature of Logarithms and their Indices, when the Numbers are Fractions, farther explain'd by the LOGARITHMIC CURVE.

1. WE have hitherto principally confidered the Nature and Properties of Logarithms of *whole Numbers* or *Integers*, and have obferv'd that in the *Decuple Series* 1, 10, 100, 1000, *&c.* the Terms have their Ratio's to Unity *affirmative* or *pofitive*, viz. 1, 2, 3, *&c.* or thus, +1, +2, +3, *&c.* the contrary of which happens when the Number of the Logarithm is not *integral*, but *fractional*, or expreffes only fome *fractionate* Part of Unity. For there the Series being continued on the *other Side*, or *below Unity*, hath the *Indices* of the Powers of the Terms of a *Quality* directly *oppofite* to the former; and therefore as *thofe Indices* were *pofitive*, fo *thefe* will be of a *negative Nature*, and import the Terms to be *below* the *State* of *Unity*, or rather of *Integrity*; and will be affected with the Sign —, as —1, —2, —3, *&c.* as is evident Chap. III. Art. 5.

2. Wherefore fince AB reprefents *Unity*, (Fig. II.) all the *Numbers* in the *Series* towards the *right Hand*, or *above Unity*, CD, EF, GH, *&c.* being *integral*, and having the *Ratio greater* than *Unity*, will have the Logarithms thereof *pofitive*, viz. +AC, +AE, +AG, *&c.* But thofe *Numbers* or *Terms* on the *Left*, or below *Unity*, cd, ef, gh, *&c.* being *fractionate*, and having the Ratio *lefs than Unity* or *decreafing*, will have the Logarithms thereof Negative, *viz.* —Ac, —Ae, —Ag, *&c.* And fo the *Indices*

 of

of thoſe *Logarithms* will be reſpectively *affirmative* or *negative.*

3. Since then (as hath been all along ſhewn) the *Indices* of the Logarithms of *Integers*, as being *affirmative*, muſt be *added*, that ſo their *Sum* may *indicate* the *Product* of the *Numbers multiplied*; as AC+AE =AG, which ſhews that its *Number* GH=CD×EF, the Numbers multiplied; ſo if the *Index* of one Logarithm be *negative*, and the *Index* of another be *affirmative*, the *Difference* of thoſe two *Indices* muſt be taken for the *Product* of the *Numbers.* Thus to multiply the *Integer* GH by the *Decimal* cd, their *Indices* being + AG, and —Ac, their Difference AG—A=AE, and ſhew the Number EF is the *Product* of the other two. And here, becauſe the greater Index GH is *affirmative*, the *Difference* alſo AE is *affirmative*, and the *Product* EF to be an *Integer* or on the *right Hand* of *Unity.*

4. But if the *Decimal* gh be to be *multiplied* by the *Integer* CD, whoſe *affirmative Logarithm* or *Index* AC is the *negative Index* of the *Decimal*, viz. Ag, their *Difference* Ag — AC = Ae, is *negative* alſo, and ſo ſhews the *Product* ef will be a *Decimal*, or *below Unity.* Again, if both the *Indices* of *Logarithms*, whoſe Numbers are to be *multiplied*, are *negative*, their *Sum* ſhall be a *negative Index* whoſe Logarithm points out the *Product* (in this Caſe) always a *Decimal*, or in the Series *below* Unity AB. For Example, to *multiply* the two *Decimals* cd, ef, the *Sum* of their *negative Indices* Ac+Ae=Ag is *negative*, and ſhews the *Decimal Product* gh, ever below Unity.

5. The Reaſon of all which is very plain; for ſince *Unity* is to the *Multiplier* as the *Multiplicand* to the *Product*, and the *Logarithm* of Unity is =0; *Unity*, any *two Numbers* and their *Product*, together with their *Indices*, will ſtand as follows:

$$1.\left\{\begin{array}{l}\text{Ind. } 0,\ AC,\ AE,\ AG,\\ \text{Num. } AB : CD :: EF : GH,\end{array}\right.$$

$$\text{therefore}\left\{\begin{array}{l}AC + AE = AG + 0 = AG.\\ CD \times EF = GH \times 1 = GH.\end{array}\right.$$

$$2.\left\{\begin{array}{l}0,\ -Ac,\ +AG,\ +AE,\\ AB : cd :: GH : EF.\end{array}\right.$$

$$\text{therefore}\left\{\begin{array}{l}AG - Ac = AE + 0 = AE,\\ GH \times cd = EF \times 1 = EF.\end{array}\right.$$

$$3.\left\{\begin{array}{l}0,\ +AC,\ -Ag,\ -Ae,\\ AB : CD :: gh : ef,\end{array}\right.$$

$$\text{therefore}\left\{\begin{array}{l}AC - Ag = 0 - Ae = -Ae,\\ CD \times gh = 1 \times ef = ef.\end{array}\right.$$

$$4.\left\{\begin{array}{l}0,\ -Ac,\ -Ae,\ -Ag,\\ AB : cd :: ef : gh,\end{array}\right.$$

$$\text{therefore}\left\{\begin{array}{l}Ae + Ac = 0 - Ag = -Ag,\\ ef \times cd = 1 \times gh = gh.\end{array}\right.$$

6. From whence 'tis evident, if the Numbers are *both Integers*, the *Product* will fall in the *Scale above* (or be *greater* than) either; if they be of *different Sorts*, the *Product* will fall between them, above Unity if the *greater Number* be an *Integer*, or *below Unity*, if it be a *Decimal*. Laſtly, if *both the Numbers* be *Decimals*, the *Product* will fall *below Unity* and *them both*, or will be *leſs* than *either* of them. Theſe things *well obſerv'd* and *underſtood*, make all the *Myſtery* of the *Arithmetic* of *Fractions* by *Logarithms* vaniſh, where it is taught with the Uſe of *negative Indices* to Logarithms of *fractional Numbers*, as it is to be found in ſome Books. *Note*, I have all along in this Chapter, ſaid *affirmative* or *negative Indices*, and not *affirmative* and negative *Logarithms*; becauſe the *Indices only* ſhew the *Quality* of the *Numbers*, viz. whether *above* or *below* Unity, *greater* or *leſſer* than it, *i. e.* whether they be *Integers*, or *wholly Fractions*; and therefore theſe *Indices* muſt be carefully *added* or *ſubſtracted*, as occaſion requires,

 and

and as before directed; but the *Logarithms* ſhewing the *Diſtances* of *Numbers* from Unity *indifferently*, or without reſpect to the *Order* of *above* or *below*, are *always* to be *added*.

7. But ſince this Method of *negative* or *different Indices* to Logarithms is attended with *Addition*, *Subſtraction*, and other *intricate Cautions* peculiar to themſelves, it can't be recommended ſo much as another Method *more approv'd*, and therefore *more generally uſed*; wherein the *Indices* of *all Logarithms*, (both of *fractional* as well as *integral* Numbers) undergo the ſame common *Management* with their *Logarithms*; and ſo can't be ſo *difficult* to *Learners* as the *other*; tho' it is alſo attended with *particular Rules*, as you'll find farther on, and is as follows:

	Numbers.		Indices.			Log^s.
Fractions.	ut =	,0000000001	o	—10	0	90.0000000
	rs =	,000000001	tr =	— 9	1.	91.0000000
	pq =	,00000001	tp =	— 8.	2.	92.0000000
	no =	,0000001	tn =	— 7.	3.	93.0000000
	lm =	,000001	tl =	— 6.	4.	94.0000000
	ik =	,00001	ti =	— 5.	5.	95.0000000
	gh =	,0001	tg =	— 4.	6.	96.0000000
	ef =	,001	te =	— 3.	7.	97.0000000
	cd =	,01	tc =	— 2.	8.	98.0000000
	ab =	,1	ta =	— 1.	9.	99.0000000
Integers.	AB =	1	tA =	0	10.	100.0000000
	CD =	10	tC =	+ 1.	11.	101.0000000
	EF =	100	tD =	+ 2.	12.	102.0000000

8. Let AB be $=1$, or Unity (Fig. IV.) and let tu be a *fractional* Number 10 times as far *below* Unity AB, as CD is *above* it; then will $At=10AC$, and ſuppoſing the Number $CD=10$, 'tis evident $tu=\frac{1}{10000000000}$, or ,0000000001; if then, inſtead of

of the Logarithms beginning at *Unity* AB (as in the former Caſe) we make 'em begin from tu= ,0000000001, then the *Index* of Logarithm of the *Fraction rs*, will be 1; of *pq*, 2; of *no*, 3; and ſo of Unity AB, 10; and ſince CD=10, EF=100; the *Indices* of their Logarithms will be tC=11, and tD=12, &c. as is evident in the foregoing Table; where you ſee the *fractional* and *integral Numbers* with *Indices* of *three ſeveral Kinds* fitted to the Logarithms of the ſaid Numbers.

9. That every thing might be made plain, I have made the *Index* of the Logarithm of *Unity*, =0, in the *firſt Rank* of *Indices*; and according (as we have ſeen above) the *Indices* of all the Logarithms of Numbers *above* AB will be *poſitive*, as +1, +2, &c. and of thoſe *below* it *negative*, as —1, —2, —3, &c. as in the Table. But if (as in the ſecond Rank of *Indices*) the Index of the Logarithm of Unity AB be made =10; then ſhall the Logarithms begin (as before ſuppoſed) from the 10*th Place below* it, *viz.* from the Number ut=,0000000001, and then all the Indices of all the *fractional Numbers* between $\frac{1}{10000000000}$, or 0,0000000001 and 1, will be *affirmative* or *poſitive*, and not *negative* as before; as is ſeen in the Table. So that by this Method we have got *poſitive Indices* for the Logarithms of *Decimal Numbers* as well as for *Integral Numbers*; and ſo both capable of the ſame Management, which could not be with *negative Indices*, as before ſhewn.

10. Alſo in the third Rank (which is here given, not for *Variety* only, but becauſe 'tis oftentimes of *Uſe*) the *Index* of the Logarithm of Unity AB is made =100; and conſequently that theſe Logarithms begin from the *Fraction* which is the 100*th Proportional below* Unity, in the Ratio of 1 to 10, or $\frac{1}{10}$ to 1. Whence the *Indices* of Logarithms of *all Fractions* between 0, 000000000000000000000 0000000000-0000000000 0000000000 0000000000 0000000000
0000000000

000000000 000000000 000000000 1 and 1, will of courſe be *poſitive*; and ſo in this Claſs, the *Indices* of *Logarithms* of all *decimal Numbers* (*greater* by far than *Uſe*, or even *Curioſity*, can deſire) will be poſitive alſo, as well as thoſe of the Logarithms of *integral Numbers*; and therefore in ſhort, we have obtain'd *poſitive Indices* for the Logarithms of *all Numbers in general*, and ſo the *Trouble* of *Addition* and *Subſtraction* of *different Parts* of Logarithms at the ſame time, is avoided, as was propoſed.

11. But (as I have before obſerved, Chap. II. Art. 16.) the *Indices* of the Logarithms of *Decimals* ſhould be diſtinguiſhed from the *Indices* of Logarithms of *Integers* in *Operations*; and this I think is beſt done as there directed, *viz.* by fixing a *Dott* on *each* Side the ſaid *Indices*. And thus manifeſtly appears the *Reaſon* of all that is deliver'd in Chap. II. from Art. 13. to the End.

12. Now in *multiplying*, *dividing*, &c. of *Fractions*, the *Factors* may be either *both Decimal*; or *one Decimal*, and the *other* an *Integer*; if *both* the *Factors* are *Decimal*, the *Product* will be *Decimal*, in Multiplication. If the *Factors* be of *different Sorts*, and the *Ratio* of *Unity* to the *Decimal* be either *leſs*, *equal to*, or *greater* than the *Ratio* of *Unity* to the *Integer*; then ſhall the Product be *greater*, *equal to*, or *leſs* than *Unity*; i. e. it will be *Integer*, Unity, or *Decimal*, in multiplying; all which will be evident by the following *Examples*, ſerving as ſo many *Rules* for the *right ordering* of *Indices* in *Operations*.

13.

$$1.\left\{\begin{array}{l}\left.\begin{array}{llll}\mathrm{AB} : \mathrm{cd} :: \mathrm{ab} : \mathrm{ef} & & & \\ \mathrm{tA}. & .\mathrm{tc}. & .\mathrm{ta}. & .\mathrm{te}. \\ 10. & .8. & .9. & .7.\end{array}\right\}\text{theref.}\left\}\begin{array}{l}\mathrm{cd}\times\mathrm{ab}=1\times\mathrm{ef}=\mathrm{ef}.\\ .\mathrm{tc}.+.\mathrm{ta}.=\mathrm{tA}+\mathrm{te}.\\ .8.+.9.=10.+.7.\end{array}\right.\\ \text{That is}\ \ldots\ldots\ldots\ .8.+.9.-10.=.7.=\textit{Index}\ \text{of ef}.\end{array}\right.$$

$$2.\left\{\begin{array}{l} \left.\begin{array}{llll} AB : ab :: & EF : & CD \\ tA. \ \ .ta. & tE. & tC. \\ 10. \ \ .9. & 12. & 11. \end{array}\right\} \text{theref.} \left\{\begin{array}{l} ab \times EF = 1 \times CD = CD. \\ .ta. + tE = tA. + tC. \\ .9. + 12. = 10. + 11. \end{array}\right. \\ \text{And consequently} \ldots\ldots\ .9. + 12. - 10 = 11. = \\ \hfill \text{Index of } CD. \end{array}\right.$$

$$3.\left\{\begin{array}{l} \left.\begin{array}{llll} AB : ab :: & CD : & AB \\ tA. \ \ .ta. & tC. & tA. \\ 10. \ \ .9. & 11. & 10. \end{array}\right\} \text{theref.} \left\{\begin{array}{l} ab \times CD = 1 \times 1 = 1 \\ .ta. + tC = tA + tA. \\ .9. + 11 = 10 + 10. \end{array}\right. \\ \text{Therefore} \ldots\ldots\ldots\ .9. + 11. - 10 = 10 = \\ \hfill \textit{Index} \text{ of } 1. \end{array}\right.$$

$$4.\left\{\begin{array}{l} \left.\begin{array}{llll} Ab.\ cd :: & CD : & ab. \\ tA. \ \ .tc. & tC. & .ta. \\ 10. \ \ .8. & 11. & .9. \end{array}\right\} \text{theref.} \left\{\begin{array}{l} CD \times cd = 1 \times ab = ab. \\ tC. + .tc. = tA. + .ta. \\ 11. + .8. = 10. + .9. \end{array}\right. \\ \text{and therefore again} \ldots\ 11. + 8. - 10 = .9. = \\ \hfill \text{Index of } ab. \end{array}\right.$$

14. From theſe Obſervations well conſider'd, 'tis eaſy to apprehend the Truth of what is deliver'd in Art. 12. above. And ſince AB, in theſe *Examples* of the *Products*, hath for the *Index* of its Logarithm 10, 'tis *equally obvious* what the *Indices* of the *Logarithms* of the *Products* would be, were the ſaid *Index* of the Logarithm of AB made 100. In this *preſent Caſe* were tA=10, if we reject 10, the *Indices* of the Logarithms of *integral Products* will be the ſame as if the *Logarithms began* at AB or Unity; as is plain in the 2d and 3d *Examples*; and accordingly if tA=100, we reject 100, and the Caſe is the ſame.

15. But ſince 'tis moſt convenient to have the *Indices* of all Logarithms of *integral Numbers* to begin from *Unity* in the ſimple Order, 0, 1, 2, 3, *&c.* as if the *Logarithms* did really begin from thence; ſo 'tis but rejecting 10 from the *ſaid Indices* in their preſent State, and what we deſire is obtain'd.

Thus

Thus the foregoing Cases
$$\left\{\begin{array}{llllll} 1. & .8. + & .9. - & 10 = & .7. \\ 2. & .9. + & 12. - & 10 = & 11. \\ 3. & .9. + & 11. - & 10 = & 10. \\ 4. & 11. + & .8. - & 10 = & .9. \end{array}\right.$$

by rejecting 10 from the *Indices* of the Logarithms of *Integers*, becomes
$$\left\{\begin{array}{llll|l} .8. + & .9. - & 0 = & .17. & .7. \\ .9. + & 2 - & 0 = & 11. & 1. \\ .9. + & 1 - & 0 = & 10. & 0. \\ 1 + & 8 - & 0 = & .9. & .9. \end{array}\right.$$

Then from the *Sums* .17. 11. 10. .9. again rejecting 10 (where 'tis found) we have the Remainders .7. 1. 0. .9. the *true Indices* of the *Logarithms* of the *Products*, as required.

16. If *either Factor*, or the *Product* of them, exceeds the Limit of $\frac{1}{10000000000}$, or is lefs than ,0000000001, we ſhall find it moſt convenient to uſe the *Indices* of thoſe Logarithms of which the *Index* of the Logarithm of *Unity* or AB is =100, *viz.* thoſe Indices in the 3d Claſs in Table of Art. 8. hereof. And if, in all the foregoing Caſes, inſtead of *rejecting* 10, we now reject 100, we ſhall have the *Indices* of the Logarithms of the Products the ſame kind as before.

Thus
$$\left\{\begin{array}{llllll} 1. & .98. + & .99. - & 100 = & .97. \\ 2. & .99. + & 102 - & 100 = & 101. \\ 3. & .99. + & 101 - & 100 = & 100 \\ 4. & 101. + & .98. - & 100 = & .99. \end{array}\right.$$

by rejecting 100, becomes
$$\left\{\begin{array}{llll|l} .98. + & .99. - & 0 = & .197. & .97. \\ .99. + & 2 - & 0 = & 101. & 1. \\ .99. + & 1 - & 0 = & 100. & 0. \\ 101. + & .98. - & 0 = & 199. & .99. \end{array}\right.$$

Thus here you ſee the Effect the ſame as above, *Art.* 15. if .97. .99. be deducted from 100, as .7. .9. were from 10, the *two middle Indices* being the *ſame* in *both Caſes*.

17. In *Division*, the *Divisor* is to *Unity* as the *Dividend* is to the *Quotient*; and so the Distance between Unity and the *Divisor* is equal to that between the *Dividend* and *Quotient*. If then the Fraction ef be divided by ab, because aA is $=$ec, therefore cd is the *Quotient*, the Index of whose Logarithm is tc; but tc$=$tA$+$te$-$ta. Also if the *Integer* CD be divided by the *Fraction* ab, because aA$=$CE, therefore EF is the integral *Quotient* whose Logarithm is tE; but tE$=$tA$+$tC$-$ta. Again, let the *Fraction* ab be divided by the *Integer* CD, because CA$=$ac, therefore the *Fraction* cd is the *Quotient*; whose Logarithm tc$=$tA$+$ta$-$tC; that is, in every Case, *the Logarithm of the Divisor being taken from the Logarithm of the Dividend*, if to the Remainder, you *add the Logarithm of Unity, the Sum will be the Logarithm of the Quotient*: which is but the *Converse* of the *Rules* for the Logarithms in *Multiplication*, as is evident by Inspection of Art. 13. foregoing. And the *Methods* there mention'd for duely adjusting the *Indices*, are to be equally observed here.

18. In *Involution*, or raising the *Powers* of *Fractions*, 'tis evident that the *Distance* between *Unity* and the *Root*, is equal to the *Distance* between the *Root* and the *first Power*, the *first* and *second Power*, the *second* and *third Power*, and so on. Therefore since Aa$=$ab$=$ce$=$eg, *&c.* cd$=$ *Square* of ab, ef $=$ *Cube*, gh$=$ *Biquadrate*, &c. *Power* of the *Root* ab. But since AB : ab :: ab : cd :: cd : ef :: ef : gh, *&c.* therefore the Logarithms tA$+$tc$=$2ta, and so tc$=$2ta$-$tA. That is, the Logarithm of the *Square* of the *Root* is equal to *double* the Logarithm of the Root *less* the Logarithm of *Unity*. Again, $\frac{\text{ta}+\text{te}}{2}=tc=$2ta$-$tA, and so ta$+te=$4ta$-$2tA; that is, te$=$3ta$-$2tA; in Words, the Logarithm of the *Cube* is equal to *triple* the Logarithm of the *Root*, less *double* the Lo-

garithm of *Unity*. Moreover, $\frac{tc+tg}{2}$ te=3ta—2tA, therefore tc + tg=6ta —4tA; but tc=2ta—tA, ſubduct this from the *laſt Equation*, there will remain tg=4ta—3ta; or the Logarithm of the *Biquadrate*.

19. And *univerſally*, if the *Power* of a *Fraction* be *x*, and the *Logarithm* = L, the Logarithm of the *Power* *x* ſhall be $=xL-\overline{x-1}\times tA$, or $= xL-x\,tA+tA$. Thus, ſuppoſe you would know the Logarithm of the Square of the *Fraction* dc; here *x*=2, and L=tc, therefore 2tC—tA=tg, the Logarithm of the *Square* gh, as required. If the Logarithm of the Cube be deſired of the *Fraction* ef, we have *x* = 3, L = te, and ſo 3te—2tA=tr, the Logarithm of the *Cube* rs, as deſir'd. And thus you proceed for the *Logarithms* of *other Powers*.

N. B. The *Indices* of Logarithms of *all Powers* of *Fractional Numbers* (I mean ſuch as are *purely* ſo) muſt be *doubly dotted*, ſince thoſe *Powers* always fall below the *Root*, which is ſuppoſed a *pure Fraction*.

20. *Evolution*, or the *Extraction* of the *Roots* of *Powers*, is juſt the *reverſe* of the foregoing *Proceſs*. For ſuppoſe the *Fraction* cd were given, whoſe *ſquare Root* was required. Becauſe AB : ab :: ab : cd, therefore AB × cd=ab×ab; and ſo $\sqrt[2]{AB\times cd}$=ab; therefore the Logarithm $\frac{tA+tc}{2}$=ta, is the Logarithm of ab the *ſquare Root* ſought. Alſo if the *Cube Root* of the *Fraction* ef be required; becauſe AB : ab :: cd : ef, we have AB × ef =ab × cd; and ſo the Logarithms tA + te= ta+tc. But tA+te—ta=tc=2ta—tA, that is, 3ta=2tA—te, and conſequently $\frac{2tA+te}{3}$=ta the Logarithm of ab the *Cube Root* required.

21.

21. And univerſally, if the Logarithm L of the *Root* of any *Power* x, of any Fraction rs, be required, we have this *Theorem* $\frac{\text{tr}+x\text{tA}-\text{tA}}{x}=\text{L}$, or $\frac{\text{tr}+\overline{x-1}\times\text{tA}}{x}=\text{L}=\text{te}$, that is, in Words, the Number $x-1\times\text{tA}$ added to the *Index* of the Logarithm of the *Fraction*, the Logarithm thus augmented being divided by x, the *Quotient* ſhall be the Logarithm of the *Root* ſought. Or, ſince tA=10, 100, the Number $x-1$ prefix'd to the *Index* of the Logarithm of the Power, and the Logarithm thus divided by x, the *Quotient* ſhall be the Logarithm of the Root ſought.

CHAP. V.

The original Conſtruction of Logarithms by the Lord Neper, *and the Alteration thereof to the preſent Form by himſelf and Mr.* Briggs, *explain'd and illuſtrated by the* LOGARITHMIC CURVE.

1. THE noble Inventer of *Logarithms*, the Lord *Neper*, having duely contemplated their wonderful Nature, firſt conſtructed and publiſh'd a *Canon* thereof; but thoſe far different from what we now commonly uſe. And this was no Wonder, ſince ſcarcely any thing receives its *Invention*, and *utmoſt Perfection* at the ſame time.

2. In the firſt kind of Logarithms that *Neper* publiſhed, the *firſt Term* of the *continual Proportionals*, was placed only *ſo far* diſtant from *Unity*, as *that Term* exceeded Unity. Thus, for Example, if on be the *firſt Term* of the Series from *Unity* AB (ſee Fig. I.) the Logarithm thereof, or the Diſtance A n, or B y, was by him put equal to vy or the *Increment* of

of the Number n y above Unity. If then we ſuppoſe vn=1.0000001, the Exceſs of this Number above AB=1, is 0.0000001, which therefore, by him, was made its *Logarithm*; that is, An=0,0000001.

3. From hence, by Computation, the Number 10 will be the 23025850th *Term* of the *Series*; which Number therefore is the *Logarithm* of 10 in *this Form of Logarithms:* and expreſſes its Diſtance from Unity in *ſuch Part* whereof vy, or An is one. Alſo the Logarithm of 2 (in this Form) is 6931471; of 3, is 10986122; of 4, is 13862943, &c.

4. But *this Poſition* of the *Ratio* of the Terms is entirely *at Pleaſure*; for the *Diſtance* of the *firſt Term* may have any *given Ratio* to *its Exceſs* above *Unity*; that is, An may be indifferently *leſs*, *equal to*, or *greater than* vy; and according to that *various Ratio* (which may be ſuppoſed at pleaſure) between An, or By and vy, *i. e.* the *Increment* of the *firſt Term* above *Unity*, and the *Diſtance* of the ſame from *Unity*, there will be produced *different Forms* of Logarithms.

5. The *Logarithms* of this *firſt Form*, were found by the *ſagacious Inventer* not to anſwer the Deſign in the beſt manner as could be wiſh'd; and therefore he changed 'em into another *more convenient Form*, wherein he put the Number 10, not as the 23025850th *Term* of the Series, but the 10000000th *Term:* And after *Neper*'s Death, the learned and indefatigable Mr. *Briggs*, with *great pains*, made and publiſh'd a *Canon* of Logarithms according to this *new Form*. Now ſince in this Canon the Logarithm of 10 is 1.0000000, and ſince 1, 10, 100, 1000, &c. are *Proportionals*, they ſhall be *equidiſtant* from each other; wherefore the Logarithm of 100 ſhall be 2.0000000; of 1000, 3.0000000, and the Logarithm of 10000 will be 4.0000000, and ſo on. And this Form of Logarithms hath been ever ſince in uſe, and are thoſe in preſent Uſe; the Nature and Properties

Properties of which we have been hitherto explaining.

6. The *Rationale* of the Method by which Mr. *Briggs* computed his Logarithms, is beſt explain'd from the *Logarithmetic Curve*, according to Dr. *Keil*, as follows. In the *Logarithmetic Curve* HBD (Fig. V.) let there be three *Proportionals* AB, *ab*, *qs*, very nearly equal to one another; that is, let their Differences have a *very ſmall Ratio* to the ſaid *Ordinates*, (for ſuch are thoſe Proportionals,) and then the *Differences* of the *Logarithms* will be *proportional* to the *Differences* of the *Ordinates*; that is, it will be sr : bc :: Br : Bc :: A*q* : A*a*. For ſince the Ordinates AB, *ab*, *qs*, are *nearly equal* to one another, they will be *very nigh* to one another; and ſo the Parts of the Curve B*s*, B*b*, intercepted between them, will nearly *coincide* with a *Right Line*; for it is poſſible that the Ordinates may be ſo near to each other, that the *Difference* between the *Part of the Curve* and the *Right Line* ſubtending it, may have to that *Subtenſe*, a *Ratio leſs* than any *given Ratio*. Conſequently the Triangles Bcb, Brs, may be taken for *Right-lined* ones, and will be *equiangular:* and therefore, ſince a b is parallel to qs, they will be *ſimilar*, and their *homologous Sides proportional*; viz. rs : *bc* :: Br : B*b*, or A*q* : A*a*.

7. From hence, by the way, appears alſo the *Reaſon* of the *Correction* of *Numbers* and *Logarithms* by *Differences* and *proportional Parts*. For putting AB=1, or Unity, 'tis evident, that the Logarithms Bc, Br, are *proportional* to the *Differences cb*, *rs*, of the *Numbers* AB, *ab*, *rs*; as we ſhall hereafter prove by *Facts*, in the *practical Part*.

8. If a *mean Proportional* be found between 1 and 10, and then again another *Mean* between that and *Unity*; and if proceeding thus, you continually find a *mean Proportional* between the *Mean laſt found* and *Unity*, biſecting the Logarithms ſtill as you proceed (in

(in the manner of the *Example* in Chap. II. Art. 6.) you will at last get a Number whose *Distance* from Unity shall be less than the $\frac{1}{100000000000000}$ Part of the Logarithm of 10.

9. After Mr. *Briggs*, in this manner, had made 54 *Extractions* of the *Square Root*, he arrived to the Number 1.00000 00000 00000 12781 91493 20032-3442, and its Logarithm was 0.00000 00000 00000-05551 11512 31257 82702. Suppose this Logarithm be equal to Aq or Br; and let the Number found by this Extraction, be $=qs$; and then its *Excess* above Unity will be $=rs$.

That is, $\begin{cases} Aq = 0.00000\ 00000\ 00000\ 05551\ 11512\text{-}31257\ 82702. \\ qs = 1.00000\ 00000\ 00000\ 12781\ 91493\text{-}20032\ 3442. \\ rs = 0.00000\ 00000\ 00000\ 12781\ 91493\text{-}20032\ 3442. \end{cases}$

10. Now by means of *these Numbers* the Logarithms of all *other Numbers* may be found in the following Manner. Between the *given Number* (whose Logarithm is to be found) and *Unity*, find (by the Extraction of Roots, as above) so many *mean Proportionals* till at last a Number be obtain'd *so little exceeding Unity*, that there be 15 *Cyphers* next after it, and as many *significant Figures* after those. Suppose the small Number thus found be $a\,b$, and let the *significant Figures* with the 15 *Cyphers prefix'd* before them, denote the Difference bc; then say, as the *Difference* rs is to the *Difference* bc, so is the *given Logarithm* Br, to Bc the *Logarithm sought* for the Number $a\,b$. If now this Logarithm Bc or Aa, be *continually doubled* the *same number of times* as there were *Extractions of the Square Root*, you'll have at last the Logarithm of the Number propos'd as required.

11. If the *Tangent* TB be drawn to touch the *Curve* in B, then may the *Subtangent* AT be found by

by the Numbers above in *Art.* 9. For ſince AB, B*r*, are parallel to *rs*, AT; therefore the *Right-lin'd Triangles* B*rs* and BAT, are ſimilar, and ſo as *sr* : *r*B :: AB : AT, the Subtangent; but ſince AB=1, therefore $\frac{rB}{sr}$=AT; or thus;

As *rs* =0.00000 00000 00000 12781 91493 20032-3442,

Is to B*r*=0.00000 00000 00000 05551 11512 31257-8270,

So is AB=1.00000 00000 00000 00000 00000 00000 0000,

to AT=0.43429 44819 03251 82765 11289 18916-6051.

12. If the *proportional Right Lines* GH, EF, AB, CD, (Fig. V.) are Ordinates to the Axis CV of the *Logarithmetic Curve*, and if their *Ends* FH, DB, be join'd by *Right Lines*, which produced meet the *Axis* in the Points P and K, then the Right Lines GP, KA, will be *always equal.* For ſince GH : EF :: AB : CD; it will be, as GH : Ft :: AB : DR (by Chap. II. Art. 8.) But becauſe of the *ſimilar Triangles* PGH HtF, as alſo KAB BRD, we have PG : Ht (:: GH : Ft :: AB : DR) :: KA : BR. But Ht=BR, and therefore PG=AK; which was to be demonſtrated.

13. If the Right Lines CD, EF *equally* accede to AB, GH, ſo that the Point D at laſt may coincide with B, and the Point F with H, then the Right-Lines DBK, FHP, which did *before* cut the Curve, will *now* only *touch* it in the Points B and H; that is, they will be chang'd into the *Tangents* BT, and HV. And ſo the Right-Lines AT, GV, will *always be equal* to each other; and ſo the *Subtangent* AT, or GV, in whatever Part of the Axis it be, is always *one conſtant given* Length; and this is one of the moſt remarkable and uſeful Properties of the *Logarithmic*

garithmic Curve: For the *different Species* or *Forms* of those Curves are determined by the *Subtangents*.

14. If the Excess cb of any Number ab extremely near Unity, or but a *small matter* exceeding it, be given, the *Logarithm* of its Distance from Unity Aa or Bc, will be known by means of the constant *Subtangent* AT; for by Art. 11. we have $bc : Bc :: AB : AT$; therefore $AT \times bc = Bc \times AB = Bc$, the Logarithm required. Thus also $AT \times rs = Br$, the Logarithm of the Number qr; and so the Logarithm of any *prime Number* 2, 3, 7, 11, 13, *&c.* may be found independently of the Logarithm of any other Number.

CHAP. VI.

A Method of constructing the Logarithms derived and demonstrated from the Nature of Numbers only, by Dr. Edm. Halley.

1. THE *admirable Method* now before us, is one of the many great and wonderful Inventions and Discoveries of the celebrated Dr. *Halley*, the present *Astronomer Royal*, and *Fellow of the Royal Society*; and whose *Name* amongst the *Literati* will be had in *everlasting Remembrance.* This Method not only comprehends all the Improvements made by others by *means* of the *Hyperbola* and other *Geometrical Figures*, but shews with great Accuracy from the *pure Properties* of Numbers (as most natural and agreeable in a *Business purely Arithmetical*, as the *Logarithmotechny* is) how the Logarithms may be produced to *any desired Number of Places*, with far greater *Ease* and *Expedition* than by any Method before known. According to him, therefore,

2.

2. Suppose an *infinite Number* of *equal Ratio's* or *Ratiunculæ* between any two Terms in a continued *Scale* or *Series* of *Proportionals*; and those *Ratiunculæ* expreſs the *Ratio* of thoſe two Terms, as of 1 to $1+x$. If then between *Unity* (1) and *any Number* propoſed ($1+x$) there be taken *any Infinity* (n) of *mean Proportionals*, the infinitely little *Augment* or *Decrement* of the *firſt* of theſe *Means* from *Unity* will be a *Ratiuncula* or *Fluxion* ($\dot{x}$) of the *Ratio* of *Unity* to the ſaid Number; and the Terms of the Series will ſtand thus; *viz.* 1. $1+\dot{x}$. $\overline{1+\dot{x}}^2$. $\overline{1+\dot{x}}^3$. $\overline{1+\dot{x}}^4$. &c. to $\overline{1+\dot{x}}^n$; and the

$$\text{Exponents}\begin{cases} 0. \ 1. \ 2. \ 3. \ 4. \ \&c. \text{ to } n; \text{ or thus,} \\ 0. \ \dot{x}. \ 2\dot{x}. \ 3\dot{x}. \ 4\dot{x}. \ \&c. \text{ to } n\dot{x}. \end{cases}$$

From whence 'tis evident that not only the Number (n) of the *Proportionals* or *Ratiunculæ*, but alſo their *Sum* ($n\dot{x}$) may be put for the Logarithm of $1+x$. And thus alſo $N\dot{y}$ may be put = Logarithm of $1+y$, and conſequently it will be, as L, $1+x$: L, $1+y$ :: $n\dot{x} : N\dot{y}$.

3. If $\dot{x}=\dot{y}$, that is, if the *Ratiunculæ* compoſing divers *Ratio's* have the ſame Magnitude, then are thoſe *Ratio's* proportional to the Numbers of *Ratiunculæ* contain'd between their Terms, *viz.* L, $1+x$: L, $1+y$ :: $n : N$. For if $x=1$, and $y=2$; then $1+x=2$, and $1+y=3$; and if $n=30103$, &c. N will be found $=47712$, &c. that is, if there be the *infinite Number* 30103, &c. of *Ratiunculæ* in the *Ratio* of 1 to 2; there ſhall be the *infinite Number* 47712, &c. of the ſame *Ratiunculæ* in the *Ratio* of 1 to 3.

4. On the contrary, if $n=N$, then L, $1+x$: L, $1+y$ :: $\dot{x} : \dot{y}$, that is, ſuppoſe the infinite Number of Ratiunculæ in *one Ratio* equal to the *infinite Number* of *Ratiunculæ* in any other *Ratio*, then are the Logarithms of thoſe Ratio's directly as their *Fluxions*,

or as the *Magnitudes* of the *Ratiunculæ* respectively. For instance, let the Ratio of 10 to 1, 100 to 1, 1000 to 1, &c. all and every of them be supposed to consist of 5 Ratiunculæ; as follows: Thus,

Ratio's	0.	1.	2.	3.	4.	5.	&c.
Terms	1.	10.	100.	1000.	10000.	100000,	&c.
Ratiunc.	$5v$.	$5w$.	$5x$.	$5y$.	$5z$,	&c.	

'Tis plain the Ratiuncula v of the whole Ratio of 10 to 1 is $\frac{1}{5}$; of 100 to 1, is $\frac{2}{5}$, &c. that is, the *Ratiunculæ* are $v=\frac{1}{5}$, $w=\frac{2}{5}$, $x=\frac{3}{5}$, $y=\frac{4}{5}$, $z=\frac{5}{5}$; but those *Fractions* are as the *natural Numbers* 1, 2, 3, 4, 5. Wherefore the Logarithms of the Ratio's of 10 to 1, 100 to 1, 1000 to 1, &c. are directly as the natural Numbers 1, 2, 3, 4, 5, &c. and so may be expressed by them.

5. Since then the *Logarithms* of Ratio's are as their *Fluxions*, therefore the Logarithm of any Number is found by taking the *Difference* of *Unity* and the *infinite Root* of that Number; that is, because $1+\dot{x}$ is the first Term from Unity, or *Ratiuncula*, $\overline{1+\dot{x}}^{n}=1+x$, is the *infinite Power* to be resolv'd; and $\sqrt[n]{1+x}$, or $\overline{1+x}^{\frac{1}{n}}=1+\dot{x}$, and so $\overline{1+x}^{\frac{1}{n}}-1=\dot{x}=$ Logarithm of $1+x$.

6. In order to extract the Root of the infinite Power $1+x$, (which, to some, may seem *strange* and next to *impossible*) we must make use of Sir *Isaac Newton*'s celebrated Theorem for that purpose. Supposing then the Power be $1+x$, according to his Theorem, $\overline{1+x}^{\frac{1}{n}}=1+\frac{1}{n}x+\frac{1-n}{2nn}xx+\frac{1-3n+2nn}{6n^3}x^3+\frac{1-6n+11nn-6n^3}{24n^4}x^4$, &c. the Root of the Power $1+x$, when the *Index* (n) is *finite*; but (n) being in the present Case *infinite*, all the Terms of the *Coefficients*, wherein (nn) is found a Divisor (as being *infinitely infinite*) will vanish, as being *infinitely* less than

than nothing. But the *Co-efficient* $\frac{1-n}{2nn}=\frac{1}{2nn}-\frac{1}{2n}=-\frac{1}{2n}$; and $\frac{1-3n+2nn}{6n^3}=\frac{-1}{6nn}+\frac{1}{3n}=\frac{1}{3n}$; Also the *Coefficient* $\frac{1-6n+11nn-6n^3}{24n^4}=\frac{1}{12nn}-\frac{1}{4n}=\frac{1}{4n}$; &c. Wherefore the foregoing Root will become $1+\frac{1}{n}x-\frac{1}{2}nx^2+\frac{1}{3}nx^3-\frac{1}{4}nx^4+\frac{1}{5}nx^5$, &c. That is, $\frac{1}{n}\times\overline{x-\frac{1}{2}xx+\frac{1}{3}xxx-\frac{1}{4}x^4-\frac{1}{4}x^5}$, &c. $=\overline{1+x}^{\frac{1}{n}}-1=\dot{x}=$ Logarithm of $1+x$.

7. And whereas the *infinite Index* (n) may be taken at pleasure, an *Infinity* of *different Scales* of Logarithms may be produced; and those different Logarithms will be to one another as $\frac{1}{n}$, or *reciprocally* as the *Indices* (n). And as it hath been shewn (in Chap. V.) that in making the *first kind of Logarithms* by *Neper*, the *infinite Index* of the Logarithm of 10 would be 23025850, *&c.* But in making the second sort after by *Briggs*, the said *infinite Index* was put $=10000000$, *&c.* Consequently in the foregoing Series for Logarithms, if $n=10000000$, *&c.* the Lord *Neper*'s Logarithms will be produced; and the Series will be *simply* $x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\frac{1}{5}x^5$, &c. On the contrary, if $n=23025850$, *&c.* then *Briggs*'s *Logarithms* will arise from the Series; and because $\frac{1}{n}=\frac{1}{2.3025850}$, *&c.* $=0.434229448$, *&c.* therefore $\frac{1}{n}$ ($=$AT) is the *Subtangent* of the *Logarithmic Curve* for the *Briggian Logarithms*, as is plain from Chap. I. V. Art. 11. Whence if a Logarithm of *Neper*'s *Form* be multiplied by 0.43429448, *&c.* or divided by 2.3025850, *&c.* it is converted into a Logarithm of *Briggs*'s *Sort*, or those in present Use.

8. If the Logarithm of a decreasing Ratio be sought, as of 1 to $1-x$, the Power being $1-x$, its *infinite Root* will be $\overline{1-x}^{\frac{1}{n}}=1-\frac{1}{n}x-\frac{1}{2}nx^2-\frac{1}{3}nx^3-$ $\frac{1}{4}nx^4-$

$\frac{1}{4}nx^4 - \frac{1}{5}nx^5$, &c. that is, $\frac{1}{n} \times \overline{x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5}$, &c. $= 1 - \overline{1 - x}^{\frac{1}{n}} = \dot{x} =$ the Logarithm of $1 - x$; the *firſt Term* next below Unity, or Root of this infinite decreaſing Series being $1 - \dot{x}$. And ſo in this Caſe, according as the *infinite Index* (n) is made $= 10000000$, *&c.* or 2.3025850, *&c.* ſo *Neper*'s or *Briggs*'s Logarithms of thoſe negative Numbers will be produced.

9. Inſtead of the Terms $1 : 1 + x$, let $a : b$ expreſs the Terms of any *Ratio* univerſally; and make $a + b = s$, and $a - b = d$; and ſince it is $1 : 1 + x :: a : b$, therefore $a + ax = b$, and ſo $x = \frac{b - a}{a} = \frac{d}{a}$. Again becauſe (in the *decreaſing Ratio*) it is $1 : 1 - x :: b : a$; therefore $b - bx = a$, and ſo we have again $x = \frac{b - a}{b} = \frac{d}{b}$. Whence the Logarithm of the ſame Ratio $a : b$, may be *doubly expreſs'd*; viz. for the *encreaſing Ratio*, the Series will be

$$\frac{1}{n} \times \frac{d}{a} - \frac{d^2}{2a^2} + \frac{d^3}{3a^3} - \frac{d^4}{4a^4} + \frac{d^5}{5a^5}, \text{ \&c. or,}$$

$$\frac{1}{n} \times \frac{d}{b} + \frac{d^2}{2bb} + \frac{d^3}{3b^3} + \frac{d^4}{4b^4} + \frac{d^5}{5b^5}, \text{ \&c. for}$$

the decreaſing Ratio; all which is evident from the three laſt Articles.

10. But if we ſuppoſe the Ratio of a to b, viz. $\frac{a}{b}$ compoſed of *two Parts*; viz. of the Ratio of a to the *Arithmetical Mean* between the Terms a and b, and of the Ratio of the ſaid *Arithmetical* Mean to the other Term b; that is, ſuppoſe $\frac{a}{b} = \frac{a}{\frac{1}{2}s} \times \frac{\frac{1}{2}s}{b}$, (for $\frac{1}{2}s = \frac{a + b}{2}$ is the *Arithmetical Mean* between a and b;) then the *Sum* of the Logarithms of thoſe two Ratio's, $\frac{a}{\frac{1}{2}s}$, $\frac{\frac{1}{2}s}{b}$, will be equal to the Logarithm of the *Ratio* of a to b. Or, L, $\frac{a}{\frac{1}{2}s}$ + L, $\frac{\frac{1}{2}s}{b}$ = L, $\frac{a}{b}$.

And

And thus alſo we have L, $\frac{\frac{1}{2}s}{a}$ + L $\frac{b}{\frac{1}{2}s}$ = L $\frac{b}{a}$.
Now becauſe the Ratio of $\frac{1}{2}s$ to b is encreaſing, therefore $1 : 1 + x :: \frac{1}{2}s : b$; and ſo $1 + x = \frac{b}{\frac{1}{2}s}$; conſequently $x = \frac{b}{\frac{1}{2}s} - 1 = \frac{b - \frac{1}{2}s}{\frac{1}{2}s} = \frac{\mathrm{d}}{\mathrm{s}}$ a-new; again, becauſe the *Ratio* of $\frac{1}{2}s$ to a is *decreaſing*; therefore $\frac{1}{2}s : a :: 1 : 1 - x$, and ſo $1 - x = \frac{a}{\frac{1}{2}s}$; and $x = 1 - \frac{a}{\frac{1}{2}s} = \frac{\frac{1}{2}s - a}{\frac{1}{2}s} = \frac{\mathrm{d}}{\mathrm{s}}$.

11. Therefore ſince $x = \frac{\mathrm{d}}{\mathrm{s}}$ for both Ratio's, *viz.* of a to $\frac{1}{2}s$, and $\frac{1}{2}s$ to b, we ſhall have (by the foregoing Rule, Art. 9.) $\frac{1}{n} \times \frac{\mathrm{d}}{\mathrm{s}} + \frac{\mathrm{d}^2}{2\mathrm{s}^2} + \frac{\mathrm{d}^3}{3\mathrm{s}^3} + \frac{\mathrm{d}^4}{4\mathrm{s}^4} + \frac{\mathrm{d}^5}{5\mathrm{s}^5}$, &c. = (A) L, a to $\frac{1}{2}s$, and $\frac{1}{n} \times \frac{\mathrm{d}}{\mathrm{s}} - \frac{\mathrm{d}^2}{2\mathrm{s}^2} + \frac{\mathrm{d}^3}{3\mathrm{s}^3} - \frac{\mathrm{d}^4}{4\mathrm{s}^4} + \frac{\mathrm{d}^5}{5\mathrm{s}^5}$, &c. = (B) L, $\frac{1}{2}s$ to b. Then $\frac{1}{n} \times \frac{2\mathrm{d}}{\mathrm{s}} * + \frac{2\mathrm{d}^3}{3\mathrm{s}^3} * + \frac{2\mathrm{d}^5}{5\mathrm{s}^5}$, &c. = (A+B) L, a to b.
Thus you have a Series expreſſing the Logarithm of the Ratio of a to b, whoſe Sum is $s = a + b$, and Difference $\mathrm{d} = a + b$: and *this Series* converges twice as ſwift as the former in Art. 8. and therefore is more proper for *making* or *examining* of Logarithms, which it performs with great Expedition, when d the Difference is but the 100th Part of s the Sum; the firſt Step $\frac{2\mathrm{d}}{\mathrm{s}}$ ſufficing for 7 Places of the Logarithm, and the ſecond $\frac{2\mathrm{d}^3}{3\mathrm{s}^3}$ for 12 Places.

12. Becauſe the Difference of the Logarithms of the *Ratio's* of a to $\frac{1}{2}$s, and $\frac{1}{2}$s to b is the Logarithm of the Ratio of $a\,b$ to $\frac{1}{4}$ss; or thus, becauſe

$\frac{\frac{1}{2}s}{b}$)

$\frac{\frac{1}{2}s}{b})\frac{a}{\frac{1}{2}s}(=\frac{ab}{\frac{1}{4}ss}$, therefore the L, $\frac{a}{\frac{1}{2}s}$ — L, $\frac{\frac{1}{2}s}{b}$ = L, $\frac{ab}{\frac{1}{4}ss}$ $(A-B) = \frac{1}{n} \times \frac{2d^2}{2s^2} * + \frac{2d^4}{4s^4} * + \frac{2d^6}{6s^6}$, &c. but half the Ratio $\frac{ab}{\frac{1}{4}ss}$ is the Ratio $\frac{\sqrt{ab}}{\frac{1}{2}s}$ (for $\frac{\sqrt{ab}}{\frac{1}{2}s} \times \frac{\sqrt{ab}}{\frac{1}{2}s} = \frac{ab}{\frac{1}{4}ss}$), that is, the Ratio of the *Geometrical Mean* $\sqrt{ab}$ to the Arithmetical Mean $\frac{1}{2}s$; consequently the Logarithm of $\frac{\sqrt{ab}}{\frac{1}{2}s} = \frac{1}{n} \times \frac{d^2}{2s^2} * + \frac{d^4}{4s^4} * + \frac{d^6}{6s^6}$, &c. which is a Theorem of good Dispatch for finding the Logarithm of $\frac{1}{2}s$.

13. But the same Logarithm may yet be much more advantageously obtained by a Method like the former. For if we make the Terms of the Ratio $\frac{ab}{\frac{1}{4}ss} = \frac{A}{B}$, and put $S = ab + \frac{1}{4}ss$, the Sum of the Terms, and $D = ab - \frac{1}{4}ss$; 'tis evident the Logarithm of $\frac{A}{B}\left(=\frac{ab}{\frac{1}{4}ss}\right) = \frac{1}{n} \times \frac{2D^2}{S} + \frac{2D^3}{3S^3} + \frac{2D^5}{5S^5} + \frac{2D^7}{7S^7}$, &c. by Art. 11. But because $\frac{1}{4}ss = \frac{1}{4}a^2 + \frac{1}{2}ab + \frac{1}{4}b^2$, therefore $D = ab - \frac{1}{4}ss = ab - \overline{\frac{1}{4}a^2 + \frac{1}{2}ab + \frac{1}{4}bb} = \frac{1}{4}a^2 - \frac{1}{2}ab + \frac{1}{4}bb = \sqrt{\frac{1}{2}a - \frac{1}{2}b} = \frac{1}{4}d^2 = 1$, in the present Case of finding the Logarithms of Prime Numbers; for suppose the Logarithm of 23 be sought, then $a = 22$, $b = 24$, $\frac{1}{2}s = 23$, and $d = 2$; also $A = ab = 528$, and $B = \frac{1}{4}ss = 529$, and therefore $ab - \frac{1}{4}ss = A - B = D = \frac{1}{4}d = 1$. Wherefore, since $D = 1$, the Series above becomes $\frac{1}{n} \times \frac{2}{S} + \frac{2}{3S^3} + \frac{2}{5S^5} + \frac{2}{7S^7}$, &c. = the Logarithm of the Ratio of $\frac{ab}{\frac{1}{4}ss}$; and so the half of it, *viz.* the Series $\frac{1}{n} \times \frac{1}{S} + \frac{1}{3S^3} + \frac{1}{5S^5} + \frac{1}{7S^7}$, &c. = the Logarithm

rithm of the Ratio of $\sqrt{ab}$ to $\frac{1}{2}$s. And this converges much ſooner than any Theorem hitherto invented, and beyond which nothing better can be hoped for, in the *great Author's* Opinion.

14. The *Logarithm given* to find what *Ratio* it expreſſes, is a Problem ſolv'd with *like Eaſe*, and demonſtrated by a *like Proceſs* to that foregoing for finding the Logarithm of a *given Ratio*. For as the Logarithm of the Ratio of 1 to $1+x$ was proved to be $\overline{1+x}^{\frac{1}{n}}-1$, (by Art. 5.) and that of the Ratio of 1 to $1-x$ to be $1-\overline{1-x}^{\frac{1}{n}}$ (by Art. 8.) ſo the Logarithm, which we will call L, being given, ſince $L=\overline{1+x}^{\frac{1}{n}}-1$, therefore $L+1=\overline{1+x}^{\frac{1}{n}}$ in the firſt Caſe; and $1-L=\overline{1-x}^{\frac{1}{n}}$, in the latter: Conſequently $\overline{1+L}^{n}=1+x$, and $\overline{1-L}^{n}=1-x$. That is, according to Sir *Iſaac*'s *Theorem*, $1+nL+\frac{1}{2}n^2L^2+\frac{1}{6}n^3L^3+\frac{1}{24}n^4L^4+\frac{1}{120}n^5L^5$, *&c.* $=1+x$; and alſo $1-nL+\frac{1}{2}n^2L^2-\frac{1}{6}n^3L^3+\frac{1}{24}n^4L^4-\frac{1}{120}n^5L^5$, *&c.* $=1-x$; conſequently the Number $1+x$ or $1-x$ is readily known by thoſe Series, be the *Species* of the Logarithm what it will. That is, whether it be *Neper*'s Logarithm, where $n=100000$, &c. and ſo $1\pm x=1\pm L+\frac{1}{2}L^2\pm\frac{1}{6}L^3+\frac{1}{24}L^4\pm\frac{1}{120}L^5$, *&c.* or whether $n=2302585$0, &c. for *Briggs*'s Logarithm.

15. If one Term of the *Ratio*, whereof L is the Logarithm, be given, the other Term will eaſily be had by the ſame Rule. Let $a=$ the *leaſt Term* of the *Ratio*, and $b=$ the greateſt; then becauſe it is $1:1+x::a:b$; and ſo $1+x=\frac{b}{a}=1+L+\frac{1}{2}L^2+\frac{1}{6}L^3+\frac{1}{24}L^4$, *&c.* if $n=1000000$, &c. and therefore $b=a+aL+\frac{1}{2}aL^2+\frac{1}{6}aL^3$, &c. if a were given; but if b were given, becauſe $1:1-x::b:a$,

therefore

therefore $1 - x = \frac{a}{b}$; and so $a = b - bL + \frac{1}{2}bL - \frac{1}{6}bL^2 + \frac{1}{24}L$, &c. Wherefore by the Help of the *Tables*, the Number belonging to any Logarithm will be *exactly had* to the utmost Extent of the Tables.

16. Suppose $\frac{b}{a} = N$, the Number belonging to the given Logarithm L of the Ratio $\frac{b}{a}$, then $\frac{b}{N} = a$, let the Logarithm of the Ratio $\frac{b}{N}$ be ɜ, and let the Term b be known; then (*per* Rule, Art. 15.) we have $1 : 1 - x :: b : N$, and so $1 - x = \frac{N}{b}$, and $N = b - b\text{ɜ} + \frac{1}{2}b\text{ɜɜ} - \frac{1}{6}b\text{ɜ}^3$, &c. if ɜ be *Neper*'s Logarithm, but $N = b - bn\text{ɜ} + \frac{1}{2}bn^2\text{ɜɜ} - \frac{1}{6}n^3b\text{ɜ}^3$, &c. if $n = 2.3025$, &c. as in *Briggs*'s Logarithm. But if the Ratio be $\frac{a}{b} = N$, then $\frac{a}{N} = b$, and so $1 + x = \frac{N}{a}$, therefore $N = a + a\text{ɜ} + \frac{1}{2}a\text{ɜɜ} + \frac{1}{6}a\text{ɜ}^3$, &c. Or $N = a + an\text{ɜ} + \frac{1}{2}an^2\text{ɜ}^2 + an^3\text{ɜ}^3$, &c. Note, here a and b denote the Number belonging to the nearest *next lesser* or *next greater* Logarithm than the *given Logarithm* L, and the Logarithm ɜ is the *Difference* of those *Logarithms*; wherefore as ɜ is less, the Series converges the swifter; and finds the Number N of the Logarithm L, much sooner and easier than the Rule in Art. 15.

17. In the foregoing Series $a + a\text{ɜ} + \frac{1}{2}a\text{ɜɜ} + \frac{1}{6}a\text{ɜ}^3$, &c. $= N$; the three first Steps may be abridg'd thus, $a + \frac{a\text{ɜ}}{1 - \frac{1}{2}}\text{ɜ} = a + a\text{ɜ} + \frac{1}{2}a\text{ɜɜ}$, very nearly, and may serve with *Exactness* enough for Numbers not exceeding 14 Places, which is more than sufficient for *common Use*. Therefore we may take $a +$

$a + \frac{a\mathfrak{z}}{1-\frac{1}{2}\mathfrak{z}} = N$, or $b - \frac{b\mathfrak{z}}{1+\frac{1}{2}\mathfrak{z}} = N$; and if the *Index* n taken for *Briggs*'s Logarithms, we ſhall have $a + \frac{a\mathfrak{z}}{\frac{1}{n}-\frac{1}{2}\mathfrak{z}} = N$, or $b - \frac{b\mathfrak{z}}{\frac{1}{n}+\frac{1}{2}\mathfrak{z}} = N$; that is (putting $z = \frac{1}{n} =, 43429$, &c.) $\frac{za+\frac{1}{2}a\mathfrak{z}}{z-\frac{1}{2}\mathfrak{z}} = N$, or $\frac{zb-\frac{1}{2}b\mathfrak{z}}{z+\frac{1}{2}\mathfrak{z}} = N$; which Equation may be reſolv'd into the following Analogies;

$$viz. \begin{cases} z-\frac{1}{2}\mathfrak{z} : z+\frac{1}{2}\mathfrak{z} :: a : N; \text{ or,} \\ z+\frac{1}{2}\mathfrak{z} : z-\frac{1}{2}\mathfrak{z} :: b : N. \end{cases}$$

18. If more Steps of *this Series* be deſir'd, it will be found as follows, *viz.* $a + \frac{a\mathfrak{z}}{1-\frac{1}{2}\mathfrak{z}} - \frac{\frac{1}{24}a^3\mathfrak{z}}{1-\mathfrak{z}} + \frac{\frac{1}{35}a\mathfrak{z}^5}{1-2\mathfrak{z}}$, &c. $= N$; alſo the Rule $1+n\mathrm{L}+\frac{1}{2}nn\mathrm{LL}+\frac{1}{6}n^3\mathrm{L}^3$, *&c.* may be thus contracted, *viz.* $1+\overline{2+n\mathrm{L}+\frac{1}{3}nn\mathrm{LL}} \times \frac{1}{2}n\mathrm{L} = N$. What is ſaid concerning this Method of making either *Logarithms* or *Numbers*, I preſume is ſufficient to render it very intelligible to any common Capacity, and to ſhew the admirable *Uſefulneſs* and *Excellency* thereof beyond any other, hitherto invented.

CHAP. VII.

The *Logarithmic Series *aforegoing, demonstrated also by* *FLUXIONS, *and from the Nature of the* *Hyperbola.

1. THE preceding Series for the Logarithms, which has been demonstrated *purely* from *Arithmetical Principles*, or the *Properties* of *Numbers*, may also be prov'd from the Doctrine of the *Fluxions of Logarithms*. For the Writers on *Fluxions* variously demonstrate *the Fluxion of the Logarithm of any Number* is equal *to the Fluxion of that Number (whose Logarithm it is) divided by the said Number it self.*

2. Let the Number proposed be $1+x$, the Fluxion of which is $\dot{x}$, therefore the *Fluxion of its Logarithm* will be $= \frac{\dot{x}}{1+x}$; from whence the foregoing infinite Series for the Logarithm of the Number $1+x$ may be derived, as follows. The Logarithm of $1+x$ is equal to the *flowing Quantity* or *Fluent* of the said Fluxion $\frac{\dot{x}}{1+x}$. But $\frac{\dot{x}}{1+x} = \dot{x} \times \frac{1}{1+x}$; and $\frac{1}{1+x} = 1+x)\ 1\ (= 1-x+xx-x^3$, &c.

$$\begin{array}{l}
1+x \\
\hline
\cdot\ -x \\
-x-xx \\
\hline
\cdot\ +xx \\
+xx+x^3 \\
\hline
\cdot\ \cdot\ -x^3 \\
-x^3-x^4 \\
\hline
\cdot\ +x^4,\ \&c.
\end{array}$$

3. The Quotient then $1-x+x^2-x^3+x^4$, &c. $=\frac{1}{1+x}$; but $\dot{x}\times 1-x+x^2-x^3+x^4$, &c. $=\dot{x}-x\dot{x}+x^2\dot{x}-x^3\dot{x}+x^4\dot{x}$, &c. $=\frac{\dot{x}}{1+x}$. The *Fluent* therefore of that *infinite fluxionary* Series (by the *inverse Method* of *Fluxions*) is found to be $x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\frac{1}{5}x^5$, &c. which therefore is the Logarithm of the Number $1+x$; and is the very same with that Series in the foregoing Chap. VI. Article 7. for *Neper*'s *Logarithms*.

4. Again, if the Ratio be *decreasing*, or the Number be $1-x$, the Fluxion of this also is $\dot{x}$, and therefore the Fluxion of its Logarithm $\frac{\dot{x}}{1-x}$. But $\frac{\dot{x}}{1-x}=\dot{x}\times\frac{1}{1-x}=1+x+x^2+x^3$, &c. $\times\dot{x}=\dot{x}+x\dot{x}+x^2\dot{x}+x^3\dot{x}$, &c. The *Fluent* of which is $x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$, &c. the same Series as that in Article 8. of the *preceding Chapter*, for *Neper*'s Logarithm of $1-x$.

5. The *same Series* is likewise deduced from the Nature or *Equation* of the *Hyperbola*. For let FCH be an *Hyperbola* (Fig. VI.) AE, AI, the *Asymptotes*; draw BC, DC parallel to AI and AE; also draw the Ordinate EF parallel to the Ordinate BC, or Asymptote AI. Let $AB=a$, $EF=y$, and $BE=x$. The Equation $aa=ay+xy$ expresses the Nature of the *Hyperbola* between the *Asymptotes*. Now the Fluxion of the Space between the *Abscissa*, *Ordinate*, and *Curve*, is always equal to the *Product* of the Ordinate into the *Fluxion* of the *Abscissa*; that is, in this Case, $=y\dot{x}$. Therefore to determine the Fluxion of the Asymptotic Space contain'd between the Abscissa BE, the Ordinates EF, BC, and the *Curve* of the Hyperbola FC, we have $\frac{aa}{a+x}=y$, therefore $\frac{aa\dot{x}}{a+x}=y\dot{x}=$ Fluxion of the said Space FCBE.

6. But $\frac{aa\dot{x}}{a+x} = \frac{aa}{a+x} \times \dot{x}$, and $\frac{aa}{a+x} = a+x)\, aa\, (= a - x + \frac{xx}{a} - \frac{x^3}{aa} + \frac{x^4}{a^3}$, &c. Wherefore $\frac{aa\dot{x}}{a+x} = a\dot{x} - x\dot{x} + \frac{x^2\dot{x}}{a} - \frac{x^3\dot{x}}{aa} + \frac{x^4\dot{x}}{a^3}$, &c. But the *flowing Quantity* of this *fluxionary Series* is $ax - \frac{1}{2}x^2 + \frac{x^3}{3a} - \frac{x^4}{4aa} + \frac{x^5}{5a^3}$, &c. = the Space FCBE. Suppose $a=1$, then $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$, &c. = the said *Asymptotic Space* FCBE, as before. But 'tis evident this is again the very same Series as was invented by Dr. *Halley* for the *Logarithm of the Number* $1+x$.

7. The *Asymptotic Spaces*, then, are with respect to the *Abscissæ*, as *Logarithms* in *respect* to *Numbers*. That is, since AB=1, and the Logarithm of 1 is=0, the Spaces B*ab*C, B*cd*C, B*eg*C, B*hk*C, *&c.* are the *Logarithms* of the Numbers A*a*, A*c*, A*e*, A*h*, &c. Again, because the *Abscissæ* are in a *reciprocal* Proportion of the *Ordinates*, that is, AB : AE :: EF : BC; therefore the *Asymptotic Spaces* are in respect of the Ordinates as *Logarithms* in respect of *Numbers*: Yet so, that while the *Ordinates* BC, *ab*, *cd*, *eg*, *hk*, EF, *decrease* in a *Geometrical Ratio*, the Spaces B*ab*C, B*cd*C, *&c.* may increase in an *Arithmetical Ratio*. And since in *Neper's Logarithms* $\frac{1}{n} \times x - \frac{1}{2}xx + \frac{1}{3}xxx - \frac{1}{4}x^4$, &c. $n=100000$, &c. 'tis plain his Logarithms become the same with the *Hyperbolic Logarithms* just now consider'd.

CHAP.

CHAP. VIII.

The Method of constructing Logarithms by the Infinite Series, exemplified and illustrated.

1. THE Manner of raising *Theorems* for the Construction of Logarithms hath been sufficiently explain'd; it therefore remains that we illustrate the same by proper Examples. The Theorems for doing this directly are,

Theorem I. $\frac{1}{n} \times x \mp \frac{1}{2}x^2 + \frac{1}{3}x^3 \mp \frac{1}{4}x^4 + \frac{1}{5}x^5$, &c. $=$ L, $\overline{1 \pm x}$.

Theorem II. $\frac{1}{n} \times \frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5} + \frac{2d^7}{7s^7}$, &c. $=$ L, $\frac{a}{b}$.

$$\left.\begin{array}{l} \text{Theorem III. } \frac{1}{n} \times \frac{d^2}{2s^2} + \frac{d^4}{4s^4} + \frac{d^6}{6s^6} + \frac{d^8}{8s^8}, \\ \quad \text{\&c.} = \text{L.} \\ \text{Theorem IV. } \frac{1}{n} \times \frac{1}{S} + \frac{1}{3S^3} + \frac{1}{5S^5} + \frac{1}{7S^7}, \\ \quad \text{\&c.} = \text{L} \end{array}\right\} \frac{\sqrt{ab}}{\frac{1}{2}s}.$$

Note, in these Theorems, $\frac{1}{n}$ is all along applied to adapt them to *all sorts of Logarithms.*

2. Since $n=$ the Logarithm of 10, we must therefore first suppose $n=10000000$, &c. and thence *Neper*'s *Logarithms* will be produced; and so these are the first sort of Logarithms which Nature affords: The others, as *Briggs*'s *Logarithms*, &c. are made from them. In order then to find a *Briggian* Logarithm, 'tis necessary first to find *Neper*'s Logarithm of 10. This may be done several Ways, either by the *Number* 10 *it self*, or by its *component Parts.* If we attempt it by the Number 10 it self, then because

$1+x=$

$1+x=10$, we shall have $x=9$, which, because it is greater than 1, will occasion that the first Theorem will not *converge*; and so the second Theorem must be used. In this $\frac{a}{b}=\frac{1}{10}$, and therefore $s=1+10=11$, and $d=10-1=9$. And thus the second Series for *Neper*'s Logarithm of 10 will be $\frac{2d}{s}+\frac{2d^3}{3s^3}+\frac{2d^5}{5s^5}$, &c. $=\frac{18}{11}+\frac{1458}{3993}+\frac{118098}{805255}$, &c. $=2.30258$, &c. the Logarithm sought.

3. But this Series converging so *extremely slow*, renders the Business very tedious, and therefore the said Logarithm must be attempted from the *component Parts* of 10. And since $8\times1\frac{1}{4}=10$, and $2\times2\times2=8$, therefore 3L, 2 + L, $1\frac{1}{4}$ = L, 10. Consequently by finding the Logarithm of 2 and $1\frac{1}{4}$, we find the Logarithm of 10. Now *Neper*'s Logarithm of 2, is found either by Theorem I. which converges very slowly; or by Theorem II. which converges much faster; and therefore to be chosen. Here $\frac{a}{b}=\frac{1}{2}$, and $a+b=s=3$, $a-b=d=1$, and so the Theorem $\frac{2d}{s}+\frac{2d^3}{3s^3}+\frac{2d^5}{5s^5}$, &c. $=\frac{2d}{s}+\overline{\frac{1}{3}\times\frac{2d^3}{s^3}}+\overline{\frac{1}{5}\times\frac{2d^5}{s^5}}$, &c. $=\frac{2}{3}+\overline{\frac{1}{3}\times\frac{2}{3^3}}+\overline{\frac{1}{5}\times\frac{2}{3^5}}$, &c. = L, 2; this multiplied by 3, is $=\frac{6}{3}+\overline{\frac{1}{3}\times\frac{6}{3^3}}+\overline{\frac{1}{5}\times\frac{6}{3^5}}$, &c. = 3L, 2. Now put $\frac{6}{3^3}=A$, and because $\frac{1}{3^2}\times\frac{6}{3^3}=\frac{6}{3^5}$, and $\frac{1}{3^2}=\frac{1}{9}$, therefore also put $\frac{1}{9}A=B$, and so $B=\frac{6}{3^5}$; and thus $\frac{1}{9}B=C$, $\frac{1}{9}C=D$, and so on. Whence the Theorem will become $2+\frac{1}{3}A+\frac{1}{5}B+\frac{1}{7}C+\frac{1}{9}D+\frac{1}{11}E+\frac{1}{13}F$, &c. = 3L, 2.

4.

4. By the ſame Theorem II. we obtain *Neper*'s Logarithm of $1\frac{1}{4}$; for becauſe $\frac{5}{4} = 1\frac{1}{4}$, therefore $\frac{1}{1\frac{1}{4}} = \frac{5}{4}) 1 (= \frac{4}{5} = \frac{a}{b}$. Whence $a + b = s = 9$, and $d = a - b = 1$; and ſo the Theorem $\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$, &c. $= \frac{2}{9} + \overline{\frac{1}{3} \times \frac{2}{9^3}} + \overline{\frac{1}{5} \times \frac{2}{9^5}}$, &c. But becauſe $\frac{2}{9} = \frac{6}{3^3} = A$; $\frac{2}{9^3} = \frac{6}{3^7} = \frac{6}{3^5} \times \frac{1}{9} = \frac{1}{9}B = C$; $\frac{2}{9^5} = \frac{6}{3^{11}} = \frac{6}{3^9} \times \frac{1}{9} = \frac{1}{9}D = E$, &c. therefore the ſaid Theorem will become $A + \frac{1}{3}C + \frac{1}{5}E + \frac{1}{7}G + \frac{1}{9}I + \frac{1}{11}L$, &c. $= L, 1\frac{1}{4}$ as required. If now this Series be added to the foregoing (in Art. 3.) we ſhall have the Theorem $2 + 1\frac{1}{3}A + \frac{1}{5}B + \frac{10}{21}C + \frac{1}{9}D + \frac{16}{55}E + \frac{1}{13}F + \frac{22}{105}G + \frac{1}{17}H$, &c. $= 3L, 2 + L, 1\frac{1}{4} = L, 10$. See the Operation in the Table below.

	2. = 2.0000000000000
A = 0.2222222222222	$1\frac{1}{3}$A = . .2962962962962
B = . . 246913580246	$\frac{1}{5}$B = . . . 49382716049
C = . . . 27434842249	$\frac{10}{21}$C = . . . 13064210595
D = ... 3048315805	$\frac{1}{9}$D = ... 338701756
E = ... 338701756	$\frac{16}{55}$E = ...98531420
F = ...37633529	$\frac{1}{13}$F = ...2894887
G = ... 4181503	$\frac{22}{105}$G = ...876124
H = ... 464612	$\frac{1}{17}$H = ...27330
I = ... 51624	$\frac{28}{171}$I = ...8454
K = ... 5763	$\frac{1}{24}$K = ... 273
L = ... 637	$\frac{34}{253}$L = ... 85
M = ...71	$\frac{1}{25}$M = ... 3

Thus *Neper*'s Log. of 10 = 2.302585092994 0, &c.

5. The Logarithm thus found (if continued on) will be 2.30258509299404568401799145468436420760110148862877297603332 8, &c. $= n$; and therefore

fore in making the *Briggian Logarithms*, we shall have $\frac{1}{n}=0.4342944819032518276511289189166050822943970058036665661144\text{54}$, &c. the Reciprocal of the former; which henceforth let be call'd z; that is, let $\frac{1}{n}=z$. And now we are prepar'd to find the Logarithm (of any Form) of any other Number.

6. For Example, let it be required to find *Briggs's* Logarithm of 2, to 10 Places of Figures. In order to this, the *Index* $\frac{1}{n}$ must be assum'd of *a Figure or two* more than the intended Number of Places in the Logarithm. The *second* Theorem is most proper for this Purpose; for here again $\frac{a}{b}=\frac{1}{2}$, $d=1$, and $s=3$, and also $z=0.434294481903$; and the Theorem $\frac{1}{n}\times\overline{\frac{2d}{s}+\frac{2d^3}{3s^3}+\frac{2d^5}{5s^5}}$, *&c.* $=z\times\overline{\frac{2}{3}+\frac{1}{3}\times\frac{2}{3^3}+\frac{1}{5}\frac{2}{3^5}}$, *&c.* $=$L, 2, and therefore $\frac{z}{3}+\frac{1}{3}\times\frac{z}{3^3}+\frac{1}{5}\times\frac{z}{3^5}$, &c. $=\frac{1}{2}$L, 2. Here put $\frac{z}{3}=$A, and because $\frac{z}{3^3}=\frac{z}{3}\times\frac{1}{3^2}=\frac{1}{9}\times\text{A}=\text{B}$, and thus $\frac{z}{3^5}=\frac{z}{3^3}\times\frac{1}{3^2}=\frac{1}{9}\text{B}=\text{C}$, and so on. Therefore the said Theorem will here again become $\text{A}+\frac{1}{3}\text{B}+\frac{1}{5}\text{C}+\frac{1}{7}\text{D}+\frac{1}{9}\text{E}+\frac{1}{11}\text{F}$, *&c.* $=\frac{1}{2}$L, 2, as is evident from the following Operation.

$\frac{1}{3}z=$

$\frac{1}{3}z$ = A =	0.144764827301	A =	0.144764827301
B =	1608498081 1	$\frac{1}{3}$B =	5361660270
C =	1787220090	$\frac{1}{5}$C =	357444018
D =	198580010	$\frac{1}{7}$D =	28368572
E =	22064445	$\frac{1}{9}$E =	2451605
F =	2451605	$\frac{1}{11}$F =	222873
G =	272400	$\frac{1}{13}$G =	20953
H =	30266	$\frac{1}{15}$H =	2017
I =	3362	$\frac{1}{17}$I =	197
K =	373	$\frac{1}{19}$K =	19
L =	41	$\frac{1}{21}$L =	1
The Sum is		$\frac{1}{2}$L, 2 =	0.150514997826
Multiplied by			2
The *Briggian* Logarithm of		2 =	0.301029995652

7. This Logarithm may yet be much eaſier and ſooner obtain'd by this Conſideration, *viz.* That $\frac{1}{2}^{10} = \frac{1}{1024}$, and $\frac{1000}{1024} \times \frac{1}{1000} = \frac{1}{1024}$; therefore $\frac{L\,\frac{1000}{1024} + L\,\frac{1}{1000}}{10} = \frac{L\,\frac{1}{1024}}{10} = L\,\frac{1}{2} = L\,2$. But $L\frac{1000}{1024} = L\frac{125}{128}$. Wherefore put $\frac{125}{128} = \frac{a}{b}$ a-new, then $a+b=s=253$, and $a-b=d=3$; thus Theorem II. will converge much faſter, and will become, in Numbers, $z \times \overline{\frac{2}{1} \times \frac{3}{253} + \frac{2}{3} \times \frac{3^3}{253^3} + \frac{2}{5} \times \frac{3^5}{253^5}}$, &c. Or if $\frac{d}{s} = y$, the ſaid Theorem, in *Species*, is $2zy + \frac{2}{3}zy^3 + \frac{2}{5}zy^5$, &c. Suppoſe $2zy = A$, then $\frac{2}{3}zy^3 = \frac{1}{3} \times 2zy \times y^2 = \frac{1}{3}Axy^2 = B$, alſo $\frac{2}{5}zy^5 = \frac{1}{5} \times 2zy \times y^2 \times y^2 = \frac{3}{5}Bxy^2 = C$, and ſo on; and thus the Theorem is $A + \frac{1}{3}Ay^2 (=B) + \frac{3}{5}By^2 (=C) + \frac{5}{7}Cy^2 (=D) + \frac{7}{9}Dy^2$, &c.

8. Wherefore $\begin{cases} 2zy = A = & 0.010299473879 12 \\ \frac{1}{3}Axyy = B = .. & 48271995 \\ \frac{3}{5}Bxyy = C = .. & 4072 \end{cases}$

The Sum is the Logarithm of $\frac{1000}{1024}(=L\frac{125}{128})=$ } .. 0.01029995663980

Add the Logarithm of $\frac{81}{1000}$ . .. 3.00000000000000

And $\frac{1}{10}$ of that Sum will be $L2=$ } .. 0,30102999566398.0

Thus you see 3 Steps of the Series thus ordered, are sufficient for 14 Places of Figures, whereas before (Art. 6.) 11 Steps produced the Logarithm true only to 10 Places.

9. Let the next Example be to find the *Briggian Logarithm* of 3. This may also be done by Theorem II. where $\frac{a}{b} = \frac{1}{3}$, and $d=2$, and $s=4$; also $\frac{d}{s} = \frac{1}{2}$, $z=0,43429$, &c. as before. Then $z \times \frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$, *&c.* $= \frac{2}{1} \times \overline{\frac{z}{2}} + \frac{2}{3} \times \overline{\frac{z}{8}} + \frac{2}{5} \times \overline{\frac{z}{32}}$, &c. $= L\,3$. The half thereof $\frac{z}{2} + \frac{1}{3} \times \overline{\frac{z}{8}} + \frac{1}{5} \times \overline{\frac{z}{32}}$, &c. $= \frac{1}{2} L\,3$. Put $\frac{z}{2} = A$, then $\frac{z}{8} = \frac{z}{2} \times \frac{1}{4} = \frac{1}{4}A = B$, and $\frac{z}{32} = \frac{z}{8} \times \frac{1}{4} = \frac{1}{4}B = C$, and so on. Whence $A + \frac{1}{3}B + \frac{1}{5}C + \frac{1}{7}D + \frac{1}{9}E$, *&c.* $= \frac{1}{2}L\,3$. But this Series converges so very slow, that as many Steps will be necessary as you intend Places of Figures in the Logarithm, and more; therefore Recourse must be had to Theorem IV. which here comes into play, because the Logarithms on each Side of it are known, *viz.* the Logarithm of 2 and 4.

10. Therefore (according to Theorem IV.) $a=2$, $b=4$, $\frac{a+b}{2} = \frac{1}{2}s = 3$, $ab=8$, consequently $\frac{1}{4}ss=9$, and so $\frac{1}{4}ss + ab = S = 17$. Wherefore $\frac{z}{S} + \frac{1}{3} \times \overline{\frac{z}{S^3}}$

+

$+\frac{1}{5}\times\frac{z}{S^5}+\frac{1}{7}\times\frac{z}{S^7}$, &c. $=L\sqrt{\frac{8}{3}}$, put $\frac{z}{S}=A$, then $\frac{1}{3}\times\frac{z}{S^3}=\frac{1}{3}\times A\times\frac{1}{S^2}=B$, $\frac{1}{5}\times\frac{z}{S^5}=\frac{3}{5}\times B\times\frac{1}{S^2}=C$, *&c.* Alſo $\sqrt{\frac{8}{3}}\times\frac{1}{\sqrt{8}}=\frac{1}{3}$, wherefore $L\sqrt{\frac{8}{3}}+L\frac{1}{\sqrt{8}}=L\,3$. Therefore $L\frac{1}{\sqrt{8}}+A+B+C+D$, *&c.* $=L\,3$. See the Operation following.

$$\text{Thus}\begin{cases} L\frac{1}{\sqrt{8}}=\frac{L2+L4}{2}=\ldots\ldots & 0.4515449934959\\ \frac{1}{1}\times\frac{z}{S}=A=\ldots\ldots & 255467342296\\ \frac{1}{3}\times\frac{1}{S^2}\times A=B=\ldots\ldots & 294656680\\ \frac{3}{5}\times\frac{1}{S^2}\times B=C=\ldots\ldots & 611744\\ \frac{5}{7}\times\frac{1}{S^2}\times C=D=\ldots\ldots & 1511\end{cases}$$

The Sum is the Log. of . . . . $3=0.4771212547190$

11. But this Logarithm may yet much ſooner and with leſs Trouble be found, by the Artifice uſed in Art. 7. For the Ratio $\frac{2^{15}}{5\times3^8}=\frac{32768}{32805}=\frac{a}{b}$, and ſo $a+b=s=65573$, and $a-b=s=37$; and ſince $\frac{2^{15}}{5\times3^8}\times\frac{5}{2^{15}}=\frac{1}{3^8}$, therefore $L\frac{2^{15}}{5\times3^8}+L\frac{5}{2^{15}}=L\,3^8$; but $L\frac{5}{2^{15}}=L\,2^{15}-L\,5$. And $L\frac{2^{15}}{5\times3^8}=\frac{2zd}{s}+\frac{2zd^3}{3s^3}$, *&c.*

$$\text{Therefore}\begin{cases} L2^{15}-L5=\ldots\ldots & 3.81647993062\\ \frac{2zd}{s}=\ldots\ldots\ldots\ldots & 0.00049010708\end{cases}$$

The Sum is the Logarithm of $3^8=3.81697003770$

And $\frac{1}{8}$ thereof is the Log. of $3=0.477121254712\frac{1}{2}$; thus the *firſt Step* of Theorem II. gives the Logarithm true to 11 Places.

12. The next Example ſhall be that which Dr. *Halley* has given for finding the Logarithm of 23, which is done by Theorem IV. In this Caſe, $a=22$, $b=24$, $\frac{1}{2}s=23$, $\frac{1}{4}ss=529$, $ab=528$, and $\frac{1}{4}ss+ab=1057=S$. And $\frac{z}{S}+\frac{z}{3S^3}+\frac{z}{5S^5}$, &c. $=\mathrm{L}\frac{\sqrt{ab}}{\frac{1}{2}s}$. But $\frac{\sqrt{ab}}{\frac{1}{2}s}\times\frac{1}{\sqrt{ab}}=\frac{1}{\frac{1}{2}s}$; therefore $\mathrm{L}\,\frac{\sqrt{24+22}}{\frac{1}{2}s}+\mathrm{L}\,\frac{1}{\sqrt{24+22}}=\mathrm{L}\,\frac{1}{\frac{1}{2}s}$; and becauſe $2\times2\times2\times3=24$, and $2\times11=22$, therefore alſo $3\,\mathrm{L},2+\mathrm{L}\,3=\mathrm{L}\,24$; and $\mathrm{L}\,2+\mathrm{L}\,11=\mathrm{L}\,22$. And $\frac{\mathrm{L}24+\mathrm{L}22}{2}=\mathrm{L}\sqrt{ab}$; therefore, (proceeding in the Operation according to Art. 10.) we have

$$\left\{\begin{array}{lr}
\frac{\mathrm{L}24+\mathrm{L}22}{2}=\ldots & 1.36131696126690612945009172669805 \\
\frac{z}{s}=A=\ldots & 41087462810146814347315886368 \\
\frac{1}{3}\times\frac{1}{ss}\times A=B= & 12258521544181829460074 \\
\frac{3}{5}\times\frac{1}{ss}\times B=C= & 6583235184376175 \\
\frac{5}{7}\times\frac{1}{SS}\times C=D= & 4208829765 \\
\frac{7}{9}\times\frac{1}{SS}\times D=E= & 2930
\end{array}\right.$$

$$\text{The Sum}=\mathrm{L}\,23=\ldots 1.36172783601759287886777711225117$$

which Logarithm is true to 32 Places of Figures, and thus you may proceed for any other.

13. In making Logarithms for *Prime Numbers*, the *Artifice*, or *greateſt Advantage* conſiſts in finding ſuch a *Ratio* or *Fraction*, whoſe Terms are the *greateſt* poſſible and their *Difference* the leaſt; and the *Number* whoſe Logarithm is ſought, or ſome *Power* thereof, is an *aliquot Part* or *Submultiple* of one of the Terms of the ſaid *Ratio* or *Fraction*. For this once obtain'd, the Logarithm is ſoon acquired by Theorem

rem II. with eaſe. Thus the Fraction $\frac{2321}{2320} = \frac{211 \times 11}{80 \times 29} = \frac{b}{a}$; where $d=1$, and $s=4641$ the Series will converge very ſwift for the Logarithm of $\frac{211 \times 11}{80 \times 29}$ but $\frac{211 \times 11}{80 \times 29} \times \frac{80 \times 29}{11} = \frac{211}{1} = 211$, therefore L80+L29—L11 added to the Series, gives the Logarithm of 211. But the *Ratio* or *Fraction* $\frac{5387041}{5387040} = \frac{121 \times 211^2}{80 \times 54 \times 29 \times 43}$ will make the Series converge very much ſooner than before; for here $d=1$, and $s=10774081$. For L80+L54+L29+L43—L121 added to the Series (or Theor. II.) gives the Logarithm of 211^2, half which is the Logarithm of 211. Laſtly, the Fraction $\frac{1982119441}{1982119440} = \frac{211^4}{60 \times 28 \times 53 \times 113 \times 197} = \frac{b}{a}$, where $d=1$, and $s=3964238881$, converges to that Degree that the firſt Step of the Series quotes the Logarithm of the Fraction to 29 Places, to which add the Logarithms of the 5 Numbers in the *Denominator*, and it gives the Logarithm of 211^4, then $\frac{1}{4}$ of that is the Logarithm of 211, as before.

14. The greateſt Difficulty conſiſts in finding out proper Numbers for producing ſuch Fractions as aforeſaid; and the beſt Method of this is by *prudent Tryals.* An Example of which is here ſubjoin'd. Suppoſe I would procure a convenient Fraction for the Logarithm of 223, I make tryal thus;

$$\begin{array}{lll} \text{First } 223\left\{\begin{array}{l}\times\ 7 = \\ \times\ 8 = \end{array}\right. & \begin{array}{r}1561 \\ \underline{1784}\end{array} \\ \text{Then } 223\left\{\begin{array}{l}\times\ 87 = \\ \times\ 2\ \ = \end{array}\right. & \begin{array}{r}19401 \\ \underline{446}\end{array} \\ \text{Therefore } 223 \times 387 = & 64001 \end{array}$$

Having thus obtain'd the Term 64001, 'tis eaſy to obſerve the other may be 64000, wherefore the Fraction is $\frac{64001}{64000} = \frac{287 \times 223}{64000}$, and finds the Logarithm of

223

223 with good Dispatch. Or thus, to find a Fraction of *larger Terms*; suppose I assume the Numerator $159000 = 1000 \times 53 \times 3$, then to find the other Term as near this as may be, I try thus;

$$223\begin{cases}\times\ 3 = & 669\\ \times\ 1 = & 223\end{cases}$$

$$223\begin{cases}\times\ 13 = & 2899\\ \times\ 7 = & 1561\end{cases}$$

Therefore $223 \times 713 = 158999$, which is within Unity as great as the other Term 159000, and consequently the Fraction $\frac{159000}{158999}$ is that required, and thus you proceed to raise the Terms of any other.

15. Let the Terms of any Fraction be represented by a=Least, and b=Greatest. Then if the *Ratio* be *increasing* it will be $\frac{a}{b}$, but if decreasing, $\frac{b}{a}$; let that Term, in which the Number sought is *ingredient*, be express'd by the Product cx, where $c=$ the Number (or Product of Numbers) whose Logarithm is known, and $x=$ the Number whose Logarithm is sought. If $a=cx$, then $\frac{cx}{b} = \frac{a}{b}$, or $\frac{b}{cx}$; but if $b=cx$, then $\frac{a}{cx}$, or $\frac{cx}{a}$; also let there be put the 2d Theorem $\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5} + \frac{2d^7}{7s^7}$, &c. $= z$; then if the unknown Number x be in the *Denominator* of the *increasing Ratio* $\frac{a}{b}$, viz. $\frac{a}{cx}$; or in the *Numerator* of the *decreasing Ratio* $\frac{b}{a}$, viz. $\frac{cx}{a}$; then it will be $Z + La - Lc = Lx$. And, *vice versa*, if it be $\frac{cx}{b}$ or $\frac{b}{cx}$, the Theorem will be $Z + Lb - Lc = Lx$. From hence the Operations in the foregoing Articles for making Logarithms have their Grounds and Reason; and

and every thing there asserted is from this Process very evident.

16. To find the natural Number of any Logarithm proposed; this is best done by the Theorem in Art. 17. Chap. VII.

viz. $\left\{\begin{matrix} a \\ b \end{matrix}\right\} \times 1 \pm n\exists + \frac{1}{2}n^2\exists^2 \pm \frac{1}{6}n^3\exists^3 + \frac{1}{24}n^4\exists^4 \pm \frac{1}{120}n^5\exists^5$ *&c.* $= N$.

For Example, let it be required to find the Interest of one Pound for one Day, at the rate of 6 *l. per Cent. per Annum, Compound Interest*; which is to extract the Root of 1.06 taken as the 365th Power; thus the Logarithm of 1.06=0.025305865264770240846731186351, *&c.*

Which divide by 365, the Quotient is $L=$0.0000693311377116-99289991044346, *&c.*

The next nearest Log. & its N°. $b=$1.00016=0.0000694815587-28037517724712696, *&c.*

Their Difference is $\exists=$0.000000150421016338227-733668350, *&c.*

Mult. this by $n=$2.3025850929940456840179914-54684, &c.

The Product is

$n\exists$ = 0.000000346357189893416971322305
$n^2\exists^2$ = ... 119963302990864503
$n^3\exists^3$ = ... 41550152514
$n^4\exists^4$ = ... 14391

The Powers of $n\exists$. Then

$1+\frac{1}{2}n^2\exists^2=$1.000000000000059981651495432251
$\frac{1}{24}n^4\exists^4=$ 599

The Sum $1+\frac{1}{2}n^2\exists^2+\frac{1}{24}n^4\exists^4=$ 1.00000000000005998165-149543285 1 $=X$.

The odd Powers

$n\exists=$ 0.000000346357189893416971322305
$\frac{1}{6}n^3\exists^3=$ 6925025419

Sum $n\exists+\frac{1}{6}n^3\exists^3=$0.00000034635718989342389634-7724 $=z$.

Then

Then the Value of the Series is
$X-z=0.99999965364287008822759908 5126.$

Which multiply by $b=1.00016$, produces
$\sqrt[365]{1{,}06}=1.000159653587452947441715500980$, *&c.* $=N$, the Number sought of the given Logarithm L, and that to 30 Places of Figures. The same Number may be seen produced to 60 Places in Mr. *Sherwin*'s Mathematical Tables.

CHAP. IX.

*Of the *Logarithmic *Spiral; and the Nature and Construction of the Table of *Meridional *Parts, or the Nautical *Meridian *Line, deduced therefrom.*

1. IF any Right-Line pW be moved with an *equable Motion* about the fix'd Point p, and at the same time the Point W be mov'd towards the Point p, with a Velocity such that the *Radii* pW, pV, pS, *&c.* form'd thereby, be in a *Geometrical Ratio* decreasing, then the Curve WVSQ, *&c.* is called the *Logarithmic Spiral*; and that for the same Reason as the Logarithmic Curve before describ'd received its Appellation. See Chap. III. Art. 8, 9, 10.

2. For suppose the Arches AC=CE=EG, *&c.* and therefore in *Arithmetical Progression*; and since, from the *Generation* of the *Spiral*, the Radius pB : pD :: pD : pF :: pF : pH, *&c.* 'tis evident the Arches AC, AE, AG, *&c.* are the *Exponents* of the *Ratio's* of the *Radii* Dp, Fp, Hp, *&c.* to the first pB; and so those *Arches* are in respect of the *Radii*, as *Logarithms* in respect of *Numbers*; as is sufficiently manifest from the preceding *Theory of Logarithms*. Wherefore

fore if Bp be = 1, 10, 100, *&c.* and PW (the 10th Proportional from pB) be = 10, 100, 1000, &c. then ſhall the Arch AC =, 1000000, AE =, 2000000, AG =, 3000000, *&c.* AW = 1,0000000; be the Logarithms of the Numbers pD = 1,259, *&c.* pF = 1,585, *&c.* pH = 1,996, *&c.* pW = 10; of Mr. *Briggs*'s Form.

3. This Spiral is alſo called the *Equiangular Spiral*; becauſe it interſects all the Radii pW, pQ, pB, at *equal Angles.* For ſuppoſe the Arches NP, TW, infinitely ſmall, and equal to each other, then may the Parts of the Spiral OQ and VW, be eſteemed Right-Lines; and ſo ſince in the Triangles pOQ, pVW, the Sides are proportional, *viz.* Op : pQ :: Vp : pW, and the Angle OpQ = VpW, thoſe Triangles are ſimilar; and conſequently the Angle pOQ = pVW, or pQO = pWV; and thus it will be every where.

4. Now let the whole Scheme be conſidered as the *Stereographic Projection* of one Quarter of a *parallel Hemiſphere*, then ſhall p be the *Pole*; WLA, a Quadrantal Arch of the *Equator*; the *Radii* pW, pT, pR, *&c.* the ſeveral *Meridians* projected on the Plane of the Equator. And ſince 'tis the Property of every Rumb Line to make equal Angles with every Meridian on the Globe, and the Angles contained between circular Arches on the Globe, are equal to the Angles between the ſame Arches in this Projection, therefore the *Logarithmic Spiral* WQB is the Projection of a *Rumb Line*; ſince it has the ſame Property on the Projection, as the *Rumb* on the Globe, as was proved Art. 3. hereof.

5. Moreover, ſince all *Right Circles*, ſuch as are the *Meridians* in this Caſe, are projected into *Right Lines* equal to the Tangents of half the Arches, the Lines pB, pD, pF, pH, *&c.* will here repreſent the *Tangents* of *half the Complements* of the Latitudes AB, CD, EF, GH, *&c.* And ſince the Arches in the Equator AC, AE, AG, *&c.* are the *Differences*

of Longitude made by ſailing from the Latitude B to the Latitudes D, F, H, *&c.* on the *Rumb* or *Spiral* BOW; and it has been ſhewn that thoſe Arches are the *Logarithms* of the *Radii* pD, pF, pH, *&c.* therefore the Difference of Longitude is the *Logarithm* of the *Tangent* of half the *Complement* of *Latitude*, reckoning from the Meridian A p whence the Logarithms begin.

6. Therefore the Difference of Longitude RT, made by ſailing from the Latitude S to the Latitude V, is equal to the Difference of the Logarithms (AT—AR) of the Tangents of the half Complements (Sp, Vp) of the Latitudes TV, RS. And ſince the Ratio of the *Progreſſion*, or of pW to pV, may be infinitely varied, 'tis plain the infinite Number of Rumbs in a Quadrant of the Compaſs determine ſo many *Scales* of *Logarithms* in the Equator of the Tangents of the half Complements of the Latitudes proper to thoſe Rumbs.

7. Since then every different Rumb is a *Logarithmic Spiral*, or determines a peculiar Scale of *Logarithms* for the Tangents of the Half-Complements of its Latitudes, therefore any Canon or Table of Logarithm-Tangents, whether of *Neper*'s, *Briggs*'s, or any *other Form* whatſoever, is the Scale of the Differences of Longitude on ſome determinate Rumb or other. And conſequently if this Rumb be inveſtigated for the Canon of *Briggs*'s *Logarithms* (now in common Uſe,) the ſaid Canon may be made to anſwer all the Purpoſes of the *Nautical Meridian Line*, in Propoſitions of *Navigation* by *Mercator's Chart.*

8. In order to this it muſt be conſidered, that the *Meridian Line* is a *Table* or *Scale* of *Longitudes* to every Degree of Latitude on the Rumb which makes an Angle of 45 Degrees with the Meridian; ſince in this Caſe the *Differences* of *Longitude* are always equal to the *Meridional, or enlarg'd Differences* of *Latitude.* And ſince there is a certain Rumb on which

which *Neper*'s or *Briggs*'s Logarithm-Tangents are the *Differences* of Longitude, and the Differences of Longitude on different Rumbs are to one another as the Tangents of the Angles of thoſe Rumbs with the Meridian; therefore by having given the Difference of Longitude on the Rumb of 45°, in Logarithms of *Neper*'s Form, and the Length of the Arch of one Minute or Degree in Parts of the Radius, we can thence find the Angle of that Rumb which determines that *Species of Logarithms.*

9. Now the *Momentary Augment* or *Fluxion* of the Tangent-Line of 45°, is exactly double to the *Fluxion* of the Arch of the Circle (as is eaſily proved), and the *Tangent* of 45° being equal to *Radius*, the Fluxion alſo of the *Logarithm-Tangent* will be double to that of the Arch, if the Logarithm be of *Neper*'s *Form*; but for *Briggs*'s Form, it will be as the ſame double Arch multiplied into $\frac{1}{n}=0.43429$, &c. or divided by $n=2.30258$, &c. the Index for *Briggs*'s *Logarithms.* See Chap. VI. Art. 7.

10. Now ſince the Radius of a Circle being put $=1$, the *Periphery* thereof will be 6.2831853, *&c.* therefore 360)6.2831853, *&c.*(0.01745329, *&c.* = the Length of the Arch of *one Degree.* Alſo 60)0.01745329, &c. (0.0002908882, &c. = the Length of an Arch of *one Minute*, in Parts of the Radius. If one Minute be ſuppoſed Unity, then the Proportion for finding the Angle of the Rumb required for *Neper*'s Logarithms, will be, as 1 : 2.908882, &c. :: Radius = 1000000, &c. : the Tangent =2908882, &c. of the Angle 71° 1′ 42″, whoſe Logarithm is 10.463726117, &c. and under that Angle is the *Meridian* interſected by that *Rumb Line*, on which the Differences of *Neper*'s Logarithm-Tangents of the Complements of the Latitudes are the true Differences of Longitude, eſtimated in Minutes and Parts, taking the firſt 4 Figures for Integers.

11. But since *Neper*'s Logarithms are to those of Mr. *Briggs*'s Form, as 2.302585, &c. is to 1.000000, &c. therefore to find the Angle of the Rumb for the Logarithms of *Briggs*'s Form; this must be the Analogy, As 2302585, &c. : 2908882, &c. :: 1000000 = Radius : 12633114, &c. = the Tangent of the Angle 51° 38′ 9″, whose Logarithm is 10.101510428, &c. Wherefore in the *Rumb* Line that makes an Angle of 51° 38′ 9″ with the *Meridian*, the *common* (viz. *Briggs*'s) Logarithm-Tangents are the true Differences of Longitude.

12. But if a Table or Scale of *Logarithm-Tangents* be made by Extraction of the Root of the *infinitest Power*, whose *Index* is the Length of the Arch you put for *Unity* in the said Scale; then such a Scale of *Logarithm-Tangents* shall be the true *Meridian Line* required. If then the *Radius* or Tangent of 45°, be put = 1; and the Difference between Radius and any other Tangent T, be called t; so that it be R±t=T; the Logarithm of the *Ratio* of *Radius* to such *Tangent* will be

$$\frac{1}{n} \times t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \frac{t^5}{5}, \text{ \&c.} = \dot{t} = \text{the}$$

Logarithm of the Tangent T, when it is R+t=T.

Or $\frac{1}{n} \times t + \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{4} + \frac{t^5}{5}$, *&c.* when it is R—t=T. All which is evident from Chap. VI. Art. 6.

13. According to the same Doctrine (Art. 9. of the same Chap.) if T be any given Tangent, and t the Difference thereof from another Tangent; then the Logarithm of their Ratio will be $\frac{1}{n} \times \frac{t}{T} - \frac{t^2}{2T^2} + \frac{t^3}{3T^3} - \frac{t^4}{4T^4} + \frac{t^5}{5T^5}$, *&c.* when T is the lesser Term. But

$$\frac{1}{n} \times \frac{t}{T} + \frac{t^2}{2T^2} + \frac{t^3}{3T^3} + \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \text{ \&c.}$$

when T is the greater Term.

14. Again, it was ſhewn in the ſame Chap. VI. Art. 10, and 11. that this Series may be made to converge twice as ſwift, omitting all the even Powers, by putting $T=$ the *Sum* of the Tangents, and $t=$ the *Difference*, as above. For thus the Logarithm will be $\frac{2}{n}\times\frac{t}{T}+\frac{t^3}{3T^3}+\frac{t^5}{5T^5}+\frac{t^7}{7T^7}$, *&c.* = the Logarithm of the Ratio of thoſe two Tangents.

15. But the Ratio of T to t, or of the *Sum* of two Tangents to their *Difference* is the ſame as that of the Sine of the Sum of thoſe Arches to the Sine of their Difference; that is, again, as the Ratio of the *Co-Sine* of *middle Latitude* (or *half Sum* of the Arches) to the Sine of half the Difference. Therefore putting $S=$ Sine-Complement of *middle Latitude*; and s for the Sine of half the Difference of Latitudes; then $\frac{t}{T}=\frac{s}{S}$; and ſo the Series will become $\frac{2}{n}\times\frac{s}{S}+\frac{s^3}{3S^3}+\frac{s^5}{5S^5}+\frac{s^7}{7S^7}$, *&c.* wherein as the Differences of Latitude are ſmaller, fewer Steps will ſuffice.

16. So that, if the *Equator* be put for *Middle-Latitude*, then ſhall $S=$Radius, and $s=$Sine of the Latitude; then the Meridional Parts reckon'd from the Equator will be $\frac{2}{n}\times\frac{s}{r}+\frac{s^3}{3r^3}+\frac{s^5}{5r^5}+\frac{s^7}{7r^7}$, *&c.* Here becauſe $r=1$, therefore $\frac{2}{n}\times s+\frac{s^3}{3}+\frac{s^5}{5}+\frac{s^7}{7}$, *&c.* the half of which is $\frac{s}{n}+\frac{s^3}{3n}+\frac{s^5}{5n}+\frac{s^7}{7n}$, *&c.* = half the Logarithm of the Ratio of $r+s$ to $r-s$; that is, of the *verſed Sines* of the Diſtances from both Poles. See Chap. VI. Art. 11.

17. I ſhall exemplify this Series by ſhewing how the *Meridional Parts* anſwering to 30° Latitude, are to be found thereby, and that by the Logarithms, as follows.

The

The Logarithm of the Sine of 30 Degrees is	$s=0{,}5000000=$	.9.6989700
Multiply by		3
The Logarithm of	$s^3=$	.9.0969100
Substract the Logarithm of	$3=$	0.4771213
There remains the Logarithm of	$\frac{s^3}{3}=0.0416667=$	.8.6197887
And proceeding thus, you'll find the other Steps of the Series by their Logarithms, as here set down.	$\frac{s^5}{5}=0.0062500=$	.7.7958800
	$\frac{s^7}{7}=0.0011160=$	.7.0476920
	$\frac{s^9}{9}=0.0002171=$	.6.3364875
	$\frac{s^{11}}{11}=0.0000444=$	.5.6472773
The Sum of $s+\frac{s^3}{3}+\frac{s^5}{5}+\frac{s^7}{7}+\frac{s^9}{9}+\frac{s^{11}}{11}$, &c. is	$=0.5492942=$	.9.7398051
To which add the Log. of	$\frac{1}{n}=3437.74677078$,&c.$=$	3.5362739
The Sum is the Log. of the *Meridional Parts* for the Arch of 30°	*viz.* 1888,334, &c. $=$	3.2760790

18. And thus you may proceed to find the Length of any other Arch, or the Distance from the Meridian of its Parallel of Latitude; and so the *Meridian Line* may be constructed *de novo*, if any one thinks it worth while. But tho' it may be done with greater Accuracy and Exactness by these *infinite Series* than what we have by the *common Method*; yet the Table of *Meridional Parts*, or *Nautical Line* made from thence, now in Use, is abundantly sufficient for all the Purposes of Sailing; and consequently

quently renders a new Calculation thereof unnecessary, and a Matter of mere Curiosity. And indeed, since it has been shewn above (Art. 9, 10, 11.) the *Meridian Line* is no other than a Scale of the Logarithm-Tangents of the Half-Complements of the Latitude on the Rumb of 51° 38′ 9″, the Propositions of Sailing by this Method are resolvable by only the Canon of Logarithm-Tangents at the End of this Treatise; so that where this Canon is at hand, neither Meridional Table or Line can be necessary, as will appear by a Chapter particularly on this Subject, in the *Practical Part.* They who would see the Theory of this Branch of the Art, may peruse N°. 219. of the *Philosophical Transactions*, where they will find a most learned Tract on this Subject, wrote by Dr. *Halley*; from whence the Substance of this Chapter is taken.

CHAP. X.

Of the Construction *of a* Large *Logarithmic Scale, *exhibiting by* Inspection *a* distinct *Idea *of the* Nature *and* Agreement *of* *Numbers and their *Logarithms.*

1. IT is an Observation of the earliest Antiquity, that we have no *Ideas* in the *Mind* which were not *first* in the *Senses*; or that the *Senses* of the *Body* are the only *Inlets* or *Entries* by which the *Idea's* of *Objects* present themselves to the *Mind.* It follows then, that the *Idea's* must needs be so much the more *clear* and *distinct* in the *Mind*, and consequently be the better understood by it, by how much the more *fully*, *compleatly*, and *obviously* they first of all affect our *Senses.* Single uniform Objects easily insinuate

ſinuate themſelves, and make ſtrong and clear Impreſſions on the *Mind*, while thoſe which involve *Multiplicity* and *Variety* in their Nature, are proportionally more difficultly apprehended by the Senſes, and conſequently affect the Mind with *imperfect*, *confuſed*, and *ſlight Impreſſions*, which therefore muſt produce a more *perplex'd*, *obſcure*, and *uncertain Notion* or *Conception* of the Things themſelves.

2. From this Conſideration we may eaſily learn the Reaſon why, of all the vaſt number of Perſons who underſtand the *practical Uſe* of Logarithms, ſo very few of them know any thing of the *Nature* and *Conſtruction* of them. The Uſe of Logarithms is very obvious to the Senſes by eaſy Examples, but their Nature and Conſtruction lead the Mind too much upon the Contemplation of *Infinities* both of *Quantity* and *Variety*, which are Subjects too vaſtly *abſtruſe* and *remote* from *Senſe*, ever to be very commonly underſtood; unleſs ſome *Expedients* be contrived, which may help to facilitate ſo difficult an Affair.

3. And as there are principally but three Ways, whereby the Nature of Logarithms are explained, *viz.* by *Numbers*, *Species*, and *Lines*, the *Expedient* aforeſaid muſt be ſought in *one* of theſe *three Methods*. But *Numbers*, of all things elſe, exhibit the moſt *complex* and *various Idea*, therefore it cannot be hoped for from them. *Species*, on the other hand, are too *ſimple and conciſe a Repreſentation* of *ſo vaſt and various Idea's* as are thoſe of *Logarithms*, and have nothing of their *Reſemblance* in their *Form*; this Expedient therefore is not to be expected from this Head. It remains then, that *Right* or *Curve Lines* be uſed for the Purpoſe of explaining the Nature of Logarithms, by making the whole Matter obvious to the Senſes. And here indeed we ſhall find all that can be deſired, or is neceſſary to the Purpoſe.

4. For Example, let the complex *Idea* of a *thousand Units* be to be express'd most advantageously to a Mind unexercised about such *complex'd Notions*; if you do it by *Numbers* or Figures, it must be by this Expression 1000; but there are but *four Characters* to form an *Idea* of a *thousand* separate Objects in the Mind. In Species, this *great* and *complex* Idea is often represented by one Character alone, as x; or two, as $1+x$; which are still more obscure and absolutely unintelligible without some *Pre-Notions* of the Matter. But a Line may be taken of a Length sufficient, that by proper Divisions, all the thousand Units may be render'd distinct and obvious to the Sense, in any variety of Magnitude almost, but especially if they are equal to each other, as in the Case of Logarithms before us. Wherefore, since by *Lines* such *great* and almost *inconceivably complex Ideas* are capable of being represented to the Senses, *distinct* and separate in their *proper Parts*, and the *Doctrine of Logarithms* depending entirely on such *Ideas*, 'tis evident that by Means of Lines of a sufficient Length, the *Nature* and *Properties* of Logarithms, and the *Operations* thereby, may be render'd more *apparent* and *compleat* to the Senses, and so be better *understood in the Mind*, than by any other Means whatsoever.

5. The Consequence of all which, is, that the young *Tyro*, and all who would have a *true Notion* and most *clear Understanding* of this *abstruse and mysterious* Doctrine, should be assisted with such a *large lineal Construction of the Logarithms*, as hath been hinted at. And this, I hope, I have effected in the *large Diagram* on the Copper-Plate before you, with considerable Exactness, which I call the *Logarithmic Scale.* Wherein all that has been said in the *General Theory* aforegoing, or may follow in the *practical Operations of Logarithms*, is evident even to *Sense it self*, to a very wonderful degree, by a bare

Inspection, or a Glance of the *Eye* only; and therefore cannot but conduce to form a very distinct and agreeable *Idea* or Notion both of the *Theory* and *Praxis* of this admirable Art.

6. The Scale consists of *three principal Lines* which bound it; the *first* is AB on the Side, which is 22 *Inches* in Length, and is divided into 1000 equal Parts which represent the *natural Numbers* from 1 to 1000, all which are visible and distinct to the *naked Eye*; which Numbers therefore are affixed to every 10th Division. The second is AC at the Bottom, divided into 300 *equal Parts*, (as being but $16\frac{6}{10}$ Inches long.) These represent the *Logarithms*; if each of these *equal Parts* be supposed to represent 10, or 100, the Logarithms, then, for all Numbers under 1000, will be exhibited by Lines only to 4 or 5 Places of Figures, including the *Indices*. The *third* principal Line is the *Logarithmetic Curve* eDEFB, in which all the Lines of *Numbers* and *Logarithms* terminate, and whose *Genesis* and *Properties* have been before described. See Chap. III, IV, V, *&c.*

7. The Scale consists (or is made up) of Lines of Numbers, and others which are the *Complements* of the Logarithms to 3.0000. The first are *perpendicular* to the *Logarithmetic Line* AC in its several Divisions, and increase in Length in a *Geometrical Ratio*; as hath been observed: thus dividing AC into 3 equal Parts CG=GH,=HA, if GD be the 10th *proportional* Term from Ce=1, or Unity, then shall GD=10Ce=10, and HE=10GD=100Ce=100. Lastly, AB=10 HE=100 GD=1000 Ce=1000; as is evident from the *Nature* of the *Curve*, and by *Inspection*. The *Complements* of the *Logarithms* are the *Lines* which run across the *Diagram*, *parallel to* the *Line* of *Logarithms* AC; these at the Curve refer the *Numbers* to their proper Logarithms, and by means of those Lines thus crossing each other

in

in every Part of the *Scheme*, the *Logarithms* of *Number*, and the *Numbers* of *Logarithms* are most easily and obviously found, for the Extent of the Scale, by *Inspection* only.

8. It is not pretended that this (or any other) *Instrument* is capable of any great Exactness in *practical Operations*; 'tis sucffiient for my Design, if it only *illustrates* and *proves the Truth* of every part of the *Doctrine* of *Logarithms* to the Sense, and thereby renders it easier to the *Intellects* of young *Learners*. If the *Theory* before deliver'd be examined by this *Scale*, it will be found to agree with it to a *sensible Exactness*; it being as it were but the same thing *at large*. In the following Part, I shall shew the *Correspondence* and *mutual Agreement* between the *fundamental Operations* by Logarithms wrote by *Numbers*, and the same performed on this *Scale*; than which nothing more, that I know of, can be said or expected.

CHAP. XI.

Of the Construction *of the* Artificial LINES *of* NUMBERS, SINES, *and* TANGENTS, *by means of the* LOGARITHMS.

1. THE *Canon* of *Logarithms* being compleated and orderly *digested* in *Books*, tho' this was a greater Advantage than the *Mathematicians* of any former Age enjoy'd, yet not content to have a *bulky Book* of Logarithms, fit to be used in *Studies* and with the *Pen* only, the restless and unsatisfy'd Faculty of *Invention* in Men put them upon Contrivances to new-model and reduce the *voluminous Art*

to *Miniature*, that so it might be render'd more *easily* manageable, and more *universally* useful.

2. In the Pursuit of this Design they also very well succeeded; for since Numbers of any kind are capable of being represented by *Right-Lines*, they were not long unappriz'd that the *whole Body of the Canon* of *Logarithms* might be laid down and express'd in the Divisions of one *strait Line*. Mr. *Gunter*, Professor of Geometry at *Gresham-College*, was the first who took this matter in hand, and constructed such an *artificial Line* of Logarithms; which therefore from him was called (ever since) *Gunter*'s *Line*, or simply, the *Gunter*. The same Person also constructed *artificial* Lines of *Sines* and *Tangents*; and all those Lines, with some others laid down on a Scale, make what we commonly call *Gunter*'s-*Scale*.

3. The *Construction* of those *artificial Lines* is easy to be understood, and is as follows. Draw the *Right-Line* AB (Fig. VII.) which divide into 10 great equal Parts, as is there denoted by 1, 2, 3, 4, *&c.* and each of these into 10 others, and so on. Conceive these *several Divisions*, or *equal Parts*, to represent the Logarithms in the Canon for the natural Numbers. Now suppose the whole Length AB=10, then the first grand Divisions will be 1, 2, 3, 4, *&c.* But if AB=100, then the *first Divisions* will be 10, 20, 30, *&c.* and the *second Divisions* 1, 2, 3, *&c.* Again if AB=1000, the *prime Divisions* will be 100, 200, 300, *&c.* and the *secondary* Divisions, 10, 20, 30, *&c.* Suppose the latter Case, *viz.* AB=1000; then draw another Right-Line CD *equal* and parallel to AB, the *natural Line* of Logarithms.

4. Now in the Line CD, such Divisions are to be made as may represent the Places of the natural Numbers 1, 2, 3, *&c.* or 10, 20, 30, *&c.* or 100, 200, 300, *&c.* But neglecting the *Indices* of Logarithms,

rithms, 'tis plain, ſince the Logarithms of the Numbers 1, 10, 100; 2, 20, 200; 3, 30, 300; *&c.* are the ſame, the Diſtances of thoſe Numbers will be the ſame on the Line or Scale CD. And therefore ſince the Logarithm of 1, is = 0, the Number 1 muſt be placed at the very Beginning of the Line CD, from whence the Logarithms begin in the Line AB. Then becauſe the whole Line AB = 1000 = Logarithm of 10 = CD, therefore againſt the Logarithm of 2, which is = 301 in the Line AB, make a Diviſion in the Line CD, and by it place the Number 2. Again, becauſe the Logarithm of 3 is = 477 in AB, therefore correſpondent to the Point in AB, make another Diviſion in CD, and by it place the Number 3. The Logarithm of 4 is 602, therefore from 602 in AB make a Diviſion in CD, by which you muſt place the Number 4; and thus you proceed to find the Diviſions for the other Numbers to 10 in the Line CD, by the Logarithms of thoſe Numbers in the Line AB.

5. If the Diviſions in the Line CD now found for the Numbers 1, 2, 3, 4, *&c.* be ſuppos'd, inſtead of them, to be for the Numbers 10, 20, 30, 40, *&c.* then each of thoſe Diviſions may be ſubdivided into 10 others, by the *Logarithmic Parts* in the Line AB. Thus, becauſe the Logarithms of 11, 12, 13, 14, *&c.* are 41, 79, 113, 146, *&c.* therefore againſt theſe latter Numbers in the Line AB, make Diviſions in the Line CD, ſo ſhall the firſt grand Diviſion from 1 to 2 be divided again into 10 others. Again, becauſe the Logarithms of 21, 22, 23, 24, *&c.* are 322, 342, 361, 380, *&c.* therefore Diviſions made in CD againſt theſe Numbers in AB will finiſh the Subdiviſions of the Space from 2 to 3, in the ſaid Line CD. And thus proceeding, you may ſubdivide all the other *grand* or *prime Diviſions*, to the End of the Line.

6.

6. If your Lines be of so great Length, that these last Subdivisions in CD, are still of a Length capable of another *tenfold Division*; then the first grand Divisions must be reputed 100, 200, 300, *&c.* and so since the Logarithms of the Numbers 101, 102, *&c.* 201, 202, *&c.* 301, 302, *&c.* are 4, 8, *&c.* 303, 305, *&c.* 478, 480, *&c.* therefore if against these Parts in AB, you make Divisions in CD, there will ensue a triple Division of the said Line CD, which is more than is necessary for *Instrumental Uses*, and indeed cannot be done but only for the two or three first Divisions.

7. Thus have you seen the Construction of the *Artificial*, *Logarithmic*, or *Gunter's* *Line*, so famous in all Parts of the Mathematics. A Line which performs the Business of the whole Logarithmic Canon; since the Divisions of this Line have all the same Properties with regard to the natural Numbers on it, as the Logarithms of the Table have to the Numbers corresponding to them. 'Tis plain the Divisions and Relation of these two Lines AB, CD, are the same as At, and AT, in Fig. IV. and III. Consequently what has been said of those Lines heretofore will help to illustrate the *Theory* and Construction of the Lines now under Consideration. But since in Use the *Gunter* CD is supposed to be divided into an 100 Parts at least, therefore you always (or mostly) observe it of a double Length of that which is expressed in Fig. VII. which Length is commonly called *Radius*; and so the *Gunter* in common Use is said to be of a *double Radius*; because else the Divisions for the *nine Digits* would be wanting, since the Distance from 1 to 10 is equal to that from 10 to 100, as is evident from the foregoing Construction, and from the *Theory* of Logarithms.

8. Having thus shewn the *Construction* of the *Line of Numbers*, the *Construction* of the Lines of artificial *Sines* and *Tangents* easily follows; since, as before observed,

obſerved, the Logarithms of *Sines* and *Tangents* are nothing more than *common Logarithms* of ſuch Numbers as expreſs the *Sines* and *Tangents* of *each Minute* of the *Quadrant*.

Deg.	N. Sine.	Log.	N. Tang.	Log.
1	17	2418	17	2419
2	34	5428	35	5430
3	52	7188	52	7194
4	69	8345	70	8446
5	87	9402	87	9419
6	104	1.0192	105	1.0216
7	121	1.0858	122	1.0891
8	139	1.1435	140	1.1478
9	156	1.1943	158	1.1997
10	173	1.2396	176	1.2463
20	342	1.5340	363	1.5610
30	500	1.6989	577	1.7614
40	642	1.8080	839	1.9238
50	766	1.8842	1191	2.0761
60	866	1.9375	1732	2.2385
70	939	1.9729	2747	2.4389
80	984	1.9933	5671	2.7536
90	1000	2.0000	Infin.	Infin.

In the little Table above, the firſt Column contains the *Degrees*, the 2d and 4th the *Natural Sines* and *Tangents*, and the 3d and 5th Columns contain the *Logarithms* of thoſe natural *Sines* and *Tangents*, the *Indices* being *omitted*, and the *Radius* ſuppoſed = 10000.

9. Let three Lines be drawn, and let L = Line of *Logarithms*, or *double Radius* of 20000 equal Parts; S = a Line for *Sines*; and T = Line for *Tangents*; the two latter muſt be drawn *equal* and *parallel* to the firſt; as in the Conſtruction of the Line of *Numbers*. Then having graduated the Line L into 20000 equal Parts,

Parts, if againſt ſuch of thoſe Parts as are expreſs'd by the Numbers in the 3d Column, you make Diviſions in the Line S, and by thoſe Diviſions you place the Numbers in the firſt Column, you will then have the *artificial Line of Sines* S graduated for the firſt great Diviſions of 1, 2, 3, 4, 5, 6, *&c.* 10, 20, 30, *&c.* Degrees: After the ſame manner by the Table of *Logarithmic Sines* you find Numbers, from whence in the Line L you find Diviſions in the Line S for *Minutes*, and *Parts* of *Minutes.* And thus the Line of *artificial* or *Logarithmic Sines* is finiſhed.

10. Again, from the ſame Parts of the graduated Line L, as are found in the fifth Column of the *Tablet*, you make Diviſions in the Line T, and by them place the Numbers of the firſt Column, the Line T ſhall be the *artificial Line of Tangents* graduated for the firſt great Diviſions of 1, 2, 3, 4, 5, *&c.* 10, 20, 30, *&c.* Degrees. And the Subdiviſions for Minutes will be found as before directed. But tho' the *double Radius* on the Line L ſuffices for graduating the Line of Sines S, to the whole Length of 90 Degrees, becauſe all *Sines* are leſs than the *Radius* of a Circle, which is the greateſt Sine ; yet becauſe the *Radius of a Circle* and the *Tangent of 45 Degrees* are equal ; therefore 'tis evident the *Logarithms* of all Tangents greater than 45 Degrees, will exceed the Length of the Line L, as is plain from the fifth Column of the foregoing Tablet.

11. But ſince *Radius* is a *mean Proportional* between the *Tangent* of any *Arch*, and the *Tangent* of that *Arch's* Complement, it follows, that the *natural Tangents* in the *Geometric Ratio or Scale* are *equally diſtant* on each Side from the *Radius* or *Tangent* of 45 *Degrees:* and therefore the Logarithms of thoſe *natural Tangents*, which are *equidiſtant* on each Side the *Radius* or Logarithm of 45°, are alſo *equidifferent* ; that is, their *Differences* are *equal.* Thus the Difference of 44° and 46°, from Radius is the ſame ; and the

the Differences of 40° and 50°, 30° and 60°, 20° and 70°, *&c.* are reſpectively equal to each other; and conſequently the firſt great Diviſions from 1 to 45° on the Line T of *artificial Tangents*, will likewiſe ſerve for the *Co-Tangents* of thoſe Degrees, that is, for all the *Tangents* from 45° to 90°, reckon'd back again to the beginning of the Line T. And this is the Reaſon why on thoſe Lines of *Tangents*, you ſee the Numbers placed at each 10th Diviſion, thus 10|80, 20|70, 30|60, 40|50, 45, at the End. For otherwiſe the ſaid Line of Tangents, muſt be continued out to *double the Length* it now is, which would not be near ſo convenient.

12. The Numbers in the 2d and 4th Columns are the Diviſions on the *Gunter*, which correſpond to the *ſimilar Diviſions* on the *Lines* of *Sines* and *Tangents*; wherefore the *former* being already made, the *two latter* may alſo eaſily be conſtructed by means of that. And theſe things are all I judge neceſſary to be ſaid here concerning the Conſtruction of thoſe excellent *Lines* of *artificial Numbers*, *Sines*, and *Tangents*; and as to their Uſes, that will be a Subject to be treated of after the *practical Uſe* of the *Logarithms* themſelves is firſt explain'd and inculcated; for then the Uſe of theſe Inſtruments will be much better conceived and underſtood.

CHAP. XII.

Of the MANNER *of* uſing *the* TABLES *of* LOGARITHMS *in* PRACTICE; *and of the* Prerequiſites *thereto.*

1. THE *Logarithms* being made for *natural Numbers* by ſome of the Methods beforegoing, the next thing neceſſary was to diſpoſe them into ſome convenient *Order* or *Form* for *practical Uſes*. And ſuch a *Digeſt* or Collection of Logarithms, we call the *Logarithmic Canon* or *Tables*.

2. Theſe Tables are of *two Sorts*; the firſt contains the *Logarithms* of all *natural Numbers* from Unity or 1 to 10000, or 101000 (as thoſe *large Tables* of Mr. *Sherwin*.) In theſe, the general Manner or Form is ſuch as here expreſs'd in the Margin, which conſiſts of *two Columns*; in the firſt are placed the *Numbers*, in the ſecond the *Logarithms correſponding* thereto, with their *Indices*. And three of theſe *double Columns* fill a Page in common Books of this Form; and this, of all others, is the moſt obvious and eaſy as to its Uſe, which therefore can need no Explanation. For by Inſpection only is ſeen what Logarithm belongs to *any Number* within the Compaſs of the Table.

Numb.	Logarithms.
997	2.9986951
998	2.9991305
999	2.9995655
1000	3.0000000
1001	3.0004341
1002	3.0008677
1003	3.0013009

3. But tho' the aforeſaid Form be the moſt natural and obvious, yet it is not the moſt *artful* and *comprehenſive*; therefore another Form or Diſpoſition of the Tables for natural Numbers has been contrived more

more concise, or which takes up less room, and is yet as perfect and useful as the other; a *Specimen* of this Form I have here annexed.

N°.	Logarithms.				
	0	1	2	3	4
173	238046	238297	238548	238799	239049
174	240549	240799	241048	241297	241546
175	243038	243286	243534	243782	244030

In which the natural Numbers are placed in the *Side-Column* to the *left hand*, all but the *Units Place*, or *first Figure* of the Numbers, which is found in a parallel Column on the top of the Page, in the Order 0, 1, 2, 3 4, *&c.* as you see in the Specimen. By this means *one Column of Numbers* suffices for *one Page*, whereas in the other Form there are three such Columns, the whole Page itself consisting entirely of the Logarithms, which in this Case admits of five Columns; but the *Indices* are here omitted as not being necessary, since they are known by the Numbers. The manner of using this Form is yet very easy. For Example, to find the Logarithm of the Number 1742; against 174 in the Side-Column, and under the Units Place 2 at the top, I find the Logarithm 241048, and since the Number has four Places, the *Index* must be 3; where 3.241048 is the Logarithm compleat for the Number 1742. Thus the Logarithm of 1753=3.243782; and so for others.

4. But the Tables of Logarithms are yet capable of a further, and much more curious Improvement with regard to their Contraction or Conciseness; for since the Differences of Logarithms decrease as the Numbers increase, 'tis plain those will grow very small as these become very large; and consequently the *two* or *three first Figures* of the Logarithms to the left will be the *same* for *divers large Numbers together* in

the Canon. Thus for Inſtance, the Logarithms of all the Numbers between 4168 and 4266, have the two firſt Figures to the left the ſame in every one, *viz.* 62. So likewiſe all the Logarithms between the Numbers 9954 and 9977 have their firſt three Figures the ſame, *viz.* 998, the Difference of the Logarithms being only in the remaining Figures. The Figures of the Logarithms therefore may be reckoned of two ſorts, *viz.* ſuch as are *permanent* or the ſame, for certain Intervals; and ſuch as are *variable* or always altering. In this Form of the Canon now under Conſideration, theſe permanent Figures are printed but once for their reſpective Intervals, and that in the firſt Column of Logarithms next the Numbers the *variable* Figures *only* pertaining to each Logarithm make the Subſtance, or fill the whole Face of each Page, as in the Specimen here ſubjoin'd.

N°.	Logarithms.				
	0	1	2	3	4
132	12.0574	0903	1231	1560	1888
133	3852	4178	4504	4830	5156
134	7105	7429	7752	8076	8399
135	13.0334	0655	0977	1298	1619
136	3539	3858	4177	4496	4814
137	6721	7037	7354	7670	7987
138	9879	*0194	0508	0822	1136
139	14.3015	3327	3639	3951	4263
945	975.432	478	524	570	616
946	.891	937	983	*029	075
947	976.350	396	442	487	533
948	808	854	900	946	991
949	977.266	312	358	403	449

5. The Numbers here, as in the laſt *Form*, are, for the three firſt Figures to the left, found in the Side-

Side-Column, the other Figure at top. The Logarithms in this Specimen are 65, yet but ſix of them need be expreſſed at length, *viz.* thoſe for the Numbers 1320, 1350, 1390, 9450, 9470, 9490. The Logarithm for 1320 is thus wrote, 12.0574, to denote the two firſt Figures 12 (ſeparated by a Dot) are permanent thro' the Interval between 1320 and 1350, that is, they belong to the Logarithms of all the intermediate Numbers between thoſe two, and therefore need be expreſs'd only for the firſt; the other Part of the ſaid Logarithm which is variable, is expreſs'd alone in all the reſt. The *upper* Part of this *Specimen* or *Tariff* conſiſts of Logarithms having two Figures in the *permanent Part*; the lower part is an Example of Logarithms having three Figures in the *permanent Part*. To find a Logarithm therefore to any given Number, will alſo be very eaſy in this Form. Thus, ſuppoſe the Logarithm be ſought for in the Number 1323, 'tis found in this Manner. Take the permanent Part either *againſt* or *next above* the three firſt Figures 132, which here is 12, then againſt 132 and under 3 at the top, you find the *variable Part* 1560, to which prefix the *permanent Part* 12, and you have the Logarithm 121560, which with the *Index*, is 3.121560. Again, to find the Logarithm of 1374, take 13 the *permanent* Part next above the three firſt Figures 137, then againſt 137, and under 4 at top, you find the *variable Part* 7987, which annexed to the former Part 13, make 137987, and with the *Index*, 3.137987, the Logarithm ſought.

6. Thus alſo you proceed when the permanent Part conſiſteth of *three Figures*, where the *Intervals* are much ſhorter. One caution only is neceſſary, and that is, that you obſerve in thoſe Lines where there is found an *Aſteriſm* *, to join all the *variable Parts* in that Line *before* the *, to the *permanent Part next above*; and all *after* it, to that *next below*.

Thus

Thus in the 7th Line, and 2d Column, you ſee an *Aſteriſm* *, therefore the *variable Part* 9879 before it muſt be join'd to 13 the *permanent Part* next above it, to form the Logarithm 3.139879 for the Number 1380: But the *variable Parts* 0194, 0508, *&c.* *following it*, are to be annexed to 14 the *permanent Part next below*, to form the Logarithms 3.140194, 3.140508, *&c.* for the Numbers 1381, 1382, *&c.* The *Reaſon* of which I preſume muſt be *ſelf-evident* to every Reader.

7. I have been the more prolix on this *laſt Form*, leſt any *Obſcurity* or *Uncertainty* ſhould remain to diſcourage or prejudice the *young Tyro* againſt ſo *rare* and ſo *advantageous* a Contrivance. I ſay, *ſo rare*; becauſe I have never ſeen (amongſt many) above one, *viz. Sherwin*'s Canon, in this *Form*; and that, by reaſon of its *great Price*, is not very common. Tho' that Gentleman ſays in his Preface, he has in this *excellent Method* followed Dr. *John Newton* in his *Trigonometria Britannica*, a Book which I have not ſeen. The Advantage alſo of this Abbreviation is next to that of the Invention it ſelf; for hereby the *prolix* and *unwieldy Tables* (in their original Form) are reduced or abridg'd to one half the Bulk nearly; all the *ſuperfluous Part* being omitted, and nothing but what was neceſſary retained in this Canon.

8. According to this moſt excellent Abridgment therefore, I have firſt of all, that I know of, publiſhed the common Canon of Logarithms for Numbers from 1 to 10000; having taken the no ſmall Pains of tranſcribing the whole with my own hand from the aforementioned large Work of Mr. *Hen. Sherwin*, which is the moſt correct of any extant.

9. In this Form or Diſpoſition of the Canon, I have alſo publiſhed the Logarithms of Sines and Tangents; which thing hath not been done before in any Work great or ſmall, that I have ever ſeen or heard of. This makes the 2d Part of the Logarithmic Tables,

Tables, as mentioned Art. 2. And ſince they are here in a different Form from all others, it may not be unneceſſary to hint to the young Learner, that the Numbers expreſs'd in the Side-Column are the Degrees, and every 10th Minute, and the Numbers in the parallel Column at top are the Minutes between the 10ths; ſee the following Tariff of the Logarithms of Sines in this *Form* for the Minutes from 72 Degrees to 73.

D. ′	0	1	2	3	4
72. 0	978.206	247	288	329	370
10	615	655	696	736	777
20	979.019	059	100	140	180
30	420	460	500	539	579
40	816	855	895	934	973
50	980.208	247	286	325	364

To give an Example, let the Logarithm be ſought for the Sine of 72° 43′. Seek in the Side-Column 72° 40′, next above which is the *permanent Part* of the Logarithm 979 in the firſt Column of Logarithms; then againſt 72° 40, and under 3′ at top you find the variable Part 934, which annexed to the other makes 979934; to which prefix the Index (which is ſet at the top of each Page) 9, and the Logarithm is compleat, *viz.* 9.979934 for the *Sine* of 72° 43′; and thus you proceed for any other.

10. I have contracted the Logarithms to ſix Places of Figures only, as being ſufficient in common Uſe; the *natural Sines* and *Tangents* are not here inſerted, ſince when their Logarithms can be uſed, they themſelves are uſeleſs. Beſides, whenever they are required, they may be immediately had from their Logarithms. For Example, ſuppoſe I would know the natural *Sine* and *Tangent* for 38° 47′, the Logarithm, Sine and Tangent of this Arch, are 9.796836, and

and 9.905009. To theſe Logarithms (neglecting their *Indices)* find the natural Numbers, by the firſt Part of the Canon, they will be 626377 and 803542, the natural Sine and Tangent ſought. (See the Method below, Art. 15. for finding the Number of a given Logarithm.)

11. The Logarithm of the *Secant* of any Arch, as of 38° 47′, is thus eaſily obtain'd:

From the double Radius	20.000000
Subſtract the Co-Sine of 38° 47′ = . .	9.891827
There remains the Log. Secant of 38° 47′ =	10.108173

And thus the Logarithm of any other Secant may be found, and conſequently the *natural Secant*, or *natural Number* belonging thereto.

12. The Reaſon why the *Indices*, on the top of the Pages, of the Logarithms of Sines and Tangents, are ſo large, *viz.* 7, 8, 9, 10, 11, *&c.* is becauſe the *Radius* of the Circle was ſuppoſed to conſiſt of 10000000000 equal Parts, whoſe Logarithm therefore is 10.000000; wherefore a Number of ſuch *equal Parts* expreſſing the Sine of one Minute 1′, will conſiſt of 7 Places, whoſe Logarithm then will have its *Index* 6. The other Sines will conſiſt of 8, 9, and 10 Places, and ſo the Indices of their Logarithms will be 7, 8, 9, as in the Tables; thus alſo the Numbers expreſſing the *Tangents* in ſuch *equal Parts* will conſiſt of 7, 8, 9, 10, 11, 12, 13, and 14 Places of Figures, whence the *Indices* of their Logarithms muſt be 6, 7, 8, 9, 10, 11, 12, 13, according to Art. 11th and 12th of Chap. I. But ſince, as before ſaid, the firſt ſix Places of the Logarithms to the left are ſufficient, the reſt are rejected as ſuperfluous.

13. In future Operations there will be frequent Occaſion for what is called the *Arithmetical Complement* of a Logarithm, which is nothing but the *Difference*

ference between that Logarithm and Logarithm-Radius 10.000000.

Thus if from	10.000000
You ſubduct the Logarithm	4.877026
There will remain the *Arithmet. Comp.* =	5.122974

And this is done *mentally* in an Inſtant, by taking every Figure from 9, except the firſt, which you take from 10.

Note. If the Logarithm be of any Sine or Tangent, add 10 to the *Index* of the Arithmetical Complement, and it will be the Logarithm of the Co-Secant of the ſame Arch. For Example,

Suppoſe the Log. Sine of $3^{\circ}\ 48' =$ . . .	8.821342
The Arithmetical Comp. thereof is . . .	1.178658
to which add	10,
The Sum is the Log. Secant of $86^{\circ}\ 12' =$	11.178658

Which is evidently the *ſame Operation* as that in Art. 11. hereof.

14. From the *Theory* of *Logarithms* we learn, *that the Differences of great Numbers are proportional to the Differences of their Logarithms.* (See Chap. V. Art. 6, 7.) Therefore tho' the Canon of Logarithms goes no farther than the Number 10000, it may by this means be extended to the Number 10000000, or the Logarithm of any Number under 10000000 may be found by the preſent Canon, according to the following Rule.

Firſt; find the Logarithm of the *four firſt Figures* of the given Number, by the *firſt Part* of the Canon.

Secondly; ſubſtract this Logarithm from the Logarithm *next greater* or *next following* in the Table; and reſerve the *Difference.*

Thirdly; multiply the *Difference*, by the *remaining Figures* of the given Number; and from the *Product* cut off to the *Right hand*, *ſo many* Figures as there were *remaining* in the given Number.

Fourthly; add the *Remainder* of the *Product* to the Logarithm firſt found, the *Sum* ſhall be the Logarithm ſought.

For Example, let the Logarithm of the Number 127053 be ſought.

The Logarithm of the firſt 4 Figures } ..	127000 =	5.103804
The next greater Log. is of. .	127100 =	5.104146
The *Differences*	100	342

Wherefore ſay, as 100 : 342 :: 53 : 181,26

$$\begin{array}{r} 53 \\ \hline 1026 \\ 1710 \\ \hline 181|26 \end{array}$$

Add the firſt Logarithm 5.103804

The Sum is . . . . 5.103985 = the Logarithm ſought for the Number 127053.

Example 2. Required the Logarithm of the Number 3567894?

The Logarithm of the firſt 4 Figures } . .	3567000 =	6.552303
The Log. next following is of	3568000 =	6.552425
The Diff. of Numb. and Log.	1000	122

Then ſay, as 1000 : 122 :: 894 : 109,068

Add the firſt Logarithm . . 6.552303

the Sum is the Log. 3567894=6.552412, as was required. And thus you proceed for the Logarithm of any other greater Number than thoſe in the Canon.

15. By a Method *reverſe* to the foregoing, you find the *Number* correſponding to a *given Logarithm*; thus, ſuppoſe the given Logarithm be 3.567026, and you would know the Number thereof. Seek this Logarithm in the Table, and becauſe you there find it *exactly*, the Number 3690 correſponding thereto, is the Number ſought. But if the given Logarithm

Logarithm be *not exactly contain'd* in the Table, and more than 4 Figures be required, proceed as follows.

First; ſeek in the Table a Logarithm the *next leſs* to the given one, for the four firſt Figures of the Number ſought.

Secondly; ſubſtract this Logarithm from the *given one*, and annex to the Remainder, *ſo many Cyphers* as you ſeek Figures more than four.

Thirdly; take the *Difference* between the Logarithm juſt found and the *next greater*, by which divide the ſaid *augmented Remainder*, the *Quotient* annexed to the four firſt Figures ſhall compleat the *Number ſought*.

Example 1. Let there be ſought the Number to the Logarithm 5.103985, to ſix Places of Figures.

The Log. next leſs is of . . . 127000 =	5.103804
The given Logarithm	5.103985
The Difference or Remainder	181
The next greater Logarithm 127100 =	5.104146
Diff. between the *greateſt* and *leaſt* Log.	342

Since the Places in the Number ſought are 6, augment the firſt Remainder 181 with *two Cyphers*, and it will be 18100; then 342)18100(=53, which annex'd to the four Figures 1270 before found, make the Number 127053 required.

Thus alſo to the given Logarithm 6.552412, you may find its proper Number 3567894, and ſo for others.

16. In the ſame manner you proceed to find any Decimal Number to a Logarithm given, only in this Caſe the *Indices* of the Logarithms are neglected till the Operation is finiſhed, and then ſo many Figures are to be cut off from the Number found for Decimals, as the Index of the given Logarithm ſhall indicate; what is here ſaid relates to *plain* or *terminate Decimals* only; but there are other ſorts of Decimals which *circulate* or *perpetually repeat* one or

more Figures *ad infinitum:* And those Figures which thus *circulate* are *Repetends*, as in these Numbers, *viz.* 235,2222, &c. 27,83333, &c. 2.383838, &c. 702,6026026, &c. 0,2672326723, &c. Now these *Repetends* need be wrote but once if we slur the first and last Figures in each, to denote them such, as thus $235,\dot{2}$; $27,8\dot{3}$; $2,\dot{3}\dot{8}$; $70\dot{2},6\dot{0}$; $0,\dot{2}672\dot{3}$, &c. See more in my *Universal System*, or *Body of Decimal Arithmetic*, printed for Mr. *Noon*.

17. The Logarithms for the Repeating 9 Digits are made by adding the *Arithmetical Complement* of the Logarithm of 9, to the Logarithms of the said Digits, and are such as here annexed. The Logarithms of *pure compound Repetends* are made by adding the *Arithmetical Complement* of *so many* 9's, as there are Figures in the *Repetend*, to the Logarithms of those Numbers considered as *terminate*. Thus the Logarithm of the *Repetend* $\dot{3}6,\dot{5}$ is found as follows:

$\dot{1}=0.045757$
$\dot{2}=0.346787$
$\dot{3}=0.522879$
$\dot{4}=0.647817$
$\dot{5}=0.744727$
$\dot{6}=0.823909$
$\dot{7}=0.890855$
$\dot{8}=0.948847$
$\dot{9}=1.000000$

To the Logarithm of . . . . 36.5=1.562293
Add the *Arith. Complement* of . . 999=0.000434
The Sum is the Log. of the Repetend $\dot{3}6,\dot{5}=1.562727$

and thus proceed for others.

18. If the *Repetend* have any prefix'd terminate Part; then from such a *mixed Repetend* substract its *terminate Part*, and to the Logarithm of the *Remainder* add the Arithmetical Complement of the Logarithm of as many 9's as there are Figures in the *Repetend*. For Example, suppose you would find the Logarithm of the *mix'd Repetend* $26\dot{8}92,\dot{7}$, proceed thus:

From

From the *Repetend*	26892,7	
Subſtract the *terminate Part*	26	
To the Log of the Rem. . .	26890,1	=4.429592
add the *Arithm. Complem.* of	9999	=0.000043
The Sum is the Logarithm of	26892,7	=4.429635

Note, the *Indices* of the *Arithmetical Complements* are here (as in theſe Caſes they always muſt be) omitted.

CHAP. XIII.

Of the Origin *and* Conſtruction *of* SHAKERLY'S *and* STREET'S LOGISTICAL LOGARITHMS.

1. THE Uſe of *Logiſtical Logarithms* is in *Aſtronomical Calculations*, or *Sexageſimal Arithmetic*; but this ſort of Arithmetic, which taught the Rules of *Addition*, *Subſtraction*, *Multiplication*, *Diviſion*, &c. of *Sexageſimal Fractions*, viz. *Degrees*, *Minutes*, and *Seconds* of *Motion* or *Time*, was, in *former times*, called *Logiſtical Arithmetic*. And ſince the Invention of the *common Canon* of *Logarithms*, Mr. *Jeremiah Shakerly*, in his *Tabulæ Britannicæ*, firſt contrived from them a ſort of Logarithms adapted to the Rules of *Logiſtical Arithmetic*; and therefore gave them the Name of *Logiſtical Logarithms*. And ſince him Mr. *Thomas Street*, in his *Aſtronomia Carolina*, has invented another and more convenient *Form* of *Logiſtical Logarithms*, than *Shakerly*'s. And ſince Tables of both theſe ſorts of Logiſtical Logarithms are extant, 'tis proper to acquaint the Reader with the *Conſtruction* of both, which is as follows.

2.

2. Since *Logiſtical Logarithms* are altogether concerned in working Proportions of *Degrees*, *Minutes*, and *Seconds*, and more eſpecially of *Minutes* and *Seconds*, together with *Integers*, 'tis evident, if thoſe Sexageſimal Fractions were reduced into the *loweſt Denomination*, viz. of *Seconds*, &c. they might then be work'd with the *Logarithms* of *common Numbers*. Thus ſuppoſe the Proportion be $60' \, 00'' : 3' \, 47''$ :: $51° \, 29'' : 3' \, 15''$; if theſe *fractional* Numbers be reduced to *Seconds*, they will ſtand thus, $3600'' : 227''$ :: $3089'' : 195''$; wherefore 'tis plain, the Proportion in this Caſe may be wrought by the *common Canon of Logarithms*, as will be hereafter ſhewn. But then as here is no *Radius*, there will ariſe *double Trouble* in the Work by Logarithms in common Uſe, in firſt *adding* the Logarithms of the two *middle Terms*, and then *ſubſtracting* the Logarithm of the *firſt* from that *Sum*, in order to have the Logarithm of the *fourth Term ſought*; or elſe the *Complement Arithmetical* of the Logarithm of the *firſt Term* muſt be taken to perform all by *Addition only*. To avoid therefore the Trouble attending perpetually either of theſe Methods,

3. Mr. *Shakerly* makes this Proportion, as $3600''$: $227''$ :: 100000, *&c.* : 0.06305, *&c.* or with the Logarithms, thus;

As the Logarithm of $3600''$ = 3.556302
to the Logarithm of $227''$ = 2.356025
ſo the Logarithm of Radius 1.00000,&c.=10.000000
to the Logarithm of 0.06305,&c.= .8.799723

Now 'tis plain, the *two laſt Logarithms* perform the ſame as the *two firſt*, their Properties being the ſame; but the firſt of the two latter Logarithms is Radius 10.000000, which therefore he called the *Logiſtical Logarithm* of $3600''$ or one Degree, and conſequently the Logarithm .8.799723 is the *Logiſtical Logarithm* of $227''=3' \, 47''$. And thus the *Logiſtical Logarithm* of

of any other *Degrees*, *Minutes* and *Seconds* may be found; *viz.* by reducing them to *Seconds*, and then by taking from the *Logarithm* of thoſe *Seconds* the *conſtant Logarithm* 3.556302 of *one Degree* in *Seconds.*

4. Thus from the Log. of . . . 1′=60″=1.778151
Take the Log. of one Degree =3600″=3.556302
there remains the Logiſt. Log. of 1′=60″=.8.221849

Again, from the Log. of } . . 1° 3′ 00″=3780″= 3.577492
take the *conſtant Log.* 1° =3600″= 3.556302
there remains the *Logiſt. Log.* of } 1° 3′ 00″=3780″=10.021190

By theſe Examples you eaſily perceive how the *Logiſtical Logarithm* may be found for any Number of *Degrees*, *Minutes* and *Seconds*, in *Shakerly*'s Form. And ſince there are 60′ or 3600″ in *one Hour*, as well as in *one Degree*, therefore a Table of theſe *Logiſtical* Logarithms ſerves equally as well in the *Computation* of *Time* as *Motion*.

5. But if the Time of a whole Day or 24 *Hours* be the *Integer*, ſince there are but 1440′ therein; and 3600″ : 1440′ :: 2″½ : 1′; alſo ſince 60′ : 24H° :: 2′½ : 1H°; therefore if, thro' the courſe of theſe Tables of *Logiſtical Logarithms*, againſt every 2′½ you place the *Hours*, and againſt every 2″½ (or rather every 3d and 5th Second) you place the *Minutes* of an *Hour*; the Table of *Logiſtical Logarithms* for *Motion* and *Time* will be compleated; a *Specimen* of which, in this Form of Mr. *Shakerley*'s,

°	Motion	Time.
′	45	H°.
″	2700	XVIII.
0	987506	0
1	987522	
2	987538	
3	987554	1
4	987570	
5	987586	2
6	987602	
7	987618	
8	987634	3
9	987651	
10	987667	4

kerley's, I have before annexed. The firſt Column contains the Degrees, Minutes or Seconds; the ſecond Column the *Logiſtical Logarithms* thereof; and the third Column contains the Minutes of Time, the Hour being expreſs'd at the top, *viz.* XVIII. anſwering to the Motion of 45′, or 2700″.

6. But becauſe theſe *Logiſtical Logarithms* of Mr. *Shakerley*, conſiſt of *many Figures* throughout the Table, it miniſter'd occaſion to Mr. *Thomas Street* to contrive a more compendious and convenient Form of theſe Logarithms; and ſuch he invented, which tho' large at the beginning of the Table, yet immediately leſſen very faſt, and ſo continue to the End of the Table, or 60′ or 3600″; whoſe *Logiſtical Logarithm* is =0.

7. The Reaſon of which is manifeſt from the Manner of their Conſtruction, which is as follows. Suppoſe any Proportion of *Sexageſimal Numbers*, as that before made uſe of, Art. 2. *viz.* 60′ : 3′ 47″ :: 51′ 29″ : 3′ 15″, which reduced to Seconds, ſtands thus, 3600″ : 227″ :: 3089″ : 195″. Now in order to obtain a *vacant Term* in this Analogy, Mr. *Street* (inſtead of *Shakerly*'s Analogy 3600″ : 227″ :: 100000 : *&c.*) inverts the *firſt Ratio*, as thus; 227″ : 3600″ :: Unity; to a fourth Number whoſe Logarithm is reputed the *Logiſtical Logarithm* of the firſt Term 227″, as the Logarithm of Unity is of the ſecond 3600″. See the Work.

The Logarithm of . . . . 3′ 47″=227″=	2.356026
The Logarithm of . . 1°=60′=3600″=	3.556302
The Logarithm of Unity	0.000000
The Logiſt. Log. of . . . 3′ 47″=227″=	1200276

8. Whence it evidently appears, that to find the *Logiſtical Logarithm* of any Number of Seconds, you need only ſubſtract the common Logarithm of the Number of Seconds from the conſtant common *Logarithm* of 3600″, for the *Remainder* or Difference will

will be the *Logiſtical Logarithm* required, in *Street*'s Form.

Thus from the Logarithm of . . 3600″ = 3.556302
Subſtract the Log. of 51′ 29″ = 3089 = 3.489818
There remains the Logiſt. Log. of 51′ 29″ = . . 66484

Again from the ſaid Log. of . . 3600″ = 3.556302
Subduct the Logarithm of 3′ 15″ = 195 = 2.290035
There remains the Logiſt. Log. of 3′ 15″ = 1266267

9. From whence 'tis evident, that the greater the Number of Seconds is, the leſs will be the Logiſtical Logarithm thereof; till you come to the Number 3600″, whoſe Logiſtical Logarithm is nothing at all, as before ſaid. And thus it appears that the Logarithms of this kind of Mr. *Street*'s Form, have in them fewer Places of Figures, and are therefore more convenient for Uſe, by much, than thoſe of *Shakerly*'s Form, before deſcrib'd. And for that Reaſon I have choſe to give the Reader a Table of *Street*'s Logarithms rather than the other; and tho' Mr. *Leadbetter* has given us Tables of both ſorts, yet I think it intirely needleſs; ſince all the principal Uſes of *Shakerly*'s, are much better perform'd in *Street*'s Logiſtical Logarithms.

10. In ſhewing the Manner of making theſe Logarithms from the common ones, I have expreſſed them at large, *viz.* 1200276, 66484, 1266267, the Logiſtical Logarithms of 3′ 47″, 51′ 29″, and 3′ 15″, as *per* Art. 7, 8. Yet here two things are to be obſerv'd: Firſt, that the Index is not diſtinguiſhed from Logarithm itſelf, with a *Point*, as in the common ſort; but the remaining Figures, both of Logarithms and *Indices*, be they more or leſs, are reputed together, the Logiſtical Logarithm. Secondly, that two Places of Figures to the Right Hand in the Examples, are ſtruck off in the Table; the other being fully ſufficient for all the Purpoſes thereof. So

that in the Table you will find the ſaid *Logiſtical Logarithms* wrote 12003, 665, 12663, &c.

11. Mr. *Leadbetter* has taken the pains to continue the Table of *Logiſtical Logarithms* in *Street*'s Form, to 120′, or 2 Degrees; but as there is little occaſion for any more than the Logiſtical Logarithms of 60′ or 1 Degree; and when there is, the ſame Logarithms are capable of anſwering it, I have therefore continued them no farther than the Inventor did, *viz.* to 60′ or 3600″.

12. As to the Form of the Table, 'tis very eaſy to be underſtood, eſpecially if thoſe of the common Logarithms of *Sines* and *Tangents* before deſcribed are: The firſt Column of the Tables contains the *Degrees* or *Minutes*, or *Minutes* and *Seconds* in *Sexageſimals*, as in the *common Tables*, in the Order of *Denaries* or 10's, the nine Digits running along on the top of the Table, under which, in the ſeveral Columns, are the *Logiſtical Logarithms* abbreviated in the ſame manner as thoſe of *Sines* and *Tangents*; and are to be taken out according to the Directions there given, which ſee. In the laſt Column are contained the Numbers of the firſt reduced to Minutes or Seconds; and are to be compleated likewiſe with the *Digits* on the top of the Table. Theſe are referred to when the *Logiſtical Logarithm* of any integral Number is ſought. An Example or two will render all eaſy.

13. Let it be required to find the *Logiſtical Logarithm* of 2° 48′ or 2′ 48″. Firſt ſeek 2 40 in the firſt Column, and againſt it and under 8 at top, you find 310, which annexed to its proper permanent Part 13 in the firſt Column of Logarithms makes 13310, the Logiſtical Logarithm of 2 48, as required. Thus the Logiſtical *Logarithm* of 37′ 59″ is found to be 1986; and of 54′ 40″ to be 404; and of 58′ 37″ to be 101; and of 59′ 53″ to be 8. And thus the *Logiſtical*

giſtical Logarithm of 60′, is $=0$; and ſo will be a vacant Term in all Analogies for Operation.

14. Let it be required to find the Logiſtical Logarithm of the Number 584. Firſt ſeek 580 in the laſt or right-hand Column, and againſt it and under 4 at top you ſee *99, which ſhews it muſt be join'd to the following permanent Part 78 in the firſt Column of Logarithms, and therewith makes 7899, the Logarithm ſought. Thus the *Logiſtical Logarithm* of 1000, is found 5563; and of 3359, to be 301; and of 3596, to be 5; and that of 3600 is nothing.

15. This Table alſo equally ſerves for Time, whether for a *Day* and *Minutes*, or *Minutes* and *Seconds*; by help of the little Table at the End of the *Logiſtical Logarithms*, which ſhews what Parts of Motion in *Degrees* and *Minutes* correſpond to Time in *Hours* and *Minutes*. Thus againſt 13′ in Time, you ſee 33′ of *Motion*; againſt 1 Hour is $2^\circ\ 30'$; and the Motion anſwering to VII^h 43′, is $17^\circ\ 30' + 1^\circ\ 48' = 19^\circ\ 18'$; and that anſwering XXI^h 21′ is $52^\circ\ 30', + 0^\circ\ 53' = 53^\circ\ 23'$; and ſo for other Parts of Time; conſequently

The Logiſtical Logarithm of	H°.	Time.	is		as being the ſame for the Motion of		
	0	13′		20378		0°	33′
	1	00		13802		2	30
	7	43		4926		19	18
	21	21		507		53	23
	24	00		0		60	00

The Reaſon of all which is evident, from Art. 5. hereof.

16. Theſe *Logiſtical Logarithms* may in like manner be render'd applicable to Computations of *Money*, *Weights*, *Meaſures*, &c. Thus ſince $60^\circ : 20^s :: 3 : 1$, therefore the Shillings in a Pound correſpond to each 3d *Degree* or *Minute* of *Motion*. And again, ſince $960^{qrs} : 3600'' :: 1 : 3'\tfrac{3}{4}$; therefore to 1, 2, 3, 4, &c. *Farthings* there anſwers 3′, 7′, 11′, 15′, &c. Minutes

 of

of *Motion*; whence a Table may be form'd to ſhew the Logiſtical Logarithm for any Number of Farthings under 960, or one Pound. And thus you may proceed to frame a Table to render theſe *Logiſtical Logarithms* for Motion uſeful for the *Ounces* and *Pounds* in an *hundred Weight*; and that with great *Eaſe*, ſince 1792, the Number of Ounces in an *hundred Weight* Averdupoiſe, is nearly half the Number 3600″, the Seconds of the Table. Conſequently, the *Logiſtical Logarithm* of the *Double* of any Number of Ounces, is that required for the *Pounds* and *Ounces* equal thereto.

17. Or, laſtly, 'tis eaſy for any one who ſhall think it worth while, to calculate Tables of *Logiſtical Logarithms*, for any Species of Computation, peculiar to it ſelf. Thus with reſpect to Money, if the common Logarithms of all the Farthings under 960, be ſubducted from the conſtant Logarithm of 960, the *Remainders* will be the Logiſtical Logarithms of the Farthings in a Pound. The common Logarithms of all the Numbers of *Ounces* under 1792 ſubſtracted from the conſtant Logarithm of the ſaid Number 1792, will leave the *Logiſtical Logarithms* of all the Ounces in an hundred Weight. And thus you proceed to make *Logiſtical Logarithms* of any Kind you pleaſe, or may have occaſion for. Which is too eaſy a matter to require an Example, beſides thoſe above, Art. 7, and 8.

L O-

LOGARITHMOLOGY.

PART II.

The PRAXIS *of* LOGARITHMS, *Common and Logiſtical:*

With its APPLICATION *to Vulgar and Duodecimal Arithmetic, Plain and Spherical Trigonometry, Navigation, Menſuration of Superficies and Solids, Gauging, Timber-Meaſure, Aſtronomy, perform'd Numerically and Inſtrumentally.*

CHAP. I.

Of the Rules of ADDITION, SUBSTRACTION, MULTIPLICATION, *and* DIVISION *of the* Indices *of* LOGARITHMS.

1. AS the *Theory* of *Logarithms* has been largely premiſed, the *Rules* for a *Practical Uſe* and *Management* of thoſe *artificial Numbers* will from thence be eaſily underſtood, and the *Rationale* of *every Operation* be apparent to the intelligent Reader. Before we proceed to the Uſe of Logarithms as applied to *common Arithmetic*, &c. we muſt

must first consider some *previous Rules* and *Methods* which regard the *due ordering* and *working* those Numbers themselves, on account of the *Indices*, which admit of divers particular Cases in the *Rules* of *Addition*, *Substraction*, *Multiplication*, and *Division*: which therefore must be *exemplified* and *illustrated* as in the *Sequel* of this Chapter.

ADDITION *of* LOGARITHMS.

2. As in all *Species* of *Arithmetic* the first *fundamental Rule* is *Addition*, so in this of Logarithms, the Rules which require this *primary Operation* come first to be consider'd, and they are as follow.

(*Note*, I shall in this Place call the *Indices* of Logarithms of *whole Numbers*, *Integral Indices*; and those of the Logarithms of *Decimal or Fractional* Numbers, *Decimal Indices.*)

Rule I. If the *Indices* be *both Integral*, add them together for the *Sum required.*

Rule II. If the *Indices* are both *Decimal*, add them as before; and observe, (1.) if the *Sum* be *above* or *just* 10 or 100 (=tA; see Chap. IV. Theory) cast away 10 or 100. (2.) If the *Sum* be under 10 or 100 (=tA) add 10 or 100 thereto; and both the *Sum* in the *latter Case* and the *Remainder* in the *former*, will be *Decimal.*

Rule III. If the *Indices* be of *different* Kinds, *viz. Integral* with *Decimal*, the *Sum*, if under 10 or 100, is *Decimal*; if *just* 10 or 100, or *above*, cast away 10 or 100, the *Remainder* is *Integral.*

Rule IV. In case the *Sum* of *two* or *more decimal Indices* be less than 10=tA, the best way will be to use the larger *decimal Indices*, where tA=100; and then their *Sum* will be greater than 100, and so the *Reason* of the Operation will be more evident.

3. All these Rules relate to the *Indices* of the Logarithms only, and are exemplified as follow:

Ex-

Examp. I. To	3.513217	Exam. II.	2.317227
add	2.303196		0.850891
Rule I. Sum =	5.816413	Sum =	3.168118
Exam. III. To	.9.849235	Exam. IV.	.97.237406
add	.7.786822		.95.072607
Rule II. Part I. Sum=	.7.636057	Sum =	.92.310013
Exam. V. To	.4.273760	Exam. VI.	.62.346174
add	.3.067247		.21.300725
Rule II. Part 2. S. =	.17.341007	Sum =	.183.646899
Exam. VII. To.	6.372458	Exam. VIII.	.88.426703
add	2.673842		5.268402
Rule III. Sum=	.9.046300	Sum =	.93.695105
Exam. IX. To	5.206737	Exam. X.	8.426735
add	.8.312046		.92.105374
Rule III. Sum=	3.518783	Sum =	0.532109

		Or rather thus, per *Rule* IV.	
Exam. XI. add	.4.203106	Ex. XII.	.94.203106
	.1.312729		.91.312729
	.2.070346		.92.070346
Rule II. Part 2. S. =	.27.586181		.77.586181

'Tis poſſible theſe two laſt Examples may ſeem ſomewhat *obſcure*; but the *Reaſon* and *Truth* of each will appear, if they are wrought *at twice*, as follows:

	To	.4.203106		And	.94.203106
	add	.1.312729			.91.312729
Rule II. Part 2.	S. =	.15.515835	Rule II. Part 1.	S. =	.85.515835
	to which add	.2.070346			.92.070346
Rule II. Part 2.	S. =	.27.586181	Rule II. Part 1.	S. =	.77.586181

From whence appears the Reaſon why, in theſe Caſes, the *larger Indices* are preferable to the *ſmaller ones*.

Substraction *of* Logarithms.

4. In the *Subſtraction* of *Indices* the following *Rules* are to be obſerv'd, *viz.*

Rule I. If they are both *Integral*, and the *higher* one the *greater*, the *Remainder* will be *Integral*. But if the *lower* one be the *greater*, then add 10 to the *higher* one, and *ſubſtract*; the *Remainder* will be *Decimal*.

Rule II. If both *Indices* are *Decimal*; and the *higher* be the *greater*; the *Remainder* is *Integral*; if not, add 10 or 100 to the *higher* one, and ſubduct the *lower*, the *Remainder* will be *Decimal*.

Rule III. If the *Indices* are of *different* ſorts, *viz.* one *Integral* the other *Decimal*; then if the *higher* be *Integral*, add 10 or 100 to it, and ſubſtract; the *Remainder* will be *Integral*. But if the *higher* be *Decimal* and the *greater*, the *Remainder* will be *Decimal*; if *leſſer*, the *larger Indices* muſt be uſed.

5. The Rules alſo are eaſily deriv'd from the foregoing *Theory*, and are illuſtrated by the following Examples.

Exam. I.	From	5.816413	Ex. II.	3.168118
	Subſtract	3.513217		2.317227
Rule I.	Rem. =	2.303196	Rem. =	0.850891

Ex.

Ex. III. From	3.513217	Ex. IV.	0.850891
ſub.	5.816413		3.168118
Rule I. Rem. =	.7.696804	Rem. =	.7.682773
Ex. V. From	.7.503617	Ex. VI.	.94.420345
ſub.	.3.467306		.92.673457
Rule II. Rem. =	4.026311	Rem. =	1.746888
Ex. VII. From	.3.467306	Ex. VIII.	.92.673457
ſub.	.7.503617		.94.420345
Rule II. Rem. =	.5.963689	Rem. =	.98.253112
Ex. IX. From	5.816413	Ex. X.	6.2067347
ſub.	.8.132700		.94.1535012
Rule III. Re. =	7.683713	Rem. =	12.0532335
Ex. XI. From	.8.132700	Ex. XII.	.94.278769
ſub.	5.816413		6.165348
Rule III. Part 2. } Re. =	.2.316287	Rule III. Part 2. } R. =	.88.113421

MULTIPLICATION *of* LOGARITHMS.

6. In Chap. IV. Art. 19th and 20th of the *Theory*, the Rules for multiplying the *Indices* of Logarithms (of *Pure Fractions* eſpecially) are demonſtrated; and are,

Rule I. If the *Index* be *Integral*, multiply as uſual; the Product ſhall be *Integral*.

Rule II. If the *Index* be *Decimal*, make the Logarithm of *Unity*, or tA=100, then ſhall the *Index*

be of the *larger Sort*, which in this Case, will be more convenient for Use; and then according as you multiply by 2, 3, 4, 5, 6, *&c.* you must reject 100, 200, 300, 400, 500, *&c.* from the *Product*, the *Remainder* thereof will be *Decimal.*

Ex. I. Mult.	3.420673	Ex. II.	5.700672
by	2		4
Rule I. Prod. =	6.841346	Product =	22.802688
Ex. III. Mult.	.96.130126	Ex. IV.	.91.034106
by	2		3
Rule II. Pr. =	.92.260252	Prod. =	.73.102318
Ex. V. Mult.	.84.034121	Ex. VI.	.70.061050
by	4		5
Rule II. Pr. =	.36.136484	Prod. =	.50.305250

In this last Example, the *Index* .70. × 5 = 350; but since from 350 you cannot reject 400, as *per* Rule II; therefore it must be 400—350 = .50. which subducted from 99, leaves 49; which shews that 49 *Cyphers* above 100, that is, 149 *Cyphers* are to be prefix'd; and this you are to understand in all Cases where xL is less than $\overline{x-1} \times$ tA. See Chap. IV. Art. 20th, of *Theory*. Or thus, in general let tA = 10 or 100; then when xL is *greater* than $\overline{x-1} \times$ tA, then it will be xL,—$\overline{x-1} \times$ tA—9 or 99, is equal to the Number of Cyphers to be prefix'd. But if $\overline{x-1} \times$ tA is *greater* than xL, then it will be $\overline{x-1} \times$ tA—xL —9 or 99, = to the said Number of Cyphers.

D i-

DIVISION *of* LOGARITHMS.

7. This is but the *Reverſe* of the foregoing Operation; and the Rule for decimal *Indices* the reverſe to that; which is alſo derived from Chap. IV. Art. 21ſt, and 22d of the *Theory*, which ſee.

Rule I. If the *Index* be *Integral*, divide as uſual; and the *Quotient-Index* will be *Integral.*

Rule II. If the *Index* be *Decimal*, uſe the *larger Sort*; and then adding to the ſaid Index 100, 200, 300, *&c.* divide by 2, 3, 4, *&c.* the Quotient-*Index* will be *Decimal.*

Ex. I.	Divide 6.841348		Ex. II.	22.802688
	by	2		4
Rule I.	Quot.=3.420673		Quotient	= 5.700672
Ex. III.	Div. .92.260252		Ex. IV.	.83.102300
	by	2		3
Rule II.	Qu.=.96.130126		Quot.	= .94.367433
Ex. V.	Divide .36.136484		Ex. VI.	.30.305250
	by	4		5
Rule II.	Qu.=.84.034121		Quot.	= .86.061050

CHAP. II.

Of MULTIPLICATION *and* DIVISION *of* WHOLE NUMBERS *and* DECIMALS *by* LOGARITHMS.

1. HAving before ſhewn the Method of finding the *Logarithms* of all kinds of *Numbers*, both *Integers* and *Decimals*, and alſo of fitting and adjuſting proper *Indices* thereto; and in the foregoing Chapter, the *Arithmetical Management* thereof in *all Varieties*: I ſhall now apply the *Uſe* of thoſe excellent Numbers in the *Rules* of *Arithmetic*; and firſt in the *Multiplication* and *Diviſion* of *Whole Numbers* and *Decimals*.

2. From the foregoing Theory (ſee Chap. I. Art. 10; and Chap. III. Art. 12.) we obtain this eaſy and obvious Rule for the *Multiplication* of all kind of Numbers by *Logarithms*,

viz. { To the *Logarithm* of the *Multiplicand*,
Add the *Logarithm* of the *Multiplier*;
The *Sum* is the *Logarithm* of the *Product*.

Examples of INTEGERS.

			Logarithms.
3. Example I. Multiply	12	=	1.079181
by	8	=	0.903090
Product	96	=	1.982271

Ex-

				Logarithms.
Example II.	Multiply	127	=	2.103804
	by	12	=	1.079181
	Product	1524	=	3.182985

Example III.	Multiply	526	=	2.720986
	by	100	=	2.000000
	Product	52600	=	4.720986

Example IV.	Multiply	9876	=	3.994581
	by	517	=	2.713490
	Product ..	5105892	=	6.708071

Example V.	Multiply ...	987600	=	5.994581
	by	517000	=	5.713590
	Product	510589200000	=	11.708071

4. *Examples of* MIX'D NUMBERS.

Example VI.	Multiply	7,5	=	0.875061
	by	10	=	1,000000
	Product	75	=	1.875061

Example VII.	Multiply	12,4	=	1.093422
	by	3,6	=	0.556302
	Product ...	44,64	=	1.649724

Ex-

				Logarithms.
Example VIII.	Multiply . .	0,762	=	.9.881955
	by	570	=	2.755875
	Product . .	434,34	=	2.637830
Example IX.	Multiply	36,5	=	1.562293
	by	0,00019	=	.6.278754
	Product	0,006935	=	.7.841047
Example X.	Multiply	473	=	.9.674861
	by	63	=	1.803705
	Product	30.1	=	1.478566
Example XI.	Multiply	6	=	0.823909
	by	,5	=	.9.744727
	Product	3,76	=	0.568636
Example XII.	Multiply . . .	21,23	=	1.326541
	by	42,6	=	1.623458
	Product	900,718	=	2.949999

5. *Examples of* Pure Decimals.

Example XIII.	Multiply	,12	=	.9.079181
	by	,8	=	.9.903090
	Product	,096	=	.8.982271

Ex-

				Logarithms.
Example XIV.	Multiply . .	,0097	=	.97.986772
	by	,00021	=	.96.322219
	Product	000002037	=	.94.308991

Example XV.	Multiply	,00420	=	.97.623458
	by . . .	,00008	=	.95.948847
	Product	,0000003735	=	.93.572305

Example XVI.	Mult.	,00000085	=	.93.929419
	by	,0000012	=	.94.079181
	Product	,000000000000102	=	.88.008600

Ex. XVII.	Mult.	,00000000075	=	.90.875061
	by	,000006	=	.94.778151
	Product	,000000000000000045	=	.85.653212

6. DIVISION *by* LOGARITHMS.

In the ſame Part of the *Theory* referr'd to (Art. 2.) for the Rule by which *Multiplication* is performed by *Logarithms*, you will likewiſe find the Demonſtration of the following Rule of *Diviſion* of Numbers by *Logarithms* ;

viz. From the Logarithm of the *Dividend*,
Subſtract the Logarithm of the Diviſor ;
The Remainder is the Logarithm of the *Quotient*.

Ex-

Examples of INTEGERS.

				Logarithms.
7. Example I.	Divide	96	=	1.982271
	by	12	=	1.079181
	Quotient	8	=	0.903090
Example II.	Divide	1524	=	3.182985
	by	172	=	2.103804
	Quotient	12	=	1.079181
Example III.	Divide	52600	=	4.720986
	by	526	=	2.720986
	Quotient	100	=	2.000000
Example IV.	Divide	5105892	=	6.708071
	by	517	=	2.713490
	Quotient ...	9876	=	3.994581
Exam. V.	Divide	510589200000	=	11.708071
	by ..	987600	=	5.994581
	Quotient	517000	=	5.713490

8. *Examples in* MIX'D NUMBERS.

Example VI.	Divide	75	=	1.875061
	by	7,5	=	0.875061
	Quotient	10	=	1.000000

				Logarithms.
Example VII.	Divide	44,64	=	1.649724
	by	12,4	=	1.093422
	Quotient ...	3,6	=	0.556302
Example VIII.	Divide	434,34	=	2.637830
	by	570	=	2.755875
	Quotient ..	,762	=	.9.881955
Example IX.	Divide ...	,006935	=	.7.841046
	by	36,5	=	1.562293
	Quotient	,00019	=	.6.278753
Example X.	Divide	30,1	=	1.478566
	by	$\cancel{6}\cancel{3}$	=	1.803705
	Quotient	,473	=	.9.674861
Example XI.	Divide	$\cancel{3},7\cancel{0}$	=	0.568636
	by	$\cancel{6}$	=	0.823909
	Quotient	$\cancel{5}$	=	.9.744727
Example XII.	Divide ...	$\cancel{9}00,71\cancel{8}$	=	2.949999
	by	$4\cancel{2},\cancel{0}$	=	1.623458
	Quotient ..	$2\cancel{1},2\cancel{3}$	=	1.326541

Q 9. Ex-

9. *Examples in* PURE DECIMALS.

				Logarithms.
Example XIII.	Divide	,096	=	.8.982271
	by	,12	=	.9.079181
	Quotient	,8	=	.9.903090
Exam XIV.	Divide	,000002037	=	.94.308991
	by	,0097	=	.97.986772
	Quotient	,00021	=	.96.322219
Ex.XV.	Divide ..	,0000003735	=	.93.572305
	by	00008	=	.95.948847
	Quotient	,00470	=	.97.623458
Ex.XVI.	Div.	,0000000000102	=	.88.008600
	by ..	,00000085	=	.93.929419
	Quotient	,0000012	=	.94.079181
Ex.XVII.	Div.	,000000000000045	=	.85.653212
	by ..	,000006	=	.94.778151
	Quotient	,0000000075	=	.90.875061

10. I think theſe Examples in the *Multiplication* and *Diviſion* of *Numbers* by *Logarithms*, are ſufficient to inſtruct any *docible Genius* in his Practice herein; and as the latter are but the Converſe of the former, ſo they mutually illuſtrate and prove the Truth of each other reſpectively.

CHAP.

CHAP. III.

Of raising POWERS, *and the* EXTRACTION *of* ROOTS *by* LOGARITHMS.

1. FROM the Theory of Logarithms (Chap. I. Art. 12. and Chap. III. Art. 13.) we have an evident Rule for the *Involution of Numbers*, or *raising* them to any proposed *Power* by means of Logarithms; which is this,

viz. { Multiply the *Logarithm* of the *given Number* by the Index of the Power, *viz.* 2, 3, 4, 5, *&c.* the *Product* shall be the *Logarithm* of the Power, *viz.* the *Square*, *Cube*, *Biquadrate*, *Surfolid*, &c. *Power* of the said *given Number*.

Examples in INVOLUTION.

2. Example I. What is the Square of the Number 32?

Multiply the Logarithm of	32=1.505150
by the Index of the Power	2
The Prod. is the Log. of the Square	1024=3.010300

Ex. II. Required the Square of	3,2=0.505150
Multiply by	2
The Prod. is the Log. of the Square	10,24=1.010300

Ex. III. Required the Square of . .	,32=.9.505150
Multiply by	2
The Product is the Answer . .	,1024=.9.010300

Example IV. Required the ſeveral Powers of the Number 1.05 to the Surſolid?

1. The Logarithm of . . . . . 1.05=0.021189
Multiply by 2

The Product is the *Square* . . . 1,1025=0.042378

2. Multiply the Logarithm of 1.05=0.021189
by 3

The Product is the *Cube* 1,157625=0.063567

3. Multiply the Logarithm of 1.05=0.021189
by 4

Product is the *Biquadrate* 1.21650625=0.084756

4. Laſtly, Multiply the ſame . . 1.05=0.021189
by 5

Product is the *Surſolid* 1.2773315625=0.105945

Example V. Required the *ſurſolid* Power of the Number ,0006?

Multiply the Logarithm of ,0006=.96.778151
By the Index of the Power 5

The *Surſol.* ,00000000000000007776=.83.890755

Example VI. What is the *Cubo-Cube* Power of ,08?

Multiply the Logarithm of . . ,08=.98.903090
by the Index of the Power . . . 6

The *Cubo-Cube* Power ,000000262144=.93.418540

Example VII. What is the 57th Power of the Number ,99?

Multiply

Multiply the Logarithm of . . .	,99=.9.995635
by the Index of the Power . . .	57
	69969445
	49978175
The 57th Power is . . . ,56389, &c.=	.9.751195

3. There is another way of raising the Powers of *Decimal Numbers* by Logarithms, and it is thus ;

viz. { Multiply the *Arithmetical Complement* of the Logarithm of the given Fraction by the Index of the Power, the *Arithmetical Complement* of the Product is the Logarithm of the Power sought.

And this in many Cases, (as when the *Index* of the *Power* is a *mix'd Number* or *pure Decimal)* will be found most certain and ready. Thus in the last Example this way ;

Example VIII. What is the 57th Power of the Number ,99 ?

The Logarithm of	,99=.9.995635
The *Arithmetical Complement*	0.004365
which multiply by . . .	57
	30555
	21825
The Product . . .	0.248805
The Arithmet. Comp. is the Log. of the Power }	,56389, &c.=.9.751195

Example IX. What is the $\frac{1}{4}$th or 25th Power of ,2 ?

The

The Logarithm of . . .	,2=.9.301030
The *Arithmetical Comp.* thereof	.0.698970
which multiply by the Index	,25
	3494850
	1397940
The Product . . .	0,17474250
The Arithmet. Comp. is the Log. of the Power } ,66874, &c.	=.9.825258

Example X. What is the 6,25th Power of ,0032?

The Logarithm of . . . ,0032	=.7.505150
The Arithmetical Complement	2.494850
which multiply by . . .	6.25
The Product is . . .	15.5928125
The Arithmetical Complem. of which is	.84.4071875

And the Number anſwering thereto, *viz.* ,000000000000000025538 is the 6.25th Power of ,0032.

4. Evolution *or* Extraction *of* Roots *by* Logarithms.

This is done by a Rule, the *converſe* of that for *Involution*, in Art. 1ſt;

viz. { Divide the Logarithm of the Power by the Index of the Root, the Quotient ſhall be the Logarithm of the Root ſought.

Examples in Evolution.

Example I. What is the *Square Root* of 1024?

Divide the Logarithm of . . . 1024	=3.010300
By the Index of the Root	2
The Quot. is the Log. of the ſquare Root 32	=1.505150

Ex-

Example II. Required the *Cube Root* of 1,157626?

Divide the Logarithm of	1,157625=0.063567
By the Index of the Root	3
The Qu. is the Log. of the *Cube R.*	1,05=0.021189

Example III. What is the *surſolid Root* of the Power ,0000000000000007776?

The Logarithm thereof is . . .	.83.890755
which divide by the Index	5
The Quot. is the Log. of the Root ſought	,0006=.96.778151

Example IV. What is the *Cubo-Cube* Root of the Power ,000000262144?

The Log. of . . ,000000262144	=.93.418540
Which divide by the Index	6
The Root ſought is . . . ,08	=.98.903090

Example V. What is the 57th Root of the Power ,56389, &c.?

The Logarithm of . . . ,56389, &c.	.9.751195
which divide by . . .	57
The Root required is . . . ,99	=.9.995635

5. Another different way to extract the Root of Decimal Numbers is the converſe of that in Art. 3d, hereof.

viz. Divide the *Arithmetical Complement* of the Logarithm of the *Decimal* given by the Index of the Root required, the Arithmetical Complement of the Quotient is the Logarithm of the Root ſought.

Example VI. What is the ,25th Root of the Power ,66874, &c.?

The Logarithm thereof is . . .	.9.825258
The Arithmetical Complement is	0.174742
which divide by . . .	,25
The Quotient . . .	0.698970
The Arith. Comp. is the Log. of the Root . .	,2=.9.301030

Example VII. Required the 6.25th Root of the Power ,0000000000000025538 ?

The Logarithm thereof is . . .	.84.407187
The Arith. Comp. of which is	15.592813
which divide by . . .	6.25
The Quotient is . . .	2.494850
the Arith. Complem. thereof is . . .	.7.505150

the Logarithm of ,0032 the Root sought.

Example VIII. What is the *Cubo-Cube* or 6th Root of the Power ,1 ?

The Logarithm of . . .	,1=.9.000000
the Arith. Comp. thereof . .	.0.999999
which divide by the Index	6
The Quotient . . .	0.166666
The Arith. Comp. is the Log. of the Root	,68129, &c. =.9.833333

6. Thus you see the great Use of Logarithms in extracting the Roots of a given Power, which tho' a thing so very difficult by the Rules of common Arithmetic, is yet render'd most easily practicable by this excellent Art; yea 'tis easy to make it appear, that the *Extraction of Roots* is not only most expeditiously performed, but hath a more universal Perfection in this Method, than in any other.

CHAP.

CHAP. IV.

Of the various RULES *of* PROPORTION, *and of finding* MEAN PROPORTIONALS *by* LOGARITHMS.

1. FROM the *Theory* 'tis evident, that the *golden Rule*, or *Rule* of *Proportionals*, is wrought in Logarithms by only the *Addition* and *Substraction* of the Logarithms of the Terms of the Proportion. And if the Proportion be direct, the Rule is thus;

viz. { Add the Logarithms of the *second* and *third* Terms, from that *Sum* substract the Logarithm of the *first*; the Remainder is the Logarithm of the *fourth* sought.

Examples in the GOLDEN RULE *Direct.*

Example I. If 12 Pounds cost 1 *l.* 15 *s.* 9 *d.* what will 173 Pounds cost?

The Logarithm of	12=1.079181
To the Log. of 1 *l.* 15 *s.* 9 *d.*=1,7875=	0.252246
Add the Logarithm of	173=2.238046
The Sum	2.490292
Substract the first, there remains 25,77 *l.*=	1.411111

Wherefore the Answer is 25,77 *l.*=25 *l.* 15 *s.* 4 *d.* $\frac{3}{4}$.

2. But since if you *divide* by any Number, or *multiply* by its *Reciprocal*, the Effect is the same; and also since the Arithmetical Complement of any Number, is but the Logarithm of the *Reciprocal* of

that Number; therefore it follows, that where the *Substraction* of a Logarithm is required in any Operation, if you take the *Arithmetical Complement* of that Logarithm, the whole may be performed by *Addition* only. Thus in the foregoing Example.

Add { the Ar. Comp. of the Log. of 12.=.8.920819
the Log. of . . . . 1,7875 *l*.= 0.252246
the Log. of . . . . 173= 2.238046

The Sum is the Anſwer . . . . 25,77 *l*.=1411111
the ſame as before.

Example II. If 2 *C*. 1 *q*. 21 *l*. 14 *oz*. coſt 5 *l*. 17 *s*. 8 *d*.¼. what will 31 *C*. 2 *q*. 26 *l*. 15 *oz*. coſt?

Then 2 *C*. 1 *q*. 21 *l*. 14 *oz*.=2.4453 A.Com. .9.611668
The Log. of 5 *l*. 17 *s*. 8 *d*. ¼ = 5.8844 = 0.769702
The Log. of 31 *C*. 2 *q*. 26 *l*. 15 *oz*. =31,7405 } . . . . =1.501614

The *Sum* is the Logarithm of 76,381 *l*.=1.882984
Therefore 76,381 *l*.=76 *l*. 7 *s*. 7 *d*.¼ is the Anſwer.

3. *Of the* Rule of Three *Inverſe*.

In this Caſe you muſt take the *Arithmetical Complement* of the *third* Term, and add it with the Logarithms of the other two as before; ſo ſhall the Sum be the Logarithm of the Anſwer.

Example. Suppoſe a Field feeds 18 Horſes for 7 Weeks, how long will it feed 42, at that rate?

Add { the Logarithm of . . . . 18=1.255272
the Logarithm of . . . . 7=0.845098
the Arith. Comp. of the Log. of 42=.8.376751

The Sum is the Logarithm of Anſwer 3=0.477121

4.

4. *Of the double Rule of Three, or Rule of* Five Numbers.

As in Queſtions of this ſort, there are always *three conditional* or *ſuppoſed Terms*; the firſt of which is the *principal Cauſe* of *Gain*, *Loſs*, *Action*, &c. the *ſecond* denotes the *Time*, *Diſtance*, &c. and the third is the *Gain*, *Loſs*, or *Action*, &c. So let theſe three Terms be denoted by the Capitals P, T, G. Alſo there are *three* other Terms (ſimilar to the three former) which make the Queſtion to be reſolv'd; and let theſe be repreſented by the ſmall Letters p, t, g. Two of which are always given, and the other is ſought. But ſince $P : G :: p : \frac{Gp}{P}$; and again, ſince $T : \frac{Gp}{P} :: t : g$; therefore $Tg = \frac{tGp}{P}$, and conſequently $PTg = tpG$; from which general Theorem we can eaſily find p, t, or g. Thus, I. $\frac{PTg}{Gt} = p$; and II. $\frac{PTg}{Gp} = t$; and III. $\frac{tpG}{PT} = g$. The Contrivance of theſe excellent Theorems we owe to the late Mr. *Ward*, of *Cheſter*.

5. I ſhall exemplify Queſtions in this Rule by Examples, as follow.

Example I. If 100 *l.* in 12 Months gain 6 *l.* what will 350 *l.* gain in 9 Months?

Here P=100, T=12, G=6; alſo p=350, t=9, to find g.

Add the Logarithm of . . . . G=6=0.778151
Add the Logarithm of . . . . . p=350=2.544068
Add the Logarithm of . . . . t=9=0.954242

The Sum is the Logarithm of . . . . Gpt=4.276461

 Add

$$\text{Add}\begin{cases}\text{the Logarithm of} \ldots\ldots & P=100=2.000000\\ \text{the Logarithm of} \ldots\ldots\ldots & T=12=1.079181\end{cases}$$

The Sum is the Logarithm of . . . . . $PT=3.079181$
Then from the Logarithm of . . . . $Gpt=4.276461$
Subduct the Logarithm of . . . . $PT=3.079181$

There rem. the Log. of $\frac{Gpt}{PT}=g=15{,}75=1.197280$

Wherefore the Anſwer is $15.75\,l.=15\,l.\ 15\,s.$

6. Example II. If 36 Buſhels will ſerve 24 Horſes 48 Days, how long will 126 Buſhels ſerve 96 Horſes?

Here $P=24$, $T=48$, $G=36$; alſo $p=96$, and $g=126$; to find t.

$$\text{Add}\begin{cases}\text{the Logarithm of} & P=24=1.380211\\ \text{the Logarithm of} & T=48=1.681241\\ \text{the Logarithm of} & g=126=2.100370\end{cases}$$

The Sum is the Log. of . . $PTg=5.161822$ from

$$\text{Add}\begin{cases}\text{the Logarithm of} & G=36=1.556302\\ \text{the Logarithm of} & p=96=1.982271\end{cases}$$

The Sum is the Log. of . . . . $Gp=3.538573$ ſubſ.

Therefore the Log. of $\frac{PTg}{Gp}=t=42=1.623249$

The Anſwer, *viz.* 42 Weeks.

7. Example III. At the Rate of $4\frac{1}{2}\,l.$ *per Cent. per Ann.* what Principal will produce $35\,l.\ 15\,s.$ in 7,5 Months?

Here $P=100$, $T=12$, $G=4{,}5$; alſo $t=7.5$, and $g=35.75$; to find p.

Add

Add $\begin{cases}\text{the Logarithm of } P=100=2.000000\\ \text{the Logarithm of } T=12=1.079181\\ \text{the Logarithm of } g=35,75=1.553276\end{cases}$

The Sum is the Log. of . . . $PTg=4.632457$ from

Add $\begin{cases}\text{the Logarithm of } G=4.5=0.653212\\ \text{the Logarithm of } t=7.5=0.875061\end{cases}$

The Sum is the Log. of . . . . $Gt=1.528273$ subs.

Whence the Log. of $\frac{PTg}{Gt}=p=1271,114=3.104184$.

Consequently, 1271,114*l.*=1271*l.* 2*s.* 3*d.*$\frac{1}{4}$ is the Answer.

Note, These *Theorems* give the Answer absolutely, without regarding whether the *Proportion* be *Direct* or *Inverse*, or both together, as in Art. 6. Exam. II.

8. *Of the Method of finding Mean Proportionals.*

In order that a clear Notion of this most useful *Problem* may be had, I shall premise the following things.

1. Between *two square Numbers* AA and BB, there will fall but one *Mean Proportional* AB; that is, $A^2 : AB :: AB : B^2$. See *Eucl.* 8. 11.

2. Between two Cubic Numbers A^3 and B^3, there will fall two *Mean Proportionals* A^2B and AB^2; that is, $A^3 : A^2B :: AB^2 : B^3$. See *Eucl.* 8. 12.

3. Between two *Biquadrate* Numbers A^4 and B^4, there will fall three *Mean Proportionals* A^3B, A^2B^2, and AB^3; that is, $A^4 : A^3B :: A^3B : A^2B^2 :: A^2B^2 : AB^3 :: AB^3 : B^4$.

4. Again, between two *sursolid* Numbers A^5 and B^5, there will fall *four Means*, viz. A^4B, A^3B^2, A^2B^3, and AB^4; that is, A^5, A^4B, A^3B^2, A^2B^3, AB^4, B^5, will be Proportionals. And thus the

Number of *Mean Proportionals* will be always lefs by *one* than the *Index* of the Power of the given Extremes.

9. But the *common Ratio* of all fuch Series is $\frac{B}{A}$

For $\overline{AA \times \frac{B}{A}} = AB$; and $\overline{AB \times \frac{B}{A}} = BB$.

And $\overline{A^3 \times \frac{B}{A}} = A^2B$; and $\overline{A^2B \times \frac{B}{A}} = AB^2$; and $\overline{AB^2 \times \frac{B}{A}} = B^3$, and fo in the others. Here I have fuppofed A to be the leaft Number, and B the greateft, and the Series to begin from A^2, A^3, *&c.* But if B be lefs than A, and the Series begin from B^2, B^3, *&c.* then the *Ratio* of the Series will be $\frac{A}{B}$.

Now $\frac{B}{A}$ is Root of $\frac{B^2}{A^2}$, $\frac{B^3}{A^3}$, $\frac{B^4}{A^4}$, *&c.* and therefore from thefe Premifes well underftood, 'tis eafy to conceive the Reafon of the following Rule for finding Mean Proportionals, *viz.*

Rule: Subftract the Logarithm of the *leaft Term* from the Logarithm of the *greateft*, and divide the Remainder by a Number greater by one than the Number of Means defired; then add the Quotient to the Logarithm of the leaft Term (or fubftract it from the Logarithm of the greateft) continually, and it will give the Logarithms of all the *Mean Proportionals* defired.

10. Example I. Let two Mean Proportionals be fought between 8 $(=A^3)$ and 28 $(=B^3.)$

The

The Logarithm of . . . . $B^3=28=1.447158$
The Logarithm of . . . . $A^3=8=0.903090$

The Difference is . . . . $\frac{B^3}{A^3}=0.544068$

which divide by 3

The Quotient is . . . . $\frac{B}{A}=0.181356$

To which add the Log. of. . . $A^3=8=0.903090$

The Sum is the Log. of 1st Mean } $A^2B=12,14=1.084446$

To which add again . . . . $\frac{B}{A}=0.181356$

The Log. of the 2d Mean $AB^2=18,44=1.265802$

Wherefore the Proportionals are } $8 : 12,14 :: 18.44 : 28$
$A^3 : A^2B :: AB^2 : B^3$.

11. Example II. Between 16 and 64 find five Mean Proportionals.

The Logarithm of . . . . $B^6=64=1.806180$
Subduct the Logarithm of . . $A^6=16=1.204120$

There remains . . . . $\frac{B^6}{A^6}=0.602060$

$\frac{1}{6}$ of which is . . . . $\frac{B}{A}=0.100343$

To which add the Log. of . . . . $A^6=16=1.204120$

The Sum is the Log. of the 1st Mean } $A^5B=20.158=1.304463$

To which add again . . . . $\frac{B}{A}=0.100343$

The Log. of the 2d Mean $A^4B^2=25.398=1.404806$

And

And thus you produce the Logarithms of

$$\begin{cases} \text{3d Mean} \ldots\ldots\ldots & A^3B^3=32=1.505150 \\ \text{4th Mean} \ldots\ldots\ldots & A^2B^4=40.317=1.605493 \\ \text{5th Mean} \ldots\ldots\ldots & AB^5=50,796=1.705836 \end{cases}$$

The Series therefore is this,

$$\begin{cases} 16. & 20,158. & 25,398. & 32. & 40,317. & 50,796. & 64. \\ A^6. & A^5B. & A^4B^2. & A^3B^3. & A^4B^2. & A^5B. & B^6. \end{cases}$$

12. If it were required to find 364 *Mean Proportionals* between 0 and 1.06; or 0 and 1.05; or 0 and 1,04, *&c.* then $A^{365}=0$, and $B^{365}=1{,}06$; 1.05, *&c.* and so $\frac{B}{A} = B = \sqrt[365]{1.06}$, or $\sqrt[365]{1.05}$, *&c.* Wherefore if you multiply the Logarithm of B, by 2, 3, 4, 5, &c. to 365, you will thereby obtain the *Mean Proportionals* required. And these will be the several *Amounts* of 1 *l.* and its *Interest* for each *Day* of the Year, at the Rates of 6, 5, 4 *l.* &c. *per Cent. per Annum, Compound Interest.* But more will be said of this hereafter.

CHAP. V.

Simple Interest *by* Logarithms.

1. MY Design being only to acquaint the Reader with the *Theory* and *practical Uses* of *Logarithms*, and not to treat of the Theory of any other Art or Branch of Mathematical Science; it will be sufficient for me barely to mention the *Theorems* or *Rules*, on which the divers Parts of Learning (I shall treat of) depend, and shew how they are most conveniently wrote by *Logarithms*.

Of

Of SIMPLE INTEREST.

2. I have more than once ſerv'd myſelf with thoſe excellent Theorems of Intereſt contrived by the late ingenious Mr. *Ward*; and ſhall once again make them ſubſervient to my Deſign in this Place. In order to which

Let $\begin{cases} \text{P=any } \textit{Principal} \text{ or } \textit{Sum} \text{ put to } \textit{Intereſt}. \\ \text{R=the } \textit{Ratio} \text{ of the } \textit{Rate, per Cent. per Ann.} \\ \text{T=the } \textit{Time} \text{ the } \textit{Principal} \text{ continues at } \textit{Intereſt}. \\ \text{A=the } \textit{Amount} \text{ of the } \textit{Principal} \text{ and } \textit{Intereſt}. \end{cases}$

3. Then any three of theſe being given, the other may be found by the following *Theorems*.

Theor. I. $TRP+P=A$. Theor. II. $\frac{A}{TR+1}=P$.

Theor. III. $\frac{A-P}{TP}=R$. Theor. IV. $\frac{A-P}{RP}=T$.

Queſt. I. What will 275*l.* 15*s.* amount to in $3\frac{1}{2}$ Years, at $4\frac{1}{2}$*l. per Cent. per Annum?*

Here P=275,75, T=3,5, R=0.045, to find A. Theor. I.

Add $\begin{cases} \text{the Logarithm of . . . P=275.75=2.440515} \\ \text{the Logarithm of . . . T= 3,5=0.544068} \\ \text{the Logarithm of . . . R=0.045=.8.653212} \end{cases}$

The Sum is the Log. of PTR=43.431=1.637795

To which add . . . . P=275,75

The Sum is the *Amount* } PTR+P=A=319,181=319*l.* 3*s.* 7*d.*

Queſt. II. What Principal or Sum being put to Intereſt, will amount to, or raiſe a Stock of 319*l.* 3*s.* 7*d.* in $3\frac{1}{2}$ Years, at the rate of $4\frac{1}{2}$*l. per Cent. per Annum?*

 The

The Log. of the *Amount* A=*l.* 319,181=2.504036

Add { the Log. of the Time . . . T=3,5=0.544068
the Log. of the Rate R=0.045=.8.653212 }

The Sum is the Log. of . . TR=0.1575=.9.197280

Then the Log. of . . . TR+1=1.1575=0.063521
Substracted from the Log. of the Amount leaves } P=275,75=2.440515

Wherefore the *Principal* required is } *l.* 275.75=275*l.* 15*s.*

Quest. III. At what Rate *per Cent.* &c. will 275*l.* 15*s.* amount to 319*l.* 3*s.* 7*d.* in $3\frac{1}{2}$ Years?

Here A—P=319,181—275,75=43.431.
The Logarithm . . . . A—P=43.431=1.637800

Add { the Logarithm of . . . P=275,75=2.440515
the Logarithm of . . . T= 3,5=0.544068 }

The Sum is the Logarithm of PT . . . =2.984583

Which subs. from the Log. A—P, there will remain } R=0,045=.8.653218

Quest. IV. In what Time will 275*l.* 15*s.* raise a Stock of 319*l.* 3*s.* 7*d.* at the Rate of $4\frac{1}{2}$*l.* *per Cent. per Annum?*

Add { the Log. of the Prin. P=275,75=2.440515
the Log. of the Ratio R=0.045=.8.653212 }

The Sum is the Log. of PR= 1.093727
Which substract from the Log. of } A—P=43,431=1.637800

There

There will remain the Log. of } $\frac{A-P}{RP}=T=3{,}5=0.544073$

Therefore the Anſwer is $3\frac{1}{2}$ Years.

4. *Of* ANNUITIES, *&c. in* ARREARS.

Put $\begin{cases} U = \text{the } \textit{Annuity, Penſion,} \text{ or } \textit{yearly} \text{ Rent.} \\ T = \text{the } \textit{Time} \text{ of its Continuance unpaid.} \\ R = \text{the } \textit{Ratio} \text{ of the } \textit{Rate} \text{ of } \textit{Intereſt.} \\ A = \text{the Amount of the Annuity and its Intereſt.} \\ \frac{2}{R} - 1 = x. \end{cases}$

Then the *Theorems* for finding each of thoſe Particulars, are as follows.

Theor. I. $\frac{TTU-TU}{2} \times R : + TU = A.$

Theor. II. $\frac{2A}{TTR-TR+2T} = U.$

Theor. III. $\frac{2A-2TU}{TTU-TU} = R.$

Theor. IV. $\sqrt{\frac{2A}{RU}+\frac{xx}{4}} : -\frac{1}{2}x = T.$

Queſtion I. If 250*l.* *yearly* Rent (Penſion, *&c.*) be forborn or unpaid 7 Years, what will it amount to in that time at 6 *l.* *per Cent.* for each Payment as it becomes due?

Here U=250, R=0.06, T=7; to find A, *per* Theor. I.

Add { the Logarithm of . . . . U=250=2.397940
{ the Logarithm of . . . . T= 7=0.845098

The *Sum* is the Log. of . . TU=1750=3.243030
add the Logarithm of . . . T= 7=0.845098

The Sum is the Log. of TTU=12250=4.088128

Then the Log. of $\frac{TTU-TU}{2}=5250=3.720159$

to which add the Log. of $R=0.06=.8.778151$

The Sum is the Log. of $\frac{TTU-TU}{2}\times R=315=2.498310$

To which add . . . . . $TU=1750$

The Sum is $\frac{TTU-TU}{2}$ $\times R:+TU=A=$ } . . . . . . 2065*l.* the Anf.

Note, if these Payments be made

$\left\{\begin{array}{l}\text{Quarterly,}\\ \text{Half-yearly,}\\ \text{Three-quarterly,}\end{array}\right\}$ then use $\left\{\begin{array}{l}\frac{1}{4}R,\ \frac{1}{4}U,\ \text{and } 4T.\\ \frac{1}{2}R,\ \frac{1}{2}U,\ \text{and } 2T.\\ \frac{3}{4}R,\ \frac{3}{4}U,\ \text{and } \frac{4}{3}T.\end{array}\right.$

Queſt. II. What *Year-Rent*, *Penſion*, &c. being forborn or unpaid ſeven Years, will raiſe a Stock of 2065*l.* allowing 6 *per Cent. per Annum*, for each Payment as it becomes due?

Here $A=2065$, $T=7$, $R=0.06$; to find U, *per* Theor. II.

Add $\left\{\begin{array}{l}\text{the Logarithm of . . . . . } T=7= 0.845098\\ \text{the Logarithm of . . . } R=0{,}06=.8.778151\end{array}\right.$

The Sum is the Log. of . . . $TR=0{,}42=.9.623249$

to which add the Log. of $T=7=0.845098$

The Sum is the Log. of . . $TTR=2{,}94=0.468337$

From which ſubduct . . . $TR=0{,}42$

There remains . . $TTR-TR=2.52$

to which add . . . . . . $2T=14$

the Sum is $TTR-TR+2T=16{,}52=1.218010$

the Logarithm of . . . . $2A=4130=3.615950$

The

The Difference of these Logarithms is U=250*l.* = 2.397940, which is the Annuity sought.

Quest. III. If 250*l.* Yearly Rent, &c. being forborn 7 years, will amount to 2065*l.* allowing simple Interest for each Payment as it becomes due, what must the Rate of Interest be *per Cent. per Annum?*

Add { the Logarithm of .... U=250=2.397940
the Logarithm of .... T=7= 0.845098

The Sum is the Log. of ... UT=1750=3.243030
to which add the Log. of ... T=7= 0 845098

The Sum is the Log. of TTU=12250=4.088128
from which subduct .... TU=1750

there remains TTU—TU=10500=4.021189
the Logarithm of 2A—2TU=630=2.799340

Subduct the former from the latter, there will remain } R=0.06=.8.778151

Wherefore, as 1*l.* : 0.06*l.* :: 100*l.* : 6*l.* the Rate required.

Quest. IV. In what time will 250*l.* Yearly-Rent, raise a Stock of 2065*l.* allowing 6 *l. per Cent.* &c. for the Forbearance of each Payment as it becomes due?

Here U=250, A=2065, R=0.06, $\frac{2}{R}$ —1= $\frac{2}{0,06}$ —1=32, *z*=*x*; to find T, *per* Theor. IV.

Add { the Logarithm of .... U=250=2.397940
the Logarithm of.... R=0.06=.8.778151

The

The Sum is the Logarithm of RU=15=1.176091
which subs. from the Log. of 2A=4130=3.615950

there remains . . . . $\frac{2A}{RU}$ =275,3=2.439859

Add $\left\{\begin{array}{l}\text{the Logarithm of . . . . . . } x=32,3=1.509658\\ \text{the Logarithm of . . . . } \frac{1}{4}x=8.083=0.907594\end{array}\right.$

The Sum is the Log. of $\frac{xx}{4}$ =261,3605=2.417252

to which add . . . . . . $\frac{2A}{RU}$=275,3333

the Sum is . . $\frac{2A}{RU}+\frac{xx}{4}$ =536,6938=2.729724

Half that Log. is the Log. $\sqrt{\frac{2A}{RU}+\frac{xx}{4}}$ =23,1668=1.364862

from which deduct . . . . $\frac{1}{2}x$=16,1668

there will remain $\sqrt{\frac{2A}{RU}+\frac{xx}{4}}$: $-\frac{1}{2}x$=T=7, the Number of Years required.

5. *Of the* PRESENT WORTH *of* ANNUITIES, PENSIONS, *&c.*

Here U, P, R, T, are used to denote the *Annuity*, *present* Worth, *Ratio* of the *Rate* of *Interest*, and *Time*, as in the former Articles; here also let $\frac{2}{R}-\frac{2P}{U}-1=x$; then the Theorems for Operation are as follows;

Theor. I. $\frac{TTR-TR+2T}{2TR+2}\times U=P=$ Present Worth.

Theor. II. $\frac{TR+1}{TTR-TR+2T}\times 2P=U,=$Annuity, *&c.*

Theor.

Theor. III. $\frac{2P-2TU}{TTU-TU-2PT}=R=$, Ratio of the Rate.

Theor. IV. $\sqrt{\frac{2P}{RU}+\frac{xx}{4}}\pm\frac{1}{2}x=T$, = the Time.

Queſt. I. What is the *preſent Worth* of 75 *l.* Yearly Rent, to continue 9 Years, at 6 *per Cent.* &c.

Here U=75, T=9, R=0.06, to find P, *per* Theor. I.

Add { the Logarithm of . . . . . . T=9=0.954242
the Logarithm of . . . . R=0.06=.8.778151

The Sum is the Log. of . . . . TR=,54=.9.732393

Alſo the Logarithm of . . TTR=4,86=0.686635
ſubſtract . . . . TR=,54

there remains . . . . TTR—TR=4.32
to which add . . . . . 2T=18.

The Sum is TTR—TR+2T=22,32=1.348694
the Logarithm of . . . 2TR+2=3.08=0.488551

the Difference is . . . . . =0.860143
To which add the Log. of . . . . U=75=1.875061
The Sum is the Logarithm of

$\frac{TTR-TR+2T}{2TR+2}\times U=P=543{,}506=2.735204$

Wherefore the preſent Worth is 543 *l.* 10 *s.* 1 *d.* $\frac{1}{2}$.

Queſt. II. What Annuity, Penſion, *&c.* may be purchaſed for 543 *l.* 10 *s.* 1 *d.* $\frac{1}{2}$, to continue 9 Years, allowing to the Purchaſer 6 *per Cent. per An.* ſimple Intereſt?

Here P=543,506; T=9; R=0,06; to find U, *per* Theor. II.

From

From the Log. of .... $TR+1=1,54=0.187521$
subduct the Log. of } $TTR-TR+2T=22.32=1.348694$

to the Difference ...... $.8.838827$
add the Logarithm of $2P=1087,012=3.036234$

The Sum is the Logarithm of

$$\frac{TR+1}{TTR-TR+2T} \times 2P = U = 75 = 1.875061$$

That is, 75*l. per Annum*, is the Annuity, *&c.* sought.

Quest. III. If 543*l.* 10*s.* 1*d.*$\frac{1}{2}$ ready Money, will purchase an Annuity, Lease, *&c.* of 75*l. per Annum*, to continue 9 Years; *Quære* the Rate of Interest *per Cent.* &c.?

Here $P=543,506$; $U=75$; $T=9$; to find R, *per* Theor. III.

From the Log. of $2P-2TU=262,988=2.419936$
subd. the Log. of } $TTU-TU-2PT=4383,108=3.641782$

the Difference is the Log. of $R=0.06=.8.778154$

Wherefore, as 1*l.* : 0,06*l.* :: 100*l.* : 6*l.* the Rate required.

Quest. IV. If for 543*l.* 10*s.* 1*d.*$\frac{1}{2}$ I purchase an Annuity, Pension, *&c.* of 75*l. per Annum*; *Quære*, how long I may enjoy it, at the Rate of 6 *per Cent.* &c. Interest?

Here $P=543,506$, $U=75$, $R=0,06$; to find T, *per* Theor. IV.

From the Log. of ... $2P=1087,012=3.036234$
subduct the Log. of .... $U=75=1.875061$

the Diff. is the Log. of .. $\frac{2P}{U}=14,493=1.161173$

Which

which ſubſtract from . . $\frac{z}{R} - 1 = 32,333$

there remains $\frac{z}{R} - \frac{2P}{U} - 1 = x = 17.840 = 1.251395$

add the Log. of . . . . $\frac{1}{4}x = 4.46 = 0.649335$

the Sum is the Log. of . . $\frac{1}{4}xx = 79,5664 = 1.900730$

to which add . . . . . . $\frac{2P}{RU} = 241,558$

the Sum is $\frac{2P}{RU} + \frac{xx}{4} = 321,1244 = 2.506659$

half which is the Log. of $\left.\right\}$ $\sqrt{\frac{2P}{RU} + \frac{xx}{4}} = 17,919 = 1.253329$

ſubſtract . . . . $\frac{1}{2}x = 8,92$

there remains $\sqrt{\frac{2P}{RU} + \frac{xx}{4}} - \frac{1}{2}x = T = 8,999 = 9,$ the Years ſought.

6. If the Queſtion be of Annuities, *&c.* in Reverſion, you muſt find the *Amount* of the *Purchaſe-Money* to the time of Commencement (together with its Intereſt) by Queſt. I. Art. 3. and make that *Amount* the Sum for the Purchaſe; and then proceed as in the Queſtions of this laſt Article. Theſe are all the *fundamental* or *original* Caſes of *ſimple Intereſt.*

CHAP. VI.

COMPOUND INTEREST *by* LOGARITHMS.

1. THE former Questions of *simple Interest* might be resolved by the Rules of *vulgar Arithmetic*; and I have only there shewn, they may also be (and that in many Cases, *most conveniently*) wrought by *Logarithms*. But in the present Affair of *compound Interest*, the Use of Logarithms is *absolutely necessary*; no other Method of the Solution of Questions in compound Interest being equal to it in Extent and Perfection. And consequently the young Student in Arithmetic is under an indispensible Obligation to be acquainted with this most excellent and useful Branch of the Science.

2. I shall here also proceed according to the Theorems of Mr. *Ward*; and therefore

Put $\begin{cases} \text{P} = \text{the } \textit{Principal} \text{ put to } \textit{Interest}. \\ \text{t} = \text{the } \textit{Time} \text{ of its } \textit{Continuance}. \\ \text{A} = \text{the } \textit{Amount} \text{ of the } \textit{Principal} \text{ and } \textit{Interest}. \\ \text{R} = \text{the } \textit{Amount} \text{ of 1 } \textit{l.} \text{ and its } \textit{Interest} \text{ for one Year, at any given } \textit{Rate}. \end{cases}$

Note, you find R thus, $\begin{cases} 100 : 106 :: 1 : 1{,}06 = \text{R, at 6 } \textit{per Ct.} \\ 100 : 105 :: 1 : 1{,}05 = \text{R, at 5 } \textit{per Ct.} \end{cases}$

The Amounts of 1 *l.* in several Years $\begin{cases} 1.\ 2.\ 3.\ 4.\ 5, \text{\&c. Years.} \\ \text{R}.\ \text{R}^2\ \text{R}^3\ \text{R}^4\ \text{R}^5, \text{\&c. Am. of 1 } \textit{l.} \end{cases}$

Therefore R^t = the Amount of 1 *l.* in the Time t for the Rate agreed on; this being premised, the Theorems for *Compound Interest* are as follow.

3. Theor. I. $\text{PR}^t = \text{A} =$ the *Amount*.

Theor.

Theor. II. $\frac{A}{R^t} = P =$ the Principal.

Theor. III. $\frac{A}{P} = R^t \begin{cases} = \text{the Time.} \\ = \text{the Amount of } 1\,l. \end{cases}$

By theſe Theorems the ſeveral Queſtions of Compound Intereſt are anſwered moſt expeditiouſly by the *Logarithms* in the Manner following.

4. Queſt. I. What will 275 *l.* 15 *s.* amount to in $3\frac{1}{2}$ Years, at $4\frac{1}{2}$ *l. per Cent. per Annum*, Compound Intereſt?

Here P=275,75; R=1,045; t=3,5; to find A, *per* Theor. I.

The Logarithm of	R=1,045=	0.019116
multiply by the Time . . .	t=	3,5
The Prod. is the Log. of	R^t=1,1665=	0.066906
To which add the Log. of	P=275,75=	2.440515
The Sum is the Log. of	PR^t=A=321,68=	2.507421

So the Amount ſought is 321 *l.* 13 *s.* 7 *d.* which is more than the Amount by ſimple Intereſt by 2 *l.* 10 *s.* See Queſt. I. Art. 3. of the foregoing Chapter.

Queſt. II. What Principal or Sum being put to Uſe at $4\frac{1}{2}$ *l. per Cent.* Compound Intereſt, will amount to 321 *l.* 13 *s.* 7 *d.* in $3\frac{1}{2}$ Years?

Here A=321,68; R=1,045; t=3,5; to find P, *per* Theor. II.

From the Logarithm of	A=321,68=	2.507421
Subduct the Logarithm of	R^t=1.1665=	0.066906
The Diff. is the Log. of	$\frac{A}{R^t}$=P=275,75=	2,440515

therefore the *Principal* ſought is 275 *l.* 15 *s.*

Queſt. III. At what rate *per Cent.* &c. Compound Intereſt, will 275*l.* 15*s.* raiſe a Stock, or amount to 321*l.* 13*s.* 7*d.* in $3\frac{1}{2}$ Years?

Here A=321,68; P=275,75; t=3.5; to find R, *per* Theor. III.

From the Logarithm of	A=321,68=2.507421
Subſtract the Log. of . . .	P=275,75=2.440515
The Difference is the Log. of R^t	0.066906
which divide by the Time . . .	t=3,5
The Product is the Log. of	R=1.045=0.019116

Then as 1 : 0,045 :: 100 : 4,5=$4\frac{1}{2}$*l.* *per Cent.* the Rate required.

Queſt. IV. In what Time will 275*l.* 15*s.* raiſe a Stock of 321*l.* 13*s.* 7*d.* at the rate of $4\frac{1}{2}$*l.* *per Cent.* Compound Intereſt?

Here P=275,75; A=321,68; R=1.045; to find t, *per* Theor. IV.

From the Logarithm of . .	A=321,68=2.507421
Subduct the Logarithm of	P=275,75=2.440515
The Diff. is the Log. of	R^t=$1,045^t$=0.66906

Then the Log. of 1.045=0.019116)0.066906(3,5=t.

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57348
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 95580
 95580
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 ....
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Thus the Anſwer is 3 Years and 6 Months.

Note, As the *Amount* of any *Sum* and its *Intereſt* is greater at *Compound Intereſt* than at *Simple Intereſt* for *any Time* above a Year; ſo it is *leſs* at *Compound*

pound than *Simple Interest* for any *Time less* than a Year, as the Learner may easily prove by the Theorems beforegoing.

5. *Of* ANNUITIES, *&c. in* ARREARS.

Theor. I. $\frac{UR^t - U}{R - 1} = A =$ the *Amount* of any Sum.

Theor. II. $\frac{RA - A}{R^t - 1} = U =$ the *Annuity*, *Pension*, &c.

Theor. III. $\frac{RA + U - A}{U} = R^t =$ the *Time*.

Theor. IV. $\frac{A - U}{U} = \frac{A}{U} R - R^t =$ the Amount of 1 *l*.

6. Quest. I. If 30 *l.* yearly Rent be forborn 9 Years, what will it amount to at 5 *l. per Cent.* &c. Compound Interest?

Here $U = 30$, $t = 9$, $R = 1{,}05$; to find A; *per* Theor. I.

The Logarithm of . . .	$R = 1{,}05 =$	0.021189
multiply by the Index . . .		9
The Product is	$R^t = 1.516 =$	0.180701
To which add the Log. of . . .	$U = 30 =$	1.477121
The Sum is the Log. of	$UR^t = 45{,}48 =$	1.657822
Subduct	$U = 30$	
There remains . . .	$UR^t - U = 15{,}48 =$	1.189771
Substract the Log. of	$R - 1 = 0.05 =$	.8.698970
The Diff. is the Log. of	$\frac{UR^t - U}{R - 1} = A = 309{,}6 =$	2.490801

The Amount therefore is 309 *l.* 12 *s.* for Answer.

Quest.

Quest. II. What Annuity, *&c.* will raise a Stock (or amount to) 309 *l.* 12 *s.* being forborn or unpaid 9 Years, at 5 *l. per Cent.* &c. Compound Interest?

Here A=309,6; t=9; R=1,05; to find U, *per* Theor. II.

Add { the Logarithm of . . . . A=309,6=2.490801
the Logarithm of . . . R=1,05=0.021189

The Sum is the Log. of . . AR=325,08=2.511990

Therefore the Log. of AR—A=15,48=1.189771
Subduct the Log. of . . . R^t—1=0.516=9.712650

The Diff. is the Log. of } . . . $\frac{AR-A}{R^t-1}$ = U = 30=1.477121

That is 30 *l. per Annum* is the *Annuity* sought.

Quest. III. In what time will 30 *l. per Annum* amount to 309 *l.* 12 *s.* at 5 *l. per Cent.* Compound Interest?

Here U=30; A=309,6; R=1,05; to find t, *per* Theor. III.

Add { the Logarithm of . . . . A=309,6=2.490801
the Logarithm of . . . . R=1,05=0.021189

The Sum is the Log. of AR=325,08=2.511990

Substract . . . . A—U=279,6

There remains AR+U—A=45,48=1.657822
Substract the Log. of U= 30=1.477121

The Diff. is the Log. of . . R^t=1,05^t=0.180701
Then the Log. of 1,050.02=1189)0.180701(9=t.
180701

. . . . .

Answer, 9 Years.

Quest.

Queſt. IV. At what Rate *per Cent.* Compound Intereſt will 30 *l.* Yearly Rent amount to 309 *l.* 12 *s.* in 9 Years?

Here U=30; A=309,6; t=9; to find R, *per* Theor. IV.

From the Log. of . . . A—U=279,6=2.446537
Subduct the Logarithm of . . . U=30=1.477121

The Diff. is the Log. of $\frac{A-U}{U}$=9,32=0.969416

Again from the Log. of . . . A=309,6=2.490801
Subſtract the Logarithm of . . U=30=1.477121

The Diff. is the Log. of $\frac{A}{U}$=10,32=1.013680

Therefore the ſaid Theorem IV. is reduced to this Equation, *viz.* 9.32=10,32R—R^9. Now this Equation may be eaſily reſolv'd by a *converging Series*; or ſtill eaſier by the Tables of the Amount of 1 *l.* for ſeveral Years ſucceſſively; for therein R^9, or the Amount of 1 *l.* in 9 Years may be tried for the ſeveral Rates *per Cent.* in the Tables. For inſtance, ſuppoſe I pitch upon the Table at 5 *l. per Cent.* Compound Intereſt, then I find R^9=1.551328, *&c.* Wherefore 9.32+1.551328=10.871328=10.32R; therefore 10.32) 10.871328 (=1,05. Now ſince 1 *l.* : 0,05 *l.* :: 100 *l.* : 5 *l.* the very Rate *per Cent.* aſſumed in the Tables, it will follow, that is the true Rate ſought.

Note, This Queſtion may be reſolved by the Rule of *falſe* Poſition; which I leave to the Learner's Exerciſe.

7. *Of the* PRESENT WORTH *of* ANNUITIES, *&c. at Compound Interest.*

Here P denotes the *present Worth* of any Annuity, Lease, *&c.* and the rest of the Letters as before, Art. 6. Then the Theorems are as follows.

Theor. I. $\frac{U-\frac{U}{R^t}}{R-1}=P=$ the *present Worth.*

Theor. II. $\frac{PRR^t-PR^t}{R^t-1}=U=$ the *Annuity*, &c.

Theor. III. $\frac{U}{P+U-PR}=R^t=$ the *Time.*

Theor. IV. $\frac{U}{P}=\frac{U}{P}R^t+R^t-R^{t+1}=$ the Value of R.

Quest. I. What is 30 *l. per Annum* to continue 9 Years, worth in ready Money, abating 5 *l. per Cent.* &c. Compound Interest to the Purchaser?

Here U=30, t=9; R=1,05; to find P, *per* Theor. I.

The Logarithm of	R=1,05=0.021189
Multiply by the Time	9
The Prod. is the Log. of	R^9=1.516=0.180701
which substract from the Log. of	U=30=1.477121
The Diff. is the Log. of	$\frac{U}{R^t}$=19,788=1.296420
which take from . . .	U=30
there remains . . .	$U-\frac{U}{R^t}$=10.212=1.009111
Subduct the Log. of . . .	R−1=0.05=.8.698970
The Diff. is the Log. of	P=204,24=2.310141

The present Worth is 204 *l.* 4 *s.* 9 *d.* ½, the Answer.

Quest.

Queſt. II. What Annuity, *&c.* to continue 9 Years may one purchaſe for 204*l.* 4*s.* 9*d*½. abating 5*l. per Cent.* &c. to the Purchaſer?

Here $P=204,24$; $t=9$; $R=1,05$; to find U, *per* Theor. II.

Add { the Logarithm of . . . $R^t=1.516=0.180701$
the Logarithm of . . . . $R=1,05=0.021189$
the Logarithm of . . $P=204,24=2.310141$ }

The Sum is the Log. of $PRR^t=325,11=2.512031$
Then from . . . . $PRR^t=3251,11$
Subſtract . . . . . $PR^t=309.628$

there remains $PRR^t-PR^t=15,482=1.189799$
Subduct the Log. of $R^t-1=0.516 = .9.712650$

The Diff. is the Log. of . . . $U=30=1.477149$
Therefore the Annuity ſought is 30*l. per Annum.*

Queſt. III. What time may one enjoy an Annuity, *&c.* of 30*l. per Annum*, for 204*l.* 4*s.* 9*d*½. ready Money, abating to the Purchaſer 5*l. per Cent.* &c.?

Here $P=204,24$; $U=30$; $R=1.05$; to find t, *per* Theor III.

Add { the Logarithm of . . $P=204,24=2.310141$
the Logarithm of . . . . $R=1,05=0.021189$ }

The Sum is the Log. of $PR=214,452=2.331330$

Then . . . . . $P\pm U=234.24$

The Diff. . . . $P+U-PR=19.788=1.296402$
which ſubſtract from the Log. of $U=30=1.477121$

The Diff. is the Log. of $R^t=1,05^t=1,516=0.180719$

U Then

Then the Log. of 1,05=0.021189)0.180719(=9=t.
180701

Anſwer, 9 Years. 18

Queſt. IV. Suppoſe an Annuity, *&c.* of 30*l.* *per Annum*, to continue 9 Years, be ſold for 204*l.* 4*s.* 9*d* $\frac{1}{2}$. ready Money, what rate of Intereſt hath the Purchaſer allow'd *per Cent.* &c. for his Money?

Here P=204,24; U=30; t=9; to find R, *per* Theor. IV.

From the Log. of the Annuity U=30=1.477121
Subduct the Log. of .... P=204,24=2.310141

The Diff. is the Log. of $\frac{U}{P}$ =0.14679=.9.166980

Then the ſaid Theor. IV. is reduced to this Equation, *viz.* $0.14679=0.14679R^9+R^9-R^{10}$; which is to be ſolv'd by the Method of Infinite Series. But for thoſe who underſtand not that Method, this Queſtion is much better anſwer'd by the *Rule of Falſe*; or yet eaſier by the Tables of Compound Intereſt. See a compleat Set, with their Uſes, in my *Syſtem of Decimal Arithmetic*, publiſhed for Mr. *Noon*, at the *White Hart*, in *Cheapſide*, *London*.

8. *Of Purchaſing* Freehold Estates.

Eſtates in *Fee Simple* (which are ſuch as we commonly call *Freehold* or *Real Eſtates*) being purchaſed *for ever*, or without *Reverſion*; 'tis plain, that in the foregoing Theorems for finding the preſent Worth of Annuities, *&c.* if thoſe Terms wherein t (= the *Time*) is found, be made to vaniſh, as being *Infinite*; the ſaid Theorems will be reduced

reduced to ſuch as ſuit the *preſent Caſe*, and are as follows.

Theor. I. $\frac{U}{R-1}=P$, the *Preſent Worth*, or *Purchaſe-Money*.

Theor. II. $PR-P=U$, the *Annuity*, or *Eſtate* per Annum.

Theor. III. $\frac{P+U}{P}=R$, the Amount of 1 *l.* at the given Rate.

Queſtion. What muſt be given for a *Freehold* Eſtate of 50 *l. per Annum*, allowing the Buyer 6 *l. per Cent.* Compound Intereſt for his Money?

Here $U=50$; $R=1{,}06$; to find P, *per* Theor. I.

The Logarithm of . . .	$U=50=1.698970$
Subduct the Log. of	$R+1=0{,}06=.8.778151$
The Diff. is the Log. of $\frac{U}{R-1}$	$=P=833{,}3=2.920819$

The Purchaſe-Money, then, is 833 *l.* 6 *s.* 8 *d.* If it had been at the Rate of 5 *l. per Cent. Compound Intereſt*, the ſaid Eſtate would be worth 1000 *l.* preſent Money; which is equal to twenty times the *Yearly Rent:* and therefore the general Method of buying *Freehold Eſtates*, is by paying 20 or 25 times the Yearly Rent; which is commonly called 20 or 25 Years Purchaſe.

The other two Theorems are wrought in the ſame Manner for U and R; which, being ſo very eaſy, need no Example.

CHAP. VII.

Of VULGAR FRACTIONS *by* LOGARITHMS.

1. TO find the *Logarithm* of a *Vulgar Fraction*, this is the

Rule $\begin{cases}\text{From the Logarithm of the } \textit{Numerator} \\ \text{Substract the Logarithm of the } \textit{Denominator}\text{;} \\ \text{the Remainder is the Log. of the Fraction.}\end{cases}$

Example I. What is the Log. of the Fraction $\frac{51}{73}$?

From the Logarithm of . . . 51=1.707570
Substract the Logarithm of . . . 73=1,863323

there remains the Log. of $\frac{51}{73}$=0.69862=.9.844247

Exam. II. What is the Logarithm of $\frac{1}{1756}$?

From the Logarithm of . . . . 1=0.000000
Substract the Logarithm of . . . 1756=3.244524

there remains the Log. of . . $\frac{1}{1756}$=,00056947=.6.755476

Exam. III. What is the Logarithm of $\frac{100}{1357}$?

From the Logarithm of . . . . 100=2.000000
Take the Logarithm of . . . 1357=3.132580

there remains the Log of $\frac{100}{1357}$=,073692=.8.867420

Exam. IV. What is the Logarithm of $\frac{5973}{1000000}$?

The

The Logarithm of . . . . 5973=3.776192
From which take the Log. of 1000000=6.000000

there rem. the Log. of $\frac{5973}{1000000}$=,005973=.7.776192

2. If the Fraction be a *mix'd one*, it muſt be reduced to an *improper Fraction*; and then proceed with it as before.

Example I. What is the Logarithm of $13\frac{5}{7}$?

Here $13\frac{5}{7}=\frac{96}{7}$;

Therefore from the Logarithm of 96=1.982271
take the Logarithm of . . . . 7=0.845098

there remains the Log. of $13\frac{5}{7}$=13.714=1.137173

Exam. II. What is the Logarithm of $193\frac{51}{73}$?

Here $193\frac{51}{73}=\frac{14140}{73}$;

Then from the Logarithm of 14140=4.150449
take the Logarithm of . . . . 73=1.863323

there rem. the Log. of $193\frac{51}{73}$=193,69862=2.287126

3. If the mix'd *Fraction* conſiſts of *large Numbers*, it may be moſt eaſily reduc'd by Logarithms, thus; ſuppoſing the Example be $2145\frac{57}{589}$;

To the Log. of the *Integral Part* 2145=3.331427
Add the Log. of the Denominator 589=2.770115

The Sum is the Log. of . . 1263405=6.101542
to which add the Numerator . . 57

Then you have $\left\{\begin{array}{l}\text{the new Num.} \\ \text{the Denomin.}\end{array}\right.$ $\frac{1263462}{57}$ $\left.\right\}$ the Fract. ſought.

4. To *multiply Vulgar Fractions* by Logarithms, add the Logarithms of the Numerators for the Logarithm of a *new Numerator*; and the Logarithms of the Denominators for the Logarithm of a new Denominator.

Example. What is the Product of $\frac{35}{173}$ into $\frac{7}{9}$?

Add the Log^s. of the Numerators	$35 = 1.544068$ $7 = 0.845098$
The Log. of the *new Numerator*	$245 = 2.389166$
Add the Log^s. of the Denominat^s.	$173 = 2.238046$ $9 = 0.954242$
The Log. of the *new Denomin.*	$1557 = 3.192288$

Therefore $\frac{35}{173} \times \frac{7}{9} = \frac{245}{1557}$, the fractional Product required.

5. The Logarithm of the Product of ſeveral Fractions multiplied into one another is thus obtain'd; *viz.* Add the Logarithms of all the *Numerators* and the *Arithmetical Complements* of the Logarithms of all the *Denominators* together; the Sum is the Logarithm required.

Example. What is the Logarithm of $\frac{87}{97} \times \frac{33}{46} \times \frac{3}{7}$?

Add together	the Logarithms of	$87 = 1.939519$ $33 = 1.518514$ $3 = 0.477121$
	the Arith. Comp. of the Logarithms of	$97 = .8.014228$ $46 = .8.337242$ $7 = .9.154902$

The Log. of $\frac{87}{97} \times \frac{33}{46} \times \frac{3}{7} = \frac{8613}{31234} = ,27639 = .9.441526$

6.

6. To divide Vulgar Fractions by Logarithms, do thus; Add the Logarithm of the *Denominator* of the *Divisor* to the Logarithm of the *Numerator* of the *Dividend*; the Sum is the Logarithm of a *new Numerator*; and the Sum of the Logarithms of the other *two Factors*, is the Logarithm of the new Denominator of the Quotient required.

Example. Divide $\frac{245}{1557}$ by $\frac{7}{9}$.

Add the Logarithms of	9	=0.954242
	245	=2.389166
The Log. of the new Numerator	2205	=3.343408
Add the Logarithms of . . .	7	=0.845098
	1557	=3.192288
The Log. of the new Denom.	10899	=4.037386

Therefore $\frac{7}{9})\frac{245}{1557}(=\frac{2205}{10899}=\frac{35}{173}$. See Art. 4.

7. The Logarithm of this Quotient may be found by one Addition, in like manneras directed in Art. 5. thus;

Add together	the Logarithm of	9	=0.954242
		245	=2.389166
	the Arith. Comp. of the Logarithms of	7	=.9.154902
		1557	=.6.807712
The Log. of . . $\frac{7}{9})\frac{245}{1557}(=\frac{35}{173}=,20231$			=.9.306022

8. The *Extraction* of the *Roots* of *Vulgar* Fractions by Logarithms is thus performed. Divide the Logarithm both of the Numerator and Denominator of the given Fraction by the *Index* of the *Root*; the *Quotients* ſhall be the Logarithms of the *Numerator* and *Denominator* of the *Fractional Root* required.

Examp.

Examp. I. What is the *Square Root* of the Fraction $\frac{1849}{10201}$?

Divide the Logarithm of . . . . . $1849 = 3.266937$
by the Index of the Root . . . 2

The Log. is the Num. of the Root $43 = 1.633468$

Again, divide the Log. of . . . $10201 = 4.008643$
by 2

The Log. of the Denom. of the Root $101 = 2.004321$

Therefore $\sqrt{\frac{1849}{10201}} = \frac{43}{101}$ the Root required.

Example II. What is the *Cube Root* of $86\frac{293}{343}$?
This reduced to an improper Fraction, is $\frac{29791}{343}$;

Therefore the Logarithm of $29791 = 4.474085$
$\frac{1}{3}$ thereof is the Log. of the *new Num.* $31 = 1.491362$

Again, the Logarithm of . . . . $343 = 2.535294$
$\frac{1}{3}$ of which is the Log. of the *new Denom.* $7 = 0.845098$

Consequently $\sqrt[3]{86\frac{293}{343}} = \frac{31}{7} = 4\frac{3}{7}$, the Cube Root sought.

9. To find the *Logarithm* of the *Root* of any Fraction; add the Logarithm of the *Numerator* to the *Arithmetical Complement* of the Logarithm of the *Denominator*, and divide that Sum by the *Index* of the *Root*; the Quotient shall be the Logarithm sought.

Example I. What is the Logarithm of the *Square Root* of the Fraction $\frac{1849}{10201}$?

The

The Log. of the Numerator . . . 1849=3.266937
Comp. Arith. of the Log. of the Denom. } . . 10201=.5.991357

The Sum is 9.258284
which divide by the Index of the Root 2

The Quot. is the Log. of } $\sqrt{\frac{1849}{10201}}=\frac{43}{101}=,42574=.9.629147$

Exam. II. What is the Logarithm of the *Cube Root* of the mix'd Fraction $86\frac{293}{343}$, or its equal $\frac{29791}{343}$?

Add { the Logarithm of . . . . 29791=4.474085
the Ar. Com. of the Log. of 343=.7.464706

The Sum is the Log. of . . . . . $68\frac{293}{343}$=1.938791
which divide by the *Index* of the Root . . 3

The Quot. is the Log. of } $\sqrt[3]{68\frac{243}{343}}=4\frac{3}{7}=4.4285=0.646263$

CHAP. VIII.

DUODECIMAL ARITHMETIC *performed by* LOGARITHMS.

1. SINCE this kind of Arithmetic is ſo very *common*, and yet in the common Way ſo very *difficult*; I hope 'twill not be unacceptable to the *young Artificer* to be convinced with how much more Eaſe and Pleaſure he may compute his Dimenſions in this Way by the help of Logarithms. And as it is proper to reduce them firſt to *Decimals*, I have here ſub-

joined a Table shewing by Inspection the *Decimal Parts* of a Foot (in this Case the *Integer*) answering to any Number of *Primes*, *Seconds*, and *Thirds*; which are the Parts or Divisions of a Foot made use of in this kind of Mensuration.

2. *The Duodecimal Table.*

Duodecimals.	Decimal Parts. Primes ′	Seconds ″	Thirds ‴
1	,08$\cancel{3}$333	,006944	,000578
2	,1$\cancel{6}$6666	,013$\cancel{8}$88	,001157
3	,25	,0208$\cancel{3}$3	,001736
4	,$\cancel{3}$33333	,02$\cancel{7}$777	,002314
5	,41$\cancel{6}$666	,0347$\cancel{2}$2	,002893
6	,5	,041$\cancel{6}$66	,003472
7	,58$\cancel{3}$333	,048611	,004051
8	,$\cancel{6}$66666	,0$\cancel{5}$5555	,004629
9	,75	,0625	,005208
10	,8$\cancel{3}$3333	,069$\cancel{4}$44	,005787
11	,91$\cancel{6}$666	,0763$\cancel{8}$8	,006365

3. The foregoing Table is too easy to need Description, I mean for any Person concern'd in *Duodecimal Mensuration*; and therefore I shall proceed to exemplify and illustrate the Operations this Way by Logarithms in all the Rules of *Multiplication*, *Division*, *Involution*, and *Extraction* of Roots.

4. *Muliplication of* DUODECIMALS *by* LOGARITHMS.

Examp. I. Suppose a Plane be $9^f\ 10'$ in Length, and $8^f\ 8'$ in Breadth; *Quære* the *Content* or *Area?*

Add { the Log. . . of $9^f.\ 10'.=9.8\cancel{3}33=0.992704$
the Log. of . . $8^f.\ 8'.=8.6666=0.937849$

The Sum is the Log. of . . . . $85,\cancel{2}22=1.930553$

That is, 85,222 Feet, $=85^f.\ 2'.\ 8''$. the Area required.

Examp.

Examp. II. What is the Product of $40^f.\ 9'.\ 10''$ by $11'.\ 9''$?

Add $\begin{cases}\text{the L. of } 40^f.\ 9'.\ 10''.=40.8194=1.610866 \\ \text{the Log. of } 11'.\ 9''=0,97916=.9.990854\end{cases}$

The Sum is the Log. of . . . . 39,969=1.601720

Therefore $39,969=39^f.\ 11'.\ 7''.\ 6'''$. the *Product*, or *Area* required.

Examp. III. What is 175 Feet $00'.\ 04''$. by $8'''$?

Add $\begin{cases}\text{the L. of } 175^f.\ 0'.\ 4''.=175,027=2.243107 \\ \text{the Log. of } 8'''.=,004629=.7.665487\end{cases}$

The Sum is the Log. of . . ,81111 =.9.908594

Therefore $0,81111=0^f.\ 9'.\ 8''.\ 9'''$ = the *Area* sought.

Examp. IV. What is $17^f.\ 9'.\ 2''.\ 6'''$ by 6^f?

Add $\begin{cases}\text{the L. of } 17^f.\ 9'.\ 2''.\ 6'''=17,76736=1.247417 \\ \text{the Log. of . . . . } 6=0.778151\end{cases}$

The Sum is the Log. of . . 106,60416=2.025568

Thus $106.60416=106^f.\ 7'\ 3''$. the Area sought.

Examp. V. What Number of *solid Feet* is in a *Cellar* $21^f.\ 2'$. long, $11^f.\ 10'.\ 8''$. broad, and $7^f.\ 3'$. deep?

Add $\begin{cases}\text{the Log. of . . } 21^f.\ 2'=21,1666=1.325659 \\ \text{the Log. of } 11^f.\ 10'.\ 8''=11,888=1.074109 \\ \text{the Log. of } 7^f.\ 3'.\ =7,25=0.860338\end{cases}$

The Sum is the Log. of . . . 1820,19=3.260106

Therefore $1820,19=1820^f.\ 2'.\ 3''.\ 4'''$, the *Solidity* required.

5. DIVISION *of* DUODECIMALS *by* LOGARITHMS.

Examp. I. What is $85^{f}.\ 2'.\ 8''$. divide by $8^{f}.\ 8'$?

The Log. of . . . . $85^{f}.\ 2'.\ 8''.=85,\not{2}2=1.930553$
Subduct the Log. of $8^{f}.\ 8'$ $=8,\not{6}66=0.937849$

The Diff. is the Log. of . . . $9.8\not{3}33=0.992704$
So that $9.8\not{3}33=9^{f}.\ 10'$, the Anſwer.

Examp. II. What is $9'.\ 8''.\ 9'''$. divided by $8'''$?

From the Log. of $9'.\ 8''.\ 9'''=0.8\not{1}111=.9.908594$
Subſtract the Log. of $8'''=,004629=.7.665487$

The Diff. is the Log. of . . . $175,02\not{7}=2.243107$
Therefore the Anſwer is $175,02\not{7}=175^{f}.\ 0'.\ 4''$.

Examp. III. Divide $39^{f}.\ 11'.\ 7''.\ 6'''$. by $40^{f}.\ 9'.\ 10''$.

From the Log. of } . . $39^{f}.\ 11'.\ 7''.\ 6'''=39.9687=1.601720$
Subd. the Log. of $40^{f}.\ 9'.\ 10''=40.8194=1.610866$

The Diff. is the Log. . . of $0,97916=.9.990854$
But $,97916=11'.\ 9''$. the Quotient required.

Examp. IV. Divide $106^{f}.\ 7'.\ 3''$. by 6.

From the L. of $106^{f}.\ 7'.\ 3''.=106,6041\not{6}=2.205568$
Subſtract the Log. of $6=0.878151$

The Diff. is the Log. of $17,76736=1.247417$

Thus $17,76736=17^{f}.\ 9'.\ 2''.\ 6'''$. the Quotient ſought.

6.

6. INVOLUTION *of* DUODECIMALS *by* LOGARITHMS.

Examp. I. What is the Area of that *Square* whoſe Side is $12^f.\ 9'.\ 7''.\ 10'''$?

The Log. of $12^f.\ 9'\ 7''.\ 10'''.=12.80439=1.107359$
Multiply by . . . 2

The Product is the Log. of $163.9524=2.214718$

Therefore $163.9524=163^f.\ 11'.\ 5''.\ 1'''.$ the *Area* required.

Examp. II. What is the Solidity of a *Cube* whoſe Side is $1^f.\ 2'.\ 9''.\ 11'''$?

The Log. of $1^f.\ 2'.\ 9''.\ 11'''.=1{,}23553=0.091854$
which multiply by . . . 3

The Product is the Log. of $1{,}8774=0.273562$

Therefore $1{,}8774=1^f.\ 10'.\ 6''.\ 4'''.$ the Solidity ſought.

7. EXTRACTION *of* ROOTS *of* DUODECIMALS *by* LOGARITHMS.

Examp. I. What is the Side of that *Square* whoſe Area is $163^f.\ 11'.\ 5''.\ 1'''$?

The L. of $163.\ 11'.\ 5''.\ 1'''.=163.9524=2.214718$
Divide by . . . 2

The Quotient is the Log. of $12.80439=1.107359$

Thus $12{,}80439=12^f.\ 9'.\ 7''.\ 10'''.$ the Side ſought.

Examp. II. What is the *Side* of that *Cube*, whoſe *Solidity* is $1^f.\ 10'.\ 6''.\ 4'''$?

The

The Log. of 1. 10′. 6″. 4‴.=1.8774=0.273562
Divide by 3

The Quotient is the Log. of 1.23553=0.091854

But 1,23553=1^f. 2′. 9″. 11‴. the *Side* of the *Cube* required.

Theſe few Examples abundantly ſhew with how much more *Eaſe*, *Brevity* and *Expedition* the Operations of *Duodecimals* are performed by *Logarithms*, than by the *ordinary Method.*

CHAP. IX.

The Operation *of the common* Rules *of* Arithmetic *by* Instruments; *viz. the* Logarithmic Scale; *and* Gunter's Line, *with the* Compasses, *and on the* Sliding-Rule.

1. Having in the *Theory* ſhewn the *Nature* and *Conſtruction* of the *Logarithmic Scale*, and *Gunter*'s *Line*; I ſhall here briefly exemplify their Uſes in the Operation of the common Rules of Arithmetic thereby; and in doing of this I ſhall obſerve this Method; firſt, to give the Operation by *Logarithms* in *Numbers*. Secondly, to perform the ſame by the *Logarithmic Scale*. Thirdly, to work the ſame Caſe on the *Gunter* with the *Compaſſes*; and fourthly, to do the ſame thing on the *Sliding-Gunter*. In this Method, the *Analogy* or *Agreement* between the *Numerical* and *Inſtrumental Operations* will more eaſily appear; and the Nature and Reaſon of the latter be much better underſtood by *young Learners*.

2.

2. MULTIPLICATION.

Examp. I. Multiply 9 by 7.

First, by Logarithms.

Add	the Logarithm of	9=0.954242
	the Logarithm of	7=0.845098
The Sum is the Log. of the Prod.		=63=1.799340

3. Secondly, by the Logarithmic Scale.

(*Note.* If the ſmalleſt Diviſions in the Line AB repreſent *Numbers*, the Logarithms begin from C e in the Line Ae; if the middle Diviſions in A B be Numbers, the Logarithms begin from G; but if the largeſt Diviſions in AB be taken for Numbers, then the Logarithms begin from H, in the ſaid Line A e. And ſince the *ſmalleſt Diviſions* are *too ſmall*, and the *largeſt Diviſions* too *large* for Examples, we muſt conſequently chuſe the *mean Diviſions* in A B to repreſent the Numbers 1, 2, 3, 4, *&c.* or 10, 20, 30, 40, *&c.* or 100, 200, 300, *&c.* and ſo the Logarithms begin from G. Therefore) Set one Foot of the Compaſſes in G, and extend the other to the Logarithm of the Multiplier 7=ag, which you'll find to be G a=845, and ſince G b=954 is the Logarithm of the Multiplicand 9=bh; therefore with the ſame Extent G a in the Compaſſes, ſet one Foot in b, the other will fall on d; therefore G d is the Logarithm of the Product dm=63 in A B, the Number ſought.

4. Thirdly, by the *Gunter* with Compaſſes.

Set one Foot of the Compaſſes in the Beginning of the Line at 1, and extend the other to 7; with that Extent in the Compaſſes ſet one Foot in 9, the other will fall on 63, the Product required.

Note.

Note. When the Numbers are *ſmall* the *larger Diviſions* may be uſed, as in the preſent Caſe; but if the Numbers be *large*, the *leſſer Diviſions* muſt be uſed.

5. Fourthly, by the *Sliding-Gunter.*

In this Caſe, there is one Line of Numbers on the *Rule*, and another on the *Slider*, both mark'd with N, at the End. And it is eaſy to conceive that by ſliding one of theſe by the other, the ſame Effects are produced as before with the Compaſſes; that is, any Part of the Line on the Rule is transfer'd to, or compar'd with any other Part of the ſaid Line by means of the ſliding Line.

Therefore ſet 1 on the Slider to 7 in the Line on the Rule; then againſt 9 on the Slider, you find 63 on the Rule, and that is the Product ſought.

6. Examp. II. By the *Gunter.* What is the Product of 27 by 18?

Here the leſſer Diviſions muſt be uſed, and the greater ones reckoned 10, 20, 30, *&c.* on the *firſt Radius*; and conſequently on the *ſecond Radius* they will be 100, 200, 300, *&c.* if the *double Radius* be uſed. For then it will be 10 : 180 :: 27 : the Product ſought. But ſince 10 : 180 :: 1 : 18; therefore if you make 1 : 18 :: 27 : the Product; the *ſingle Radius* will give the Anſwer in the ſame manner; only remembring that the *fourth Number* ſought will be of the ſame Denomination with the *ſecond*, which in this Caſe is *Hundreds.*

Therefore ſet one Foot of the Compaſſes in the Beginning of the Line, and extend the other to 18, the ſame Extent will reach from 27 to 486, the Product ſought.

And by the *Sliding-Rule*, thus; ſet 1 on the Slider to 18 on the Rule, and then againſt 27 on the Slider

you

you find 486 on the Rule, which is the ſame Product as before.

7. Examp. III. What is the Product of 257 by 34?

Take in your Compaſſes the Diſtance from 1 to 34 on the Line of Numbers, the ſame Extent will reach from 257 to 8738, the Product required.

By the Sliding-Rule, thus; Set 1 on the Slider to 34 on the Rule, and againſt 257 on the Slider, you ſee 8738 on the Rule, which is the Product as before.

8. Examp. IV. What is the Product of 215 by 108?

With the Compaſſes, take the Diſtance from 1 to 108 on the *Gunter*, the ſame Extent of the Compaſſes will reach from 215 to 23220, the Product required.

By the Sliding Rule, thus; Set 1 on the Slider to 108 on the Rule, and againſt 215 on the Slider you find 23220 on the Rule, the Product ſought.

9. When the Product becomes ſo large, it muſt be a very large Line of Numbers indeed to ſhew it near the Truth; the Uſe of theſe Lines being principally where the Numbers are ſmall; or where great Exactneſs is not required. They who underſtand the foregoing Doctrine of Logarithms can never be at any loſs to know how many Places of Figures are contained in the Number ſought, in this, or any of the following Rules.

10. DIVISION.

Examp. I. What is the Quotient of 63 divided by 9?

First, by Logarithms.

From the Logarithm of . . .	63=1.799340
Subduct the Logarithm of . . .	9=0.954242
The Diff. is the Log. of the Quot.	=7=0.845098

11. Secondly, by the Logarithmic Scale.

From Gd=1799 the Logarithm of dm=63, take Gb=954 the Logarithm of bh=9; and there will remain Ga=845, the Logarithm of ag=7, the Quotient sought.

12. Thirdly, by *Gunter*'s Line and Compasses.

Set one Foot of the Compasses in 1, and extend the other to 9, and then with that Extent of the Compasses set one Foot in 63, the other will fall (towards the beginning of the Line) on 7, the Quotient sought.

13. Fourthly, by the Sliding-Rule.

Because 9 : 63 :: 1 : the Quotient; therefore set 9 on the Slider to 63 on the Rule, and then against 1 on the Slider is 7 on the Rule, which is the Quotient sought.

14. Examp. II. What is 486 divided by 18?

By the *Gunter* and Compasses.

Extend the Compasses from 1 to 18, that Extent will reach from 486 (downward) to 27, the Quotient required.

By the Sliding-Rule.

Set 18 on the Slider to 486 on the Rule, then against 1 on the Slider you find 27 on the Rule, the Quotient sought.

All other Operations of Division being performed in the very same manner, 'tis needless to add any more Examples of this kind.

15.

15. INVOLUTION.

Examp. I. What is the Square of 9?

First, by Logarithms.

The Logarithm of . . .	9=0.954242
Multiply by the Index . . .	2
The Prod. is the Log. of the Square =81=	1,908484

16. Secondly, by the Logarithmic Scale.

Let the Logarithms begin from G in the Line A e, as before; then with the Compaſſes take the Diſtance g b=954 the Logarithm of b h=9; and with one Foot remaining in b, turn the Compaſſes, the other Foot will fall on n; then ſhall G n=(2Gb=)1908 the Logarithm of n o=81, which therefore is the *ſquare Number* ſought.

17. Thirdly, by the *Gunter* and Compaſſes.

Set one Foot in 1, and extend the other to 9, where keep it fix'd, and turn the Compaſſes, the other Foot will fall on 81, the *Square* ſought.

18. Fourthly, by the Sliding-Rule.

Becauſe 1 : 9 :: 9 : the Square required, therefore ſet 1 on the Slider to 9 on the Rule, then againſt 9 on the Slider is 81 on the Rule; which is the *ſquare Number* ſought.

19. Examp. II. What is the Cube of 9?

By the *Gunter* and Compaſſes.

Extend the Compaſſes from 1 to 9, that Extent will reach from 9 to 81, and again from 81 to 729, the Cube Number required.

By the Sliding-Rule.

Set 1 on the Slider to 9 on the Rule, then againſt 9 on the Slider is 81 on the Rule, and againſt 81 on the Slider (remaining unmov'd) is 729 on the Rule, the Cube Number required.

20. Examp. III. What is the Square and Cube of the Number 37?

By the *Gunter* and Compaſſes.

Extend the Compaſſes from 1 to 37, that Extent will reach from 37 to 1369 the *Square*; and the ſame Extent will reach from 1369 to 50653 the *Cube*; both as required.

By the Sliding-Rule.

Set 1 on the Slider to 37 on the Rule, then againſt 37 on the Slider is 1369 on the Rule, which is *Square*; and againſt 1369 on the Slider (remaining unmov'd) is 50653 on the Rule, which is *Cube* of 37; both as before.

21. EXTRACTION *of* ROOTS.

Examp. I. What is the *Square Root* of 81?

Firſt, by Logarithms.

The Logarithm of	81=1.908484
which divide by the Index	2
The Quotient is the Log. of the *Square Root*	=9=0.954242

22. Secondly, by the Logarithmic Scale.

Biſect Gn=1908 the Logarithm of no=81, in b; then ſhall Gb=954 be the Logarithm of the *Square Root*, viz. bh=9, the Number ſought.

23. Thirdly, by the *Gunter* and Compaſſes.

Take with the Compaſſes the Diſtance between 1 and 81, and biſect it; then take one Half in the Compaſſes, and it will reach from 1 to 9, the *ſquare Root* ſought.

24. Fourthly, by the Sliding-Rule.

Move the Slider forwards and backwards till you make the ſame Number on the Rule anſwer 1 on the Slider, as on the Slider anſwers 81 on the Rule; which Number will be the *ſquare Root* ſought, and in the preſent Caſe will be found 9.

25. Examp. II. What is the Cube Root of 50653?

Divide the Diſtance between 1 and 50653 into 3 equal Parts; the firſt Diviſion will fall on 37, the *Cube Root* required.

Note, In the double Line of Numbers, if the grand Diviſions be eſteem'd *Units* in the firſt Radius, thoſe in the ſecond Radius will be *Tens*; if thoſe in the firſt be *Tens*, viz. 10, 20, 30, *&c.* thoſe of the ſecond will be *Thouſands*, as 1000, 2000, 3000, *&c.* with regard to *ſquare Numbers:* and conſequently in *Extraction*; if the Number whoſe ſquare Root is ſought be leſs than 100, yet greater than 10, the Number it ſelf will be found on the *ſecond Radius*; and its Root a Number of Units on the *firſt Radius*. But if the *Square* be leſs than 10, both it ſelf and Root will be found in the *firſt Radius*. Again, if the *ſquare Number* be between 1000 and 10000, the Number it ſelf will be found on the *ſecond Radius*, and its Root a Number of *Tens* on the firſt Radius. But if it be between 100 and 1000, both the Number and its Root of Tens will be found on the *firſt Radius*. After the ſame manner you may reaſon concerning the *Cube Number* and its *Root*.

26. Since the Logarithm of the *Square* is *double* the Logarithm of the *Root*; and the Logarithm of the *Cube triple* the Logarithm of the *Root*: therefore if a Line of Numbers of a *ſingle Radius*, be equal to another of a *double Radius*, and theſe two appoſitely laid together, beginning from the ſame Point; then againſt any Number on the *ſingle Radius*, you ſee its correſpondent *Square* on the *double Radius*; and ſuch Lines you have on ſome *Sliding-Rules.*

Alſo if a Line of *ſingle Radius* were made equal to another of a *triple Radius*, and theſe exactly and properly placed together, then the Numbers on the *latter* would be the Cubes of thoſe on the former; and ſo the *Square* and *Cube Roots* of any Number; and *vice verſâ*, would be diſcoverable by *Inſpection*.

27. Moreover by means of a *ſingle* and *double Line* of *Numbers* made to ſlide by each other, 'tis very eaſy to find a *mean Proportional* between any two given Numbers; as ſuppoſe 13 and 23. Thus; ſet 13 on the *double Line* to 13 on the *ſingle one*, then againſt 23 on the *double Line* is 17,35 on the *ſingle one*, which is the Mean required between 13 and 23. Or if you ſet 23 to 23, then againſt 13 on the *double* you find 17,35 on the *ſingle Line*, the *Mean* required as before.

28. In like manner, by means of a *ſingle* and a *triple* Line of Numbers, *two mean Proportionals* may be eaſily found between any two Numbers, as 2 and 54; thus; ſet 2 on one Line to 2 on the other, then againſt 54 on the triple Line is 6 on the ſingle one, which is the *firſt Mean*; then ſet 2 on the *triple Line* to 6 on the *ſingle one*, and againſt 54 is 18, the *ſecond Mean*, on the ſingle Line; ſo the four Numbers are 2 : 6 :: 18 : 54. And thus you may find *two Means* between any other two Numbers, which in many Caſes is a moſt uſeful Problem.

29. In the foregoing Operations I have made no mention of *Decimals*, becauſe they are to be reſpected

as Whole Numbers in the Management of them by *Instrumental Operations*, in the same manner as they were by *Numerical Logarithms*; the Number of *decimal* Places in any *Product*, *Quotient*, *Power*, *Root*, &c. being always determined here, as in all other Methods of working them, by the Rules proper to *Decimal Arithmetic*.

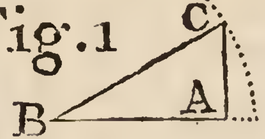
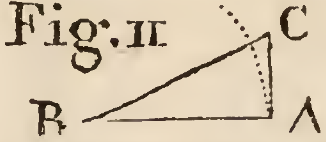
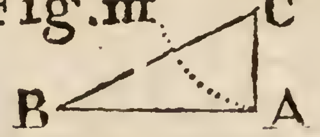
30. Thus it appears what *Similarity*, *Coherence*, and mutual Relation there is between the foregoing Methods of solving *Arithmetical Questions* by Logarithms, both *Numerical* and *Instrumental*; and that they are all one in Nature, and differ only in the *Modus operandi*, or Manner and Form of working. By this Chapter, I presume, 'twill be easy for the Learner to observe how any common Question in *Arithmetic*, or the Mensuration of *Artificers* Work, as *Joinery*, *Masonry*, *Carpentry*, *Painting*, *Timber-Measure*, *Gauging*, &c. may be most readily performed by the *Line of Numbers*, with Compasses, or by the *Sliding-Rule*, which is much the best Way.

CHAP. X.

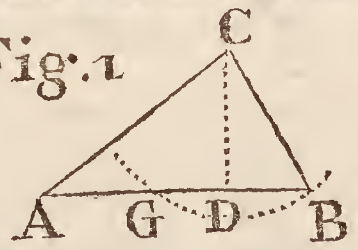
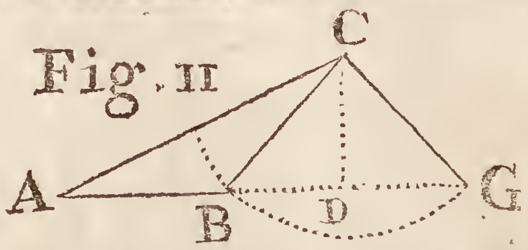
Sheweth the ANALOGIES *or* PROPORTIONS *for the* SOLUTION *of all the* CASES *of* Plain *and* Spherical TRIANGLES, *both* Right *and* Oblique *angled.*

1. IF any desire to be throughly instructed in the *Theory* of *Plain* and *Spherical Trigonometry*, I must refer them to my *Young Trigonometer's Guide*; since all I intend here is only to shew the *great* and *most excellent Use* of *Logarithms* in the practical Resolution of *Plain* and *Spherical Triangles*. A Synopsis

nopſis of all the Caſes of a Right-angled plain Triangle here follows.

Caſes	Given	Sought	Fig. I 	Fig. II 	Fig. III 
I	B AB	AC BC	sC : AB :: sB : AC. sB : AC :: R : BC.	R : AB :: tB : AC. R : AB :: seB : BC.	tC : AB :: R : AC. R : AC :: seC : BC
II	B AC	AB BC	sB : AC :: sC : AB. sB : AC :: R : BC.	tB : AC :: R : AB. R : AB :: seB : BC.	R : AC :: tC : AB. A : AC :: seC : BC.
III	B BC	AB AC	R : BC :: sC : AB. R : BC :: sB : AC.	seB : BC :: R : AB. R : AB :: tB : AC.	seC : BC :: tC : AB. tC : AB :: R : AC.
IV	AB AC	B BC	$\sqrt{ABq+ACq}=BC$.	AB : R :: AC : tB. R : AB :: seB : BC.	AC : R :: AB : tC. R : AC :: seC : BC.
V	AB BC	AC C	BC : R :: AB : sC. R : BC :: sB : AC.	AB : R :: BC : seB. R : AB :: tB : AC.	$\sqrt{BCq-ABq}=AC$.
VI	AC BC	B AB	BC : R :: AC : sB. R : BC :: sC : AB.	$\sqrt{BCq-ACq}=AB$.	AC : R :: CB : seC. R : AC :: tC : AB.

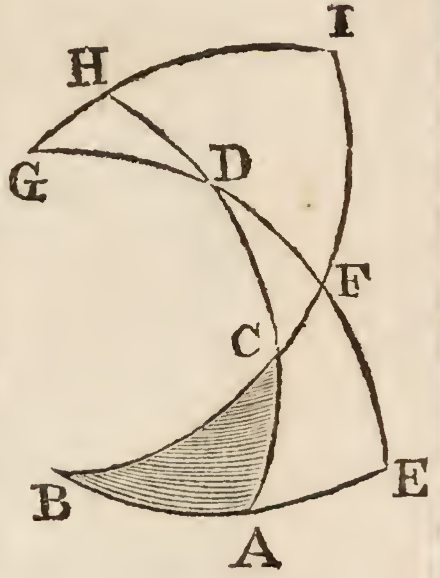
2. A Synopſis of the Analogies for the Solution of all the Caſes of Oblique-angled plain Triangles.

Caſes	Given	Sought	Fig. I  Fig. II 
I	AB C, A, B.	AC. BC.	sC : AB :: sB : AC sC : AB :: sA : BC.
II	AC BC A	AB C, B	BC : sA :: AC : sB. sB : AC :: sC : AB.
III	AC BC C	A, B, AB	$AC+CB : AC-CB :: t\,\frac{A+B}{2} : t\,\frac{A-B}{2}$. Whence the Angles are known.
IV	AB AC CB	A, B, C.	$AB : AC+CB :: AC-CB : AD\mp DB$. In the firſt Triangle, 'tis $AD-DB$ } AG in the ſecond Triangle, $AD+DB$ } AG Then $AB-AG=GB=2DB$. Thus each Triangle is reduced to two Right ones, *viz.* ADC, and BDC; in either of which two Sides are known whence the Angle C may be eaſily found.

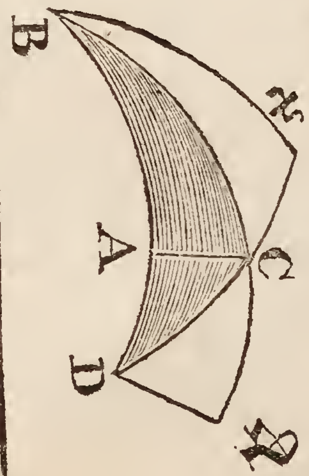
3. A Synopsis of the Canons, and the Analogies (form'd from them for the Solution of all the Cases of Right-angled Spherical Triangles.

Cases	Given	Sought	Canons.	Analogies.
I	AB B	AC BC C	sBE : sBA :: tFE : tAC. sDE : sDF :: tAE : tCF.	R : sBA :: tB : tAC. R : csB :: ctAB : ctBC. R : csAB :: sB : csC.
II	AC B	AB BC C	tEF : tAC :: sBE : sAB. sEF : sAC :: sBF : sBC. sCD : sCH :: sDF : sHI.	tB : tAC :: R : sAB. sB : sAC :: R : sBC. csAC : R :: csB : sC.
III	BC B	AB AC C	sDF : sDE :: tCF : tAE. sBF : sBC :: sEF : sAC. sCF : sCI :: tDF : tHI.	R : csB :: tBC : tAB. R : sBC :: sB : sAC. csBC : R :: ctB : tC.
IV	AB AC	BC B C	sAD : sDC :: sAE : sCF. sAB : sBE :: tAC : tEF.	R : csCA :: csAB : csBC. sAB : R :: tAC : tB. sAC : R :: tAB : tC.
V	BC AC	AB B C	sDC : sDA :: sCF : sAE. sBC : sBF :: sAC : sEF. tIF : tDH :: sGI : sGH.	csAC : R :: csBC : csAB. sBC : R :: sAC : sB. tBC : R :: tAC : csC.
VI	B C	AB AC BC	. . . . sHI : sDF :: sCH : sCD. tHI : tDF :: sCI : sCF.	sB : csC :: R : csAB. sC : csB :: R : csAC. tC : ctB :: R : csBC.

The Triangle.



A Synopſis of Canons and Analogies for the Solution of all the Caſes of que-angled Spherical Triangles.

Caſes	Given	Sought	I. Analogies.	II. Analogies.	The Triangle.
I	B D BC	CD C BD	. . . . csBC : R :: ctB : tBCA. R : csB :: tBC : tAB.	sD : sBC :: sB : sCD. csB : sBCA :: csD : sDCA. tD : tB :: sAB : sAD.	
II	B C BC	D DC BD	R : csBC :: tB : ctBCA. R : csBC :: tB : ctBCA. R : csBC :: tC : ctCBℵ.	sBCA : sDCA :: csB : csD. csDCA : csBCA :: tBC : tDC. csDBℵ : csℵBC :: tBC : tBD.	
III	BC CD B	BD C D	R : csB :: tBC : tAB. R : csBC :: tB : ctBCA.	csBC : csBA :: csDC : csDA. tDC : tBC :: csBCA : csDCA. sDC : sB :: sBC : sD.	
IV	BC BD B	CD D C	R : csB :: tBC : tBA. R : csB :: tBC : tBA. R : csB :: tBD : tB𝔐.	csBA : csBC :: csDA : csDC. sDA : sBA :: tB : tD. sC𝔐 : sB𝔐 :: tB : tC.	
V	BC BD CD	C	$sBC \times sCD : Rq :: s\frac{BD+AM}{2} \times s\frac{BD-AM}{2} : sq\tfrac{1}{2}C.$ *Note*, Here AM=BC—CD. And q = Square. Then the Angle C being known, the other Parts are eaſy found.		
VI	B C D	BD	Change the Angles into Sides (taking the Complement of the greateſt Angle C,) and this Caſe will then be the ſame with the laſt foregoing. For the Sides of this Triangle		

5. Being thus furniſhed with Proportions, we ſhall ſoon ſee with what incomparable Pleaſure and Eaſe the ſeveral Caſes of Triangles before going are reſolved by the Canon of Logarithmical *Sines*, *Tangents*, and *Secants* ; and alſo by the *Line of Numbers*, both with the *Compaſſes* and by the *Sliding-Rule* ; I ſay, we ſhall ſee in the two next Chapters, with how much more Eaſe and Pleaſure they are reſolved by *theſe two Methods*, above what is attainable by any other Way yet invented for this Purpoſe.

CHAP

CHAP. XI.

The SOLUTION *of* PLAIN TRIANGLES *by the* CANON *of* LOGARITHMICAL SINES *and* TANGENTS; *by* GUNTER'S SCALE *and* COMPASSES; *and by the* SLIDING RULE.

1. I Have already deſcribed and taught the Uſe of the Logarithmic Canon, ſo far as to find the Logarithms of any *Number*, *Sine*, *Tangent*, or *Secant* propoſed. I ſhall therefore here only obſerve, that what I call *Gunter*'s *Scale* is ſuch a *Plane Scale* as hath upon it *Gunter*'s *Line of Numbers*, and of *Artificial Sines* and *Tangents*; whoſe Nature, Conſtruction, and Deſign, have been before diſcourſed of in the Theory. The *Sliding-Rule* has the ſame Lines, which are contrived to ſlide by one another as you pleaſe; and to avoid Repetitions, I ſhall call the Line of Numbers on the Rule it ſelf, A; and that on the Slider, B; alſo I ſhall call the Lines of Sines and Tangents on the Rule Sr, Tr; and thoſe on the Slider Ss, and Ts. You muſt know alſo that the End of each Line marked 10, 90, 45, is here called Radius. Having premiſed theſe things, we proceed immediately to the Solution of

2. *Right-angled Plain Triangles.*

Caſe I. In the Right-angled Triangle ABC, there is given the Baſe AB=285, and the Angle at Baſe B=32° 48′; to find the Perpendicular AC, and the Hypothenuſe BC. The Hypoth. Radius, Fig. I.

The Analogy for AC is, sC : AB :: sB : AC.

In

In Words;

As the Sine of the Angle C=57° 12′=9.924572

is to the given Side or Base AB=285=2.454845 } add
So is the Sine of the Ang. B=32° 48′=9.733765 }

12.188610

To the Perpendicular AC=183,67=2.264038

The Analogy for BC is, sB : AC :: R : BC.

In Words;

As the Sine of . . .	B=32° 48′	=9.733765
is to the Perpendicular	AC=183,67	=2.264038
So is Radius	90° 00′	=10.000000
to the Hypothenuse . . .	BC=339,06	=2.530273

3. *By Scale and Compasses. To find* AC.

Set one Foot of the Compasses to 32° 48′=B, in Line of Sines, and extend the other upwards to 57° 12′=C; the same Extent will reach from 285 =AB downwards in the Line of Numbers to 183,67 =AC, the Perpendicular sought.

To find BC.

Extend the Compasses from 32° 48′=B to Radius 90° in the Line of Sines; the same Extent will reach (in the Line of Numbers) from 183,67=AC, to 339,06=BC, the Hypothenuse sought.

4. *By the Sliding-Rule. To find* AC.

Make the Line of Numbers slide by the Line of Numbers, then will the Lines of Sines slide by each other, and also the Line of Tangents by the Line of Tangents. Having thus prepared the Rule; set

57°

$57^\circ\ 12'$ on Ss to $32^\circ\ 48'$ on SR; then againſt 285 on B is 183,67 on A, the Perpendicular ſought.

To find BC.

Set $32^\circ\ 48'$ on Ss to Radius 90° on SR, then againſt 183,67 on B is 339,06 on A, which is the Hypothenuſe required.

5. Caſe II. Given the Angles $B=32^\circ\ 48'$, and $C=57^\circ\ 12'$, and the Side $AC=183,67$; to find AB, and BC.

The Baſe made Radius. Fig. II.

As the Tangent of the Ang.	$B=32^\circ\ 48'=9.809193$
is to the Side	$AC=183,67=2.264038$
So is Radius	$90^\circ\ 00'=10.000000$
to the Side or Baſe . . .	$AB=285=2.454845$

To find BC.

As Radius	$90^\circ\ 00'=10.$
is to the Side	$AB=285=\ 2.454845$
So is the Secant of the Angle	$B=32^\circ\ 48'=10.075428$
to the Side or Hypoth.	$BC=339,06=\ 2.530273$

Note, I have wrought this laſt in *Secants* for Variety ſake, and that the Reader may ſee the Concluſions are the ſame every Way. But this Caſe is much better reſolved by making BC Radius, as in Fig. I.

6. *By the Plane Scale and Compaſſes.*

As there is no *Line* of *Artificial Secants* on the *Scale* or *Sliding-Rule* (as being uſeleſs) ſo this Caſe will be beſt performed Inſtrumentally by the Analogies of Fig. I. where BC is made Radius.

Therefore

Therefore, extend the Compasses from 32° 48′ to 57° 12′ in the Line of Sines, the same Extent will reach from 183,76 to 285 in the Line of Numbers; thus 285=AB, the Side required.

Or thus, By the first Analogy of this Case of Fig. II. extend the Compasses from 32° 48′ to Radius 45 in the Line of Tangents, the same will reach from 183,67 to 285 =AB (as before) in the Line of Numbers.

To find BC.

Extend the Compasses from 32° 48′ to Radius 90 in the Line of Sines, the same will reach from 183,67 to 339,06=BC; in the Line of Numbers.

7. *By the Sliding-Rule.*

To find AB.

Set 32° 48′ on Ts to Radius 45° on TR; then against 183,67 on B is 285=AB, on A.

To find BC.

Set 32° 48′ on Ss to Radius 90° on SR; then against 183,67 on B is 339,06=BC, on A.

8. Case III. Given the Angles B=32° 48′ and C=57° 12′, and the Hypothenuse BC=339,06; to find the Sides AB and AC.

The Hypothenuse made Radius. Fig. I.

As Radius	90° 00′.=10.
is to the Hypothenuse	BC=339,06=2.530273
So is the Sine of the Angle	C=57° 12′=9.924572
To the Side or Base	AB=285=2.454845
And so is the Sine of the Ang.	B=32° 48′=9.733765
to the Side	AC=183,67=2.264038

Or

Or thus; AB made Radius, Fig. II. To find AC.

As Radius	90° 00=10.
is to the Side	AB=285=2.454845
So is the Tangent of	B=32° 48′=9.809193
to the Side	AC=183,67=2.264038

9. *By the Scale and Compasses.*

Extend the Compasses from Radius 90 to 57° 12′ in the Line of Sines, the same Extent will reach from 339,06 to 285=AB, in the Line of Numbers.

And then again, extend them from 90 to 32° 48′ in the Line of Sines, the same will reach from 339,06 to 183,67=AC, in the Line of Numbers.

10. *By the Sliding-Rule.*

This Case may be solved by once setting the Rule, thus; Make the Line of Sines to slide by the Line of Numbers: Then set Radius 90 on Ss to 339,06 on A; thus against 57° 12′ on Ss you see 285=AB, and against 32° 48′ is 183,67=AC, on A. Such is the great Conveniency of this small Instrument.

11. Case IV. Given the two Sides, AB=285 and AC=183,67; to find the Side BC, and the Angles B and C.

The Base made Radius, Fig. II. To find B.

As the Side	AB=285=2.454845
is to Radius	10.
So is the Side	AC=183,67=2.264038

To the Tangent of the Angle B=32° 48′=9.809193

Or thus, making AC Radius, Fig. III. To find C.

As the Side ..... AC=183,67=2.264038
is to Radius ..... 10.
So is the Side ..... AB=285=2.454845

To the Tang. of the Angle C=57°48′=10.190707

12. To find the Side BC.

This may be done directly with the Secants, or by finding the Angles first, with the Sines; as is manifest from the Synopsis. But since if the required Side BC be made Radius, it can't be found by the common Analogies of *Trigonometry*; I have given an Equation formed on the Principles of *Geometry* for that purpose, *viz.* $\sqrt{ABq+ACq}=BC$; which I shall resolve by Logarithms, as follows.

The Log. of the Side ... AB=285=2.454845
the Double thereof is the Log. of ... ABq=81225=4.909690

Also the Log. of the Side AC=183,67=2.264038
The Double of which is the Log. of ACq=33734,66=4.528076
Add the Square ..... ABq=81225

The Log. of the Sum ABq+ACq=114959.66=5.060546

Half which is the Log. of
$\sqrt{ABq+ACq}$=BC=339,06=2.530273
the Side required.

13. *By Scale and Compasses.* To find the Ang. B.

Extend the Compasses from 285 to 183,67 in the Line of Numbers, the same will reach from Radius 45° to 32° 48′, in the Line of Tangents, the Angle B required.

By

By the Sliding-Rule.

Set 285 on A to 183,67 on B, then againſt Radius 45° on TR is 32° 48′= the Angle B, on Ts.

Having thus found the Angles, the Side BC is found as in the foregoing Caſes.

14. Caſe V. Given the Hypothenuſe BC=339,06 and the Side AB=285; to find the Angles B and C, and the Side AC.

To find the Angle C. Fig. I. BC Radius.

As the Side	BC=339,06=2.530273
is to Radius	10.
So is the Side	AB=285=2.454845
To the Sine of the Angle	C=57° 12′=9.924572

To find the Side AC.

As Radius	10.
is to the Side	BC=339,06=2.530273
So is the Sine of the Angle	B=32° 48′=9.733765
to the Side	AC=183,67=2.264038

Note, The Side AC may be found *Geometrically*, as taught in Art. 12. the Equation being $\sqrt{BCq-ABq}=AC$.

15. *By Scale and Compaſſes.* To find the Angle C.

Extend the Compaſſes from 339,06 to 285 in the Line of Numbers, the ſame will reach from Radius 90° to 57° 12′=C, in the Line of Sines.

By the Sliding-Rule.

Set 339,06 on A to 285 on B, then againſt Radius 90° on SR is 57° 12′=C, on Ss.

The Angles being thus found, the Side AC may be found by *Scale* or *Sliding-Rule*, as before.

16. Caſe VI. Given the Side AC=183,67 and BC =339,06; to find the Angles B, C, and the Side AB.

As this Caſe is, in the Nature of it, the ſame as the laſt, ſo the Solution is in all reſpects the ſame, and needs not be repeated.

17. *Of Oblique-angled Plain Triangles.*

Caſe I. There is given the Angles C=82° 30′, B=60° 00′; and the Side AB=365; to find the other two Sides AC and BC.

Note, The firſt Triangle in the Synopſis of Oblique Plain Triangles is that which I have regard to here, and is acute-angled; the Difference between this and the obtuſe-angled one, Fig. II. will be taken notice of as I go along.

To find the Side AC. Fig. I. Co. Ar.

As the Sine of the Angle C=82° 30′=0.003732
is to the Side . . . . . AB=365°=2.562293
So is the Sine of the Angle B=60° 00′=9.937531

to the Side ſought . . . . AC=318,82=2.503556

To find the Side BC. Co. Ar.

As the Sine of the Angle C=82° 30′=0.003732
is to the Side . . . . . AB=365=2.562293
So is the Sine of the Angle A=37° 30′=9.784447

to the Side required . . . . BC=224,11=2.350472

18.

18. *By the Scale and Compaſſes.* To find AC.

Extend the Compaſſes from 82° 30′ to 60° in the Line of Sines, and the ſame Extent will reach from 365 to 318,82 in the Line of Numbers; therefore 318,82=AC.

To find BC.

Extend the Compaſſes from 82° 30′ to 37° 30′ in the Line of Sines, the ſame will reach from 365 to 224,11=BC, in the Line of Numbers.

19. *By the Sliding-Rule.*

To find AC.

Set 82° 30′ on SR to 60° 00′ on Ss; then againſt 365 on A, you have 318,82=AC, on B.

To find BC.

Set 82° 30′ on SR to 37° 30′ on Ss; then againſt 365 on A, you have 224,11=BC, on B.

But ſince all the Angles are known, both the unknown Sides are found at once ſetting the Rule thus;

Let the *Line* of *Numbers* ſlide by the *Line of Sines*; and ſet 182° 30′ on SR to 365 on B, then againſt 60° 00′ is 318,82=AC; and againſt 37° 30′ is 224,11=BC, on the Line of Numbers.

20. Caſe II. Given two Sides, AC=318,82 and BC=224,11; and the oppoſite Angle A=37° 30′; to find the Angle B, and the other Side AB.

To find the Angle B.	Co. Ar.
As the Side	BC=224,11=.7.649528
is to the Sine of the Angle	A=37° 30′=9.784447
So is the Side	AC=318,82=2.503556
to the Sine of the Angle . .	B=60° 00′=9.937531

The

The Angles being thus known, the Side AB is found as were the Sides AC and BC in the foregoing Caſe.

21. *By the Scale and Compaſſes.*

To find the Angle B.

Extend the Compaſſes from 224,11 to 318,82 in the Line of Numbers, the ſame Extent will reach from $37^\circ\ 30'$ to $60^\circ\ 00'$=B, on the Line of Sines.

By the Sliding-Rule.

Set 224,11 on A to 318,82 on B, then againſt $37^\circ\ 30'$ on SR is $60^\circ\ 00'$ =B, the Angle ſought, on Ss.

Note, In this Caſe of the Obtuſe-angled Triangle, Fig. II. 'tis obvious the Angle here found is the outward Angle CBG, whoſe Complement therefore to 180 Degrees is equal to the internal obtuſe Angle ABC=$120^\circ\ 00'$.

22. Caſe III. Given two Sides AC = 318,82 and BC 224,11; and the included Angle C = $82^\circ\ 30'$; to find the Angles A, B, and the Side AB.

To find the Angles A and B.

The Sum of the given Sides is AC+BC=542,93; their Difference is AC—BC=94,71; the Sum of the unknown Angles A+B=$97^\circ\ 30'$; therefore the half Sum is $\frac{A+B}{2} = 48^\circ\ 45'$; whence by the Analogy in the Synopſis, ſay;

As

		Co. Ar.
As the Sum of the Sides	... $AC+BC=542,93=$	.7.265256
is to their Difference	$AC-BC=94,71=$	1.976396
So is the Tangent of the half Sum of the unknown Angles	$\frac{A+B}{2}=48^\circ\ 45'=$	10.057012
to the Tangent of half their Difference	$\frac{A-B}{2}=11^\circ\ 15'=$	9.298664

Then to the half Sum of the Angles	$48^\circ\ 45'$
add the half Difference	$11^\circ\ 15'$
The Sum is the greater Angle	$B=60^\circ\ 00'$
But the Diff. is the leſſer Ang.	$A=37^\circ\ 30'$

23. *By Scale and Compaſſes.*

To find the Half Difference of the Angles $\frac{A-B}{2}$.

Having prepared the Work as above, proceed thus;

Extend the Compaſſes from the Sum of the Sides 542,93 to their Difference 94,71 on the Line of Numbers; with this Extent ſet one Foot of the Compaſſes in Radius 45°, and pitch the other downwards in the Line of Tangents, where fix it while you bring the other Foot from 45° to 48° 45′; then with this Extent apply one Foot in 45°, the other will reach to 11° 15′= the half Difference of the Angles A and B; which therefore may be found as before.

Or if you have a Line of Tangents continued beyond 45°, then the Extent from 94,71 to 542,93 in the Line of Numbers will reach from 48° 45′ to 11° 15′ in the Tangent Line.

24.

24. *By the Sliding-Rule.*

Set 542.93 on A to 94,71 on B; then againſt 45° on TR obſerve the Degree and Minute on Ts, and bring that Point to 48° 45′ on TR; then againſt 45° on TR you have 11° 15′ on Ts, which is the half Difference as before, of the two enquired Angles A and B.

Or thus, if the Line of Tangents be continued on the Slider beyond 45°; ſet 542,93 on A to 48° 45′ on Ts, then againſt 94,71 on A is 11° 15′ on Ts.

Having therefore the *Half Sum*, and *Half Difference* of thoſe Angles, they are found as in Art. 22. and then the Side AB will be found to be 365 as in Caſe I. hereof.

25. Caſe IV. Given all three Sides AB=365, AC=318,82, and CB=224,11, to find the Angles.

In order to this 'twill be neceſſary to reduce the oblique Triangle into two Right-angled ones ADC, and BDC, thus; Find the Sum of any two Sides AC+CB=542,93; and their Difference AC—CB =94,71; eſteeming the other Side AB the Baſe, the Difference of whoſe Segments AD—DB=AG, is firſt of all to be found by this Proportion, *viz.*

		Co. Ar.
As the Baſe	AB=365=	.7.437707
is to the Sum of the two Sides	AC+BC=542,93=	2.734744
So the Diff. of the two Sides	AC—BC=94,71=	1.976396
to the Diff. of the Segments	AD—DB=AG=141=	2.148847

Therefore

Therefore AB—AG=224=GB=2BD; therefore BD=112, and AD=253; and ſo the whole Triangle ACB is reduced to two Right-angled ones ADC and BDC, in each of which there is two Sides given AD and AC, DB and BC; by which means the Angles ACD and DCB may be found, which together are equal to the Angle ACB; and this being known, the other two are found with Eaſe by Caſe I. And thus the whole oblique Triangle is reſolved.

Note, When the Perpendicular falls without, as in the obtuſe-angled Triangle ABC, Fig. II. then it will be the Sum of the Segments AD+DB=AG; and the Difference of the Angles ACD—BCD=ACB, the Angle required.

26. And ſince the ſeveral Problems of *Navigation*, whether in the *Plain*, *Mercator's*, *Middle Latitude*, *Oblique*, or *Traverſe Sailing*; as alſo of meaſuring *Heights* and *Diſtances*, *acceſſible* and *inacceſſible*; of *Fortification*, *Gunnery*, and divers Parts of *Aſtronomy*, &c. are all reſolved by the *Doctrine* of *Plain Trigonometry*, as in the Method before-going; it muſt be very eaſy for any who underſtands the *Solution* of Plain Triangles, to apply it to any practical Caſes that may occur in any of the aforeſaid Arts, without any farther Inſtructions or Examples. Yet thoſe who would ſee the *Theory* of *Plain Trigonometry*, and its *Application*, in the largeſt Extent, may find it in the firſt Vol. of my *Young Trigonometer's Guide*.

CHAP. XII.

The SOLUTION *of* SPHERICAL TRIANGLES *by* LOGARITHMS, *by* GUNTER'*s* SCALE, *and by the* SLIDING RULE.

1. AS in the foregoing Chapter I have exemplified the Reſolution of *Plain Triangles* both by the *Canon* of *Logarithms*, and *Logarithmical Inſtruments*, ſo I ſhall purſue the ſame Method here with reſpect to *Spherical Triangles* of both kinds; in each of which there are *ſix different Caſes*, which in all their *Varieties* are reſolvable according to the *Analogies* aſſigned in the *Synopſis*; on which, and on the Figure of the Triangle there, the Reader is deſired to have his eye, thro' the whole Courſe of Examples. To begin then with

2. *Right-angled Spherical Triangles.*

Caſe I. Given the Baſe AB$=38^{\circ}\ 15'$, and Angle at Baſe B$=39^{\circ}\ 56'$ to find the Perpendicular AC, the Hypothenuſe BC, and the Angle C.

1. To find the Perpendicular AC.

As Radius	$90^{\circ}\ 00'$	$=10.000000$
is to the Sine of the Side AB	$=38^{\circ}\ 15'$	$=\ 9.791757$
So is the Tang. of the Ang. B	$=39^{\circ}\ 56'$	$=9.922787$
to the Tang. of the Perp. AC	$=27^{\circ}\ 23'\frac{3}{4}$	$=9.714544$

2.

2. To find the Hypothenuſe BC.

As Radius .... 90° 00′=10.000000
to the Co-Sine of the Angle B=39° 56′=9.884677
So is the Co-T. of the Side AB=38° 15′=10.103288

to the Co-T. of the Hypoth. BC=45° 48′=9.988065

3. To find the Angle C.

As Radius .... 90° 00′=10.000000
to the Co-Sine of the Side AB=38° 15′=9.895045
So is the Sine of the Angle B=39° 56′=9.807465

to the Co-Sine of the Angle C=59° 44′=9.702510

3. *By Scale and Compaſſes.*

1. To find AC.

Extend the Compaſſes from 90° to 39° 56′ in the Line of Sines, that will reach from 38° 15′ in the Line of Tangents to 27° 23′$\frac{3}{4}$=AC, the Side required.

2. To find the Hypoth. BC.

Extend the Compaſſes from Radius 90° to the Co-Sine of B, 50° 04′ in the Line of Sines; then apply that Extent from 45° in the Line of Tangents downwards, where fix that lower Foot, and bring the other to 51° 45′ the Co-Tangent of AB, this laſt Extent will reach from 45° to 44° 12′ the Co-Tangent of BC=45° 48′, the Side required.

3. To find the Angle C.

Extend the Compaſſes from 90° to 51° 45′ the Co-Sine of AB, the ſame will reach from the Sine 39° 56′ to 30° 16′ the Co-Sine of C=59° 44′.

4. *By the Sliding-Rule.*

1. To find AC.

Set 90° on SR to 38° 15′ on Ss; then againſt 39° 56′ on TR is 27° 23′¾=AC, on Ts.

2. To find BC.

Set 90 on SR to 50° 04′ the Co-Sine of B, on Ss; and mark the Degree and Minute on Ts againſt 45° on TR, bring that Point to 51° 45′ (the Co-Tangent of AB) on TR, on which againſt 45°, you have 44° 12′ on Ts, the Co-Tangent of BC=45° 48′, as required.

3. To find the Angle C.

Set 90° on SR to the Co-Sine of AB, 51° 45′ on Ss; then againſt 39° 56 on SR is 30° 16′ on Ss, the Co-Sine of C=59° 44′, as required.

5. Caſe II. Given the Perpendicular AC=27° 23′, and the oppoſite Angle B=39° 56′; to find the Side AB, the Hypothenuſe BC, and Angle C.

To find AB.

As the Tangent of	B=39° 56′	=9.922787
is to the Tangent of . .	AC=27° 23′¾	=9.714544
So is Radius	90° 00′	=10.
to the Sine of the Side required	AB=38° 15′	=9.791757

The Operations of this Caſe being only the Converſe of the foregoing, needs no further Examples, in Numbers.

6.

6. *By Scale and Compasses.*

1. To find AB.

Extend the Compasses from $39^\circ\ 56'$ to $27^\circ\ 23'\frac{3}{4}$ on the Tangents, the same will reach from 90° on the Sines, to $38^\circ\ 15'$=AB.

2. To find BC.

Extend the Compasses from $39^\circ\ 56'$ to $27^\circ\ 23'\frac{3}{4}$ on the Sines, the same will reach from 90° to (the Sine of BC=) $45^\circ\ 48'$, as required.

3. To find the Angle C.

Extend the Compasses from (the Co-Sine of AC) $62^\circ\ 36'\frac{1}{4}$ to 90°, the same will reach from (the Co-Sine of B) $50^\circ\ 04'$ to $59^\circ\ 44'$ the Sine of the Angle C required.

7. *By the Sliding-Rule.*

1. To find AB.

Set $39^\circ\ 56'$ on TR to $27^\circ\ 23'\frac{3}{4}$ on Ts, then against 90° on SR is $38^\circ\ 15'$=AB, on Ss.

2. To find BC.

Set $39^\circ\ 56'$ on SR to $27^\circ\ 23'\frac{3}{4}$ on Ss; then against 90° on SR is $45^\circ\ 48'$=BC, on Ss.

3. To find the Angle C.

Set (the Co-Sine of AC) $62^\circ\ 36'\frac{1}{4}$ on SR to 90° on Ss; then against (the Co-Sine of B) $50^\circ\ 04'$ on SR is $59^\circ\ 44'$=C, on Ss.

8. Case III. Given the Hypothenuse BC=$45^\circ\ 48'$, and an acute Angle B=$39^\circ\ 56'$, to find the Legs AB, AC, and the Angle C.

1.

1. To find AB.

As Radius	$90^\circ\ 00'$	$=10.000000$
to the Co-Sine of . . .	$B=39^\circ\ 56'$	$=\ 9.884677$
So is the Tangent of . .	$BC=45^\circ\ 48'$	$=10.012129$
to the Tang. of the Side sought	$AB=38^\circ\ 15'$	$=\ 9.896806$

2. To find AC.

As Radius	$90^\circ\ 00'$	$=10.$
is to the Sine of	$BC=45^\circ\ 48'$	$=\ 9.855465$
So is the Sine of	$B=39^\circ\ 56'$	$=9.807465$
to the Sine of the Side sought	$AC=27^\circ\ 23'\tfrac{3}{4}$	$=9.662930$

3. To find the Angle C.

As the Co-Sine of	$BC=45^\circ\ 48'$	$=9.843336$
is to Radius	$90\ \ 00$	$=10.$
So is the Co-Tangent of	$B=39^\circ\ 56'$	$=10.077213$
To the Tangent of	$C=59^\circ\ 44'$	$=10.233877$

9. *By Scale and Compasses.*

1. To find AB.

Extend the Compaſſes from Radius 90° to (the Coſine of B) $50^\circ\ 04'$ in the Sines, then ſet one Foot in 45° in the Tangents, and pitch the other downward; where fix it, while you bring the former from 45° to $45^\circ\ 48'$; then will this laſt Extent reach from 45° to the Tangent of $38^\circ\ 15'=AB$, the Side required.

2. To find AC.

Extend the Compaſſes from 90° to $45^\circ\ 48'$ in the Sines,

Sines, the ſame will reach from 39° 56′ to 27° 23′¾ =AC, in the ſame Line.

3. To find the Angle C.

Extend the Compaſſes from (the Co-Sine of BC) 44° 16′ to Radius 90° ; the ſame applied to (the Co-Tangent of B) 50° 04′, will reach to the Tangent 59° 44′=C, the Angle required.

10. *By the Sliding-Rule.*

1. To find AB.

Set Radius 90° on SR to 50° 04′ (the Co-Sine of B) on Ss, and againſt 45° on TR mark the Point on Ts, bring that Point to 45° 48′ on TR ; and now againſt 45° on TR you have 38° 15′=AB, on Ts.

2. To find AC.

Set Radius 90° on S to 45° 48′ on SR, then againſt 39° 56′ on Ss, you'll ſee 27° 23′¾ on SR, the Side AC required.

3. To find the Angle C.

Set (the Co-Sine of BC) 44° 16′ on Ss to Radius 90° on SR ; then againſt (the Co-Tangent of B) 50° 04′ on TR, is 59° 44′=C, on Ts.

11. Caſe IV. Given the Legs, AB=38° 15′, and AC=27° 23′¾ ; to find the reſt.

To find the Hypothenuſe BC.

As Radius	90° 00′=10.
is to the Co-Sine of . .	AC=27° 23¾=9.948388
So is the Co-Sine of . .	AB=38° 15′=9.895045
to the Co-Sine of	BC=45° 48′=9.843433

As there is nothing new in finding the Analogies B and C, I ſhall paſs them by.

12. *By Scale and Compaſſes.*

1. To find BC.

Extend the Compaſſes from 90° to (the Co-Sine of AC) 62° 37′; the ſame applied to (the Co-Sine of AB) 51°45′ will extend to (the Co-Sine of BC) 44° 16′; therefore BC=45° 48′.

2. To find the Angle B.

Extend the Compaſſes from 38° 15′ to 90° in the Sines, the ſame will reach from 27° 23′$\frac{3}{4}$ to 39° 56′ =B, in the Line of Tangents.

3. To find the Angle C.

Extend the Compaſſes from 27° 23′$\frac{3}{4}$ to 90° in the Sines; the ſame will reach from 45° to a Point in the Tangent-Line, where fix the Compaſſes while you bring the Leg from 45° to 38° 15′, then will this laſt Extent reach from 45° to 59° 44′=C, in the Tangents.

13. *By the Sliding-Rule.*

1. To find BC.

Set 90° on Ss to (the Co-Sine of AC) 62° 36′$\frac{1}{4}$ on SR, then againſt (the Co-Sine of AB) 51° 45′ on Ss is (the Co-Sine of BC) 44° 16′ on SR; wherefore BC=45° 48′, as required.

2. To find the Angle B.

Set 38° 15′ on Ss to 90° on SR; then againſt 27° 23′$\frac{3}{4}$ on Ts is 39° 56′ on TR, the Angle B required.

3. To find the Angle C.

Set $27^\circ\ 23'$ on Ss to 90° on SR; and mark the Point on Ts againſt 45° on TR; bring that Point to $38^\circ\ 15'$ on TR; then againſt 45° on TR you ſee $59^\circ\ 44'$=C, on Ts.

14. Caſe V. Given the Hypothenuſe BC=$45^\circ\ 48'$, and one Side AC=$27^\circ\ 23'\frac{3}{4}$; to find the reſt.

To find the Angle C.

The Tangent of	BC=$45^\circ\ 48'$=	19.012129
is to Radius	$90^\circ\ 00'$=	10.
As the Tangent of	AC=$27^\circ\ 23'\frac{3}{4}$=	9.714544
is to the Co-Sine of the Angle	C=$59^\circ 44'$=	9.702415

15. *By the Scale and Compaſſes.*

Extend the Compaſſes from 45° to $27^\circ\ 23'\frac{3}{4}$ in the Tangents, then ſet one Foot in $45^\circ\ 48'$, and pitch the other downward, where fix it while you move the other again from $45^\circ\ 48'$ to 45°, then ſhall this laſt extent reach from Radius 90° to (the Co-Sine of C) $30^\circ\ 16'$ in the Line of Sines; whence the Angle C=$59^\circ\ 44'$, as required.

16. *By the Sliding-Rule.*

Set 45° on TR to $27^\circ\ 23'\frac{3}{4}$ on Ts, and mark the Point in Ts againſt $45^\circ\ 48'$ on TR, then bring that Point to 45° on TR; laſtly, againſt 90° on SR, you have $30^\circ\ 16'$ on Ss, which is the Co-Sine of C= $59^\circ\ 44'$, as required.

Note, The Proportions for AB and the Angle B, contain nothing new or difficult either in *Numbers* or by *Inſtrument*, therefore ſhall give no Examples in them.

17. Cafe VI. Given the Angles B=39° 56′, and C=59° 44′, to find the Sides AB, AC, BC.

1. To find AB.

As the Sine of	B=39° 56′=9.807465
is to the Co-Sine of . . .	C=59° 44′=9.702452
So is Radius	90° 00′=10.
To the Co-Sine of the Side AB=38° 15′=9.894987	

2. To find AC.

The Analogy is the fame as for AB, by which you will find AC=27° 23′¾ nearly.

3. To find BC.

As the Tangent of . . .	C=59° 44′=10.233905
is to the Co-Tangent of	B=39° 56′=10.077213
So is Radius	90° 00′=10.
to the Co-Sine of the Side BC=45° 48′= 9.843308	

18. *By Scale and Compaffes.*

1. To find AB.

Extend the Compaffes from 39° 56′ to (the Co-Sine of C) 30° 16′, the fame will reach from Radius 90° to (the Co-Sine of AB) 51° 45′, in the Line of Sines; therefore AB=38° 15′. In the fame manner you find AC=27° 23′¾.

2. To find BC.

Extend the Compaffes from 59° 44′ to (the Co-Tangent of B) 50° 04′ in the Tangents, the fame Extent will reach from Radius 90° to (the Co-Sine of BC) 44° 12′ in the Sines; therefore BC=45° 48′.

19. *By the Sliding-Rule.*

1. To find AB.

Set 39° 56′ on Ss to (the Co-Sine of C) 30° 16′ on SR; then against 90° on Ss is (the Co-Sine of AB) 51° 45′; consequently AB=38° 15′.

In like manner, you find AC=27° 23′¼.

2. To find BC.

Set 59° 44′ on Ts to (the Co-Tangent of B) 50° 04′ on TR; then against 90° on SR is 44° 12′ (the Co-Sine of BC) on Ss. Wherefore BC=45° 48′.

20. *Of Oblique Triangles.*

Oblique Spherical Triangles admit also of six different Cases, as follow.

Case I. Given two Angles, B=34° 30′, and D=48° 00′, and an opposite Side BC=38° 45′; to find the other two Sides DC and BD; and Angle C.

Let fall a Perpendicular CA from the unknown Angle C to its opposite Side BD. Then is the oblique Triangle reduced to two Right ones, BAC and DAC. Then say by the first Analogy; To find the Angle C.

As the Co-Sine of . . . . BC=38° 45′=9.892030
is to Radius . . . . . 90° 00′=10.
So is the Co-Tangent of B=34° 30′=10.162866

to the Tangent of . . BCA=61° 48′½=10.270836

Again, by the second Analogy, say;

		Co. Ar.
As the Co-Sine of . . .	$B=34°\ 30'$	$=0.084006$
is to the Sine of . . .	$BCA=61°\ 48'\frac{1}{2}$	$=9.945159$
So is the Co-Sine of . . .	$D=48°\ 00'$	$=9.825511$
to the Sine of the Angle	$DCA=45°\ 41'\frac{1}{2}$	$=9.854676$

Now since the Perpendicular CA falls within the Triangle, 'tis plain the Sum of the two Angles now found makes the Angle sought, *viz.* $BCA+DCA=BCD=45°\ 41'\frac{1}{2}+61°\ 48'\frac{1}{2}=107°\ 30'$.

To find the Side BD.

21. By the first Analogy, say;

As Radius	$90°\ 00'$	$=10.$
to the Co-Sine of	$B=34°\ 30'$	$=9.915994$
So is the Tangent of	$BC=38°\ 45'$	$=9.904497$
to the Tangent of the Side	$BA=33°\ 29'$	$=9.820485$

Again, by the second Analogy, say;

		Co. Ar.
As the Tangent of . . .	$D=48°\ 00'$	$=.9.954437$
is to the Tangent of . . .	$B=34°\ 30'$	$=9.837134$
So is the Sine of	$AB=33°\ 29'$	$=9.741698$
to the Sine of	$AD=19°\ 57'$	$=9.533269$

Now 'tis evident, the Sum of the two Arches $AB+AD=BD=33°\ 29'+19°\ 57'=53°\ 26'$, the Side required.

22. To find the Side CD.

		Co. Ar.
As the Sine of the Angle	$D=48°\ 00'$	$=.0.128927$
is to the Sine of the Side	$BC=38°\ 45'$	$=9.796571$
So is the Sine of the Angle	$B=34°\ 30'$	$=9.753128$
to the Sine of the Side sought	$DC=28°\ 30'$	$=9.678626$

Thus

Thus the whole Triangle is resolved according to the Data of this Case.

23. Case II. Given two Angles B=34° 30′, and C=107° 30′; and a Side included BC=38° 45′; to find the rest.

You let fall the Perpendicular CA, and find the Angle BCA=61° 48′½, as in Art. 20. Then from the given Angle C=107° 30′, take the Angle BCA =61° 48′½, and there will remain the Angle DCA =45° 41′½, by which you will find the Angle D=48° 00′, according to the second Analogy of this Case. These Things being known, we may proceed

To find DC. Co. Ar.

Thus, as the Co-Sine of DCA=45° 41′½=0.155821
is to the Co-Sine of BCA=61° 48′½=9.674329
So is the Tangent of . . . BC=38° 45′=9.904491

To the T. of the Side sought DC=28° 30′=9.734641

24. To find BD.

From the given Angle B let fall the Perpendicular Bℵ to the unknown Side DC produced; then by the first Analogy, find the Angle CBℵ, thus;

As Radius . . . . 90° 00′=10.
to the Co-Sine of . . . . BC=38° 45′=9.892030
So is the Tangent of BCℵ=72° 30′=10.501278

to the Co-Tangent of CBℵ=22° 00′¼=10.393308

Then by the second Analogy, say;

As the Co-Sine of . . DBℵ=56° 30′¼=.0.258208
is to the Co-Sine of ℵBC=22° 00′¼=9.967151
So is the Tangent of . . . BC=38° 45′=9.904491

to the T. of the Side sought DC=53° 26′=10.129850

25. Case III. Given two Sides BC=38° 45′, and CD=28° 30′, and an Angle opposite to one of them, B=34° 30′; to find the rest.

This Case being but the Converse of Case I. I shall not repeat the Examples.

Case IV. Given two Sides BC=38° 45′, and BD=53° 26′, and the Angle included B=34° 30′; to find the rest.

This Case contains nothing difficult if what goes before be well understood; the Analogies being plain and easy for the Operations, I shall leave them to the Learner's Exercise; and pass on to

26. Case V. Given all the three Sides, BC=38° 45′, DC=28° 30′, and BD=53° 26′; to find the Angles.

To find the Angle C.

The Difference of the Legs containing the Angle sought, is BC—CD=AM=10° 15′; then $\frac{BD+AM}{2}$ =31° 50′½, and $\frac{BD-AM}{2}$=21° 35′½; wherefore, according to the Analogy for this Case in the Synopsis, proceed thus:

The Sine of the Side . . . BC=38° 45′	=9.796571
Add the Sine of the Side DC=28° 30′	=9.678663
The Sum is the Log. of sBC x sDC	=19.475234
Again, the Sine of $\frac{BD+AM}{2}$=31° 50′½	=9.722283
Add the Sine of $\frac{BD-AM}{2}$=21° 35′½	=9.565836
The Sum is s $\frac{BD+AM}{2}$ x s $\frac{BD-AM}{2}$	=19.288119

Then

Then ſay;

As . . . . $sBC \times sDC = 19.475234$
is to the Square of Radius . . . $Rq = 20.$
So is . . . . $s\frac{BD+AM}{2} \times s\frac{BD-AM}{2} = 19.288119$

to the ſquare Sine of $\frac{1}{2}$ the Angle ſought } . . $Sq\frac{1}{2}C = 19.812885$
The half whereof is . . . . $s\frac{1}{2}C = 53^\circ\ 44' = 9.906442$

Wherefore the Angle ſought is $C = 107^\circ\ 28'$; or more compendiouſly thus, by the Arithmetical Complement of the Sides BC, and DC.

27. The Sine of $BC = 38^\circ\ 45' = .0.203429$ Co. Ar.
The Sine of $DC = 28^\circ\ 30' = .0.321337$ Co. Ar.
The Sq. of Radius Rq $= 20.000000$
The Sine of $\frac{BD+AM}{2} = 31^\circ\ 50'\frac{1}{2} = 9.722283$
The Sine of $\frac{BD-AM}{2} = 21^\circ\ 35'\frac{1}{2} = 9.565836$

The Sum of all . . . . $sq\frac{1}{2}C = 19.812885$

Therefore . . . . $s\frac{1}{2}C = 53^\circ\ 44' = 9.906442$
Conſequently . . . $C = 107^\circ\ 28'$; the ſame as before.

Having found one Angle, the others are eaſily found by the former Caſes.

Note, By Caſe I. the Angle C was found to be $107^\circ\ 30'$, which is but $2'$ different from what it is found by this Caſe; whence the Reader may obſerve the wonderful Certainty and Agreement of the moſt different Methods of Calculation, and from the greateſt Diverſity of Data.

28. The ſixth Caſe is but the ſame with this, if the three Angles given be changed into Sides, taking for the greateſt Angle in that Triangle and greateſt Side in this, their Supplements to 180 Degrees.

29. Having thus ſhewn the beſt Methods, both for *Exactneſs* and *Eaſe*, of reſolving the ſeveral Caſes of *Right* and *Oblique-angled Triangles*, I ſhall leave the Application thereof to the various Problems of *Aſtronomy*, *Geography*, *Dialling*, *Orthodromics*, &c. for the Learner's own Recreation; as being in it ſelf very eaſy, if what is here taught be underſtood; and alſo becauſe I have both demonſtrated the *Theory*, and very cloſely applied the *Doctrine* of *Spherical Trigonometry*, in the 2d Vol. of my *Young Trigonometer's Guide*, printed for Mr. *J. Noon*, at the *White Hart*, in *Cheapſide*; and which I recommend to all unacquainted with, and who would have a good Notion of the noble and moſt uſeful Art of *Trigonometry*.

CHAP. XIII.

MERCATOR's SAILIMG *performed by the* CANON *of* LOGARITHMIC TANGENTS, *without the* MERIDIONAL PARTS.

1. THE Property of *Mercator's Chart* is its having the Degrees of Latitude increaſed in the ſame Proportion as a Degree of Longitude decreaſes from the Equator to the Pole. Which Proportion is that of *Radius* to the *Co-Sine* of *Latitude*; or, of the *Secant* of the *Latitude* to the *Radius*; which is thus demonſtrated.

2. Let AEB (Fig. VIII.) be a Sector in the Plane of the Equator, made by the Interſections of the Planes of two Meridians therewith, *viz.* AE, and BE,

BE, whose Inclination, or Angle BEA, that is, the Arch of the Equator AB is = 1 Degree. Also let EC be the Radius of any Parallel of Latitude; then shall DC be an Arch in that Parallel similar to (AB) one Degree of Longitude in the Equator, or it is that Degree diminished. Now (from the Elements) the Arch AB is to the same diminish'd in the given Parallel DC, as the Radius of the Equator AE to the Radius of that Parallel CE; but since AE=EC (AC being the given Latitude) and because of similar Triangles EcC and ESA, therefore Ec (EA) : EC :: ES : EA :: AB : DC. But EC= E c, the Co-Sine of the Latitude AC, and ES the Secant thereof; therefore, *&c.* which was to be demonstrated.

3. And because in *Mercator's Projection* or *Chart*, the Meridians and Parallels are all represented by *parallel Right-Lines*, the Arch CD in every Parallel is ever equal to that in the Equator AB; therefore that the true Proportion between Longitude and Latitude might be preserved on this Chart, as on the Globe it self, 'twas necessary the several Degrees of Latitude should be such as belong to those Circles whose Radii are severally equal to the Secants of those Latitudes.

4. Let A = the Arch of one Degree in any Latitude; a, b, c, &c. = the several enlarg'd Degrees on the Meridian; R = Radius; and S, s, s, *&c.* the Secants of those Latitudes. Then it will

$$\text{be}\left\{\begin{array}{l} \text{A 1st} : a :: \text{R} : \text{S}. \\ \text{A 2d} : b :: \text{R} : \text{s}. \\ \text{A 3d} : c :: \text{R} : s, \ \ \&\text{c}. \end{array}\right\} \begin{array}{l} \text{Therefore we shall have} \\ 3\text{A} : a+b+c :: 3\text{R} : \text{S}+ \\ \quad \text{s}+s. \end{array}$$

That is, the Sum of the Secants of 1, 2, 3 Degrees is always equal to the Distance of the Parallel of those three enlarg'd Degrees from the Equator. And consequently by a continual Addition of the Secants of $\frac{1}{2}'$ $1\frac{1}{2}'$, $2\frac{1}{2}'$, $3\frac{1}{2}'$, *&c.* or their Doubles $1'$, $3'$, $5'$ $7'$, *&c.* the Line or Table of *Meridional Parts* may be made; and by some has been made in this manner

 for

for every Minute of the Quadrant; but a truer way is that delivered in Chap. X. of the Theory, which ſee.

5. If then you draw any Line AB (Fig. X.) to repreſent an Arch of the Equator, as ſuppoſe 60 Degrees; and on every tenth Degree be erected Perpendiculars, theſe ſhall be the *Meridians*; on which if the Degrees of Latitude, enlarg'd in the Manner and Proportion above deſcrib'd, be ſet off; and thoſe Diviſions join'd by Right-Lines, repreſenting the Parallels of Latitude, *Mercator's Chart* will be conſtructed for the given Longitude and Latitude; and a *Meridian* thus graduated, is what is called the *Nautical* or *Meridian Line*. This Chart, as here drawn, includes 60° *Longitude*, and 80° *Latitude*, that the Reader may have a perſpicuous Idea thereof.

6. Now ſuppoſe F be the Latitude whence you ſail, and C the Latitude you arrive to, on the Rhumb FC; 'tis plain there will be formed a Right-angled plain Triangle FCD, wherein FD is the *enlarg'd Difference of Latitude*, and CD the *true Difference of Longitude*, and the Angle CFD the *Courſe* or *Rhumb*; and therefore any two of theſe being given, the reſt are found by the Analogies of plain Triangles foregoing.

7. Since every Degree is equal to 60 Minutes or Nautical Miles, let the *proper Difference of Latitude* be reduc'd to theſe Parts, and ſet (off the ſame Scale as AB was laid down by) from F to E, and draw EK parallel to DC; ſo ſhall there be form'd another Right-angled plain Triangle FEK; in which FE is the *proper Difference of Latitude*; EK the *Departure*; KF the *Diſtance* ſail'd on the Rhumb or Courſe EFK. Any two of which Parts being given, the others are to be found as aforeſaid.

8. And theſe two Triangles FCD, FKE, comprehending all the Particulars of *Mercator's Sailing*; 'tis obvious enough how they are all reſolv'd, either by

by *Projection* or *Trigonometrical Calculation*; but my Purpose is here to shew, that without those *common Methods*, they are all to be resolv'd by the Canon of Logarithm-Tangents only; in order to which, the following Articles must be premis'd and duly observ'd, *viz.*

9. First, That it has been shewn, the *Nautical* or *Meridian Line*, or Scale of *Mercator's Chart*, is no other than a Scale of Logarithm-Tangents of the Half-Complements of the Latitude. Secondly, that such Logarithm-Tangents of Mr. *Brigg*'s Form, (or those in common Use) are a Scale of the Differences of Longitude upon the Rhumb, which makes an Angle of 51° 38′ 9″ with the Meridian. Thirdly, the Differences of Longitude on differing Rhumbs, are as the Tangents of the Angles of those Rhumbs with the Meridians; as is evident from the Triangle FCD (Fig. X.) And fourthly, that the Logarithm-Tangent of the Angle 51° 38′ 9″, *viz.* 10.1015104 is a constant Factor in these Computations. On these Premises 'tis easy to operate the Propositions of *Mercator's Sailing*, as follows.

10. In Fig. 11. let F represent the *Lizard's Point*, whose Latitude North DF=49° 55′; also let K represent *Barbadoes* in Latitude North 13° 10′=CK; then is FE = the Difference of Latitude; CD the Difference of Longitude; and EFK the Angle of the Rhumb, FK with the Meridian. Lastly, let FO be the Rhumb making an Angle of 52° 38′ 9″ with the Meridian; or that on which the Difference of Longitude QD is the Difference of the Logarithms of the Tangents of half the Complements of the Latitude PF, PK, or PO: Then,

11. Case I. Given the Latitude of the *Lizard*, 49° 55′ N. and that of *Barbadoes* 13° 10′ N. and their Difference of Longitude 53° 00′=

 3180

3180 Nautic Miles: I demand the Course and Distance sail'd?

$$\left\{\begin{array}{l}\text{The Lat.}\\ CK=13^\circ\ 10'\\ DF=49^\circ\ 55'\end{array}\right\}\begin{array}{l}\text{Comp.}\\ PK=76^\circ\ 50'\\ PF=40^\circ\ 05'\end{array}\left\}\begin{array}{l}\text{the Halfs.}\\ 38^\circ\ 25'\\ 20^\circ\ 2'\tfrac{1}{2}\end{array}\right\}$$

Then the Log. Tangent of $\left\{\begin{array}{l}38^\circ\ 25'=9.899308\\ 20^\circ\ 2'\tfrac{1}{2}=9.562048\end{array}\right.$

The Difference of which is . . . QD=3372,60

(*Note*, The four first Figures are *Integers*, according to the Theory)

Then say, for the Course KFE;

As the Diff. of Log. Tang. QD=3372,6=3.527965

to the given Diff. of Long. CD=3180=3.502427
So is the const. Tang. of OFE=51° 38′ 9″=10.101510

3.603937

to the Tangent of the Course sought } KFE=49° 59′½=10.075972

Secondly, For the Distance FK.

The Difference of Latitude is 49° 55′—13° 10′= 36° 45′=2205 Miles;

Say, as Radius . . . . . . 10.
to the Diff. of Lat. . . . EF=2205=3.343409
So is the Secant of the Course } KFE=49° 59′ 10″=10.191807

to the Distance sail'd FK=3429,38=3.535216

By this Proposition you estimate the Course a Ship must steer, and the Distance of her Port.

12. Case II. Given the Latitude of the *Lizard* $49^\circ\ 55'$ N. and of *Barbadoes* $13^\circ\ 10'$ N. and the Course $49^\circ\ 59'\frac{1}{6}$, to find the Difference of Longitude, and Distance sailed.

Things being prepared as before, say;

As the Log. Tang. of OFE $=51^\circ\ 38'\ 9''=10.101510$

to the Tang. of the Course } KFE $=49^\circ\ 59'\ 10''=10.075972$

So is the Diff. of the Log. Tang. of the $\frac{1}{2}$ Comp. of the Latitudes } QD $=3372{,}6=3.527965$

3.603937

to the Diff. of Long. required CD $=3180=3.502427$

Which converted into Degrees makes $53^\circ\ 00'$. And so much is the Difference of Motion or Time between these two Places. The Distance on the Rhumb will be found $3429{,}38=$ FK, as before.

13. Case III. Given the Latitude of the *Lizard* $49^\circ\ 55'$ N. the Distance sailed $3429{,}38$ Miles on a Course $49^\circ\ 59'\ 10''$ Southwesterly, 'tis required to find the Latitude and Longitude of the Place to which the Ship is arrived.

As Radius . . . . $10.$

to the Distance sailed FK $=3429{,}38=3.535216$

So is the Co-Sine of the Course } KFE $=49^\circ\ 59'\ 10''=9.808193$

to the Diff. of Latitude FE $=2205=3.343409$

Then from the given Lat. of the *Lizard* $49^\circ\ 55'$

Subd. the Diff. of the Lat. $2205=36^\circ\ 45'$

There remains the Lat. sought . . . $=13^\circ\ 10'$

And

And having thus obtained the Latitudes, the Difference of Longitude will be found, as *per* Case II. to be 3180 Miles or 53 Degrees; and so the Operation needs not to be repeated.

14. If it so happen that the Ship passes the *Equator*, and consequently has one Latitude *North*, the other *South*; then observe two things; first, that both the Complements of the Latitudes are to be estimated from the same Pole of the World. And therefore, secondly; suppose you sail from a *Northern* to a Southern Latitude, you must add 90° to the *former*, and substract the latter from 90°; then substract this Sum and Remainder from 180°, and take the Difference of the Logarithm-Tangents of half the Remainders, as before.

15. Case IV. Suppose I sail from Latitude 48° 30′ N. to Latitude 23° 45′ S. on a Course 43° 50′ Southwesterly; required the *Difference, Longitude*, and *Distance* sailed?

Then $48^\circ\ 30' + 90 = 128^\circ\ 30'$; and $90 - 23^\circ\ 45' = 66^\circ\ 15'$. And

$$\left.\begin{array}{l} 180 - 66^\circ\ 15' = 113^\circ 45' \\ 180 - 128^\circ\ 30' = 51^\circ 30' \end{array}\right\} \tfrac{1}{2} \left\{\begin{array}{l} 56^\circ\ 52'\tfrac{1}{2} = 10.185410 \\ 25^\circ 45' = 9.683356 \end{array}\right.$$

The Diff. of those Tangents is . . . 5020,54

Therefore say, for the Difference of Longitude;

As the Log. Tangent of $51^\circ\ 38'\ 9'' = 10.101510$

is to the Diff. of the Log. Tang. $5020,54 = 3.700750$
So is the Tang. of the Course $43^\circ\ 50' = 9.982309$

3.683059

to the Diff. of Long. required $3815,5 = 3.581549$

That is, in Degrees, $= 63^\circ\ 35'\tfrac{1}{2}$.

To

To find the Diſtance ſail'd; ſay,

As Radius......		10.
to the Sum of the Lat.	$72^\circ\ 15' = 4335$	$= 3.636989$
So is the Secant of the Courſe	$43^\circ\ 50'$	$= 10.141849$
to the Diſt. ſail'd in Miles,	$6009{,}5$	$= 3.778838$

16. Thus you have all the *Practical Caſes* of *Mercator's Sailing* performed by the *Canon* of *Logarithm-Tangents* only, without the *Meridional Parts* or *Chart*, as in the common Way. And ſince this is the moſt exact, and natural Method of Navigation (next to the *Globular Chart* it ſelf) and wholly reſolvable by *Logarithms*, it adds not a little to the (before invaluable) Eſtimation of thoſe excellent Numbers; and renders their Uſe to Navigators more neceſſary than before.

CHAP. XIV.

Of the MENSURATION *of* SUPERFICIES *and* SOLID BODIES *by* LOGARITHMS.

1. AMongſt the Variety of Methods for meaſuring the *Surfaces* and *Solidity* of *Bodies*, I intend here to ſhew the Excellency of that by *Logarithms*; which may juſtly be allow'd the *Preference* to all others in point of *Eaſe* and *Brevity*, Advantages none of the leaſt in common Eſtimation. And ſince theſe Operations conſiſt altogether in *Multiplication* and *Diviſion*, I need not here repeat them by the *Inſtruments*, as having already largely ſhewn the Manner thereof in a *Chapter* particularly on that Head.

2.

2. *To measure a* SQUARE, Fig. XII.

The Logarithm of the Side multiplied by 2, gives the Logarithm of the *Area* or *superficial Content*.

Examp. Let the Side of the Square be AB=31,57.

Then the Logarithm of AB=31,57=1.499275
which multiply by 2

The Product is the Area of the Square } ABCD=996,66=2.998550

3. *To measure a* PARALLELOGRAM, Fig. XIII.

The Sum of the Logarithms of the Length and Breadth is the Logarithm of the Area.

Examp. The Length AB=41,5 and Breadth BC= 31,57.

To the Log. of the Length AB=41,5=1.618048
add the Log. of the Breadth BC=31,57=1.499275

The Sum is the Log. of the Parallelog. } ABCD=1312,155=3.117323

4. *To measure a* RHOMBUS ABCD, Fig. XIV.

The Sum of the Logarithms of a Side and the Perpendicular Height, is the Logarithm of the Area.

Examp. Let the Side AB=15,5; and the Perpendicular BE=13,42.

Then to the Log. of the Side AB=15,5=1.190332
add the Log. of the Perpend. BE=13,42=1.127752

The Log. of the Area of the Rhombus } . . =208,01=2.318084

5. *To meaſure a* RHOMBOIDES, ABCD, *Fig.* XV.

The Sum of the Logarithm of the longeſt Side, and perpendicular Height is the Logarithm of the Area.

Exam. Let the longeſt Side AB=19,5, and perpendicular Height BE=6,07

Then to the Log. of the Side AB=19.5=1.290035
Add the Log. of the Perpend. BE=6,07=0.783189

The Log. of the Area of the Rhomboides . . . . } =118,365=2.073224

6. *To meaſure a* TRIANGLE ABC. *Fig.* XVI.

The Sum of the Logarithm of the Baſe, and of half the perpendicular Height (or *vice verſâ*) is the Logarithm of the Area.

Exam. Let the Baſe BC=65,25; and the perpendicular Height AG=21,5; then $\frac{1}{2}$ AG=10.75. Therefore,

To the Log. of the Baſe . . . BC=65,25=1.814580
Add the Log. of half the Perp. $\frac{1}{2}$AG=10,75=1.031408

The Log. of the Area . . . . =701,4375=2.845988

7. *To meaſure a* CIRCLE ABCD. *Fig.* XVII.

In order to this, let { D=the Diameter.
P=the Periphery.
A=the Area.

Then the Rules or Theorems for finding thoſe ſeveral Parts are as follows;

Theo. I. $3.1416D=P$. Th. II. $0.7854DD=A$.
Theo. III. $0.3183P=D$. Th. IV. $0.07957PP=A$.
Theo. V. $\sqrt{1.2732A}=D$. Th. VI. $\sqrt{12.5664A}=P$.

8. Therefore the Diameter AB being given, ſuppoſe = 20,15, to find the Periphery P? *per* Theor. I.

Thus, to the Log. of the Diameter	AB=20.15=1.304275
Add the Log. of the conſtant Number	3.1416=0.497151
The Log. of the Periphery P=63,303, &c.=	1.801426

9. Given the Periphery of a Circle P=63,303, &c. to find the Diameter D? *per* Theor. III.

To the Log. of the Periphery	P=63.303, &c.=1.801426
Add the Log. of the conſtant Number . .	0.3183, &c.=9.502837
The Log. of the Diameter D or AB=20.15=	1.304263

10. Having the Diameter given, ſuppoſe=20.15, to find the Area A of the Circle? *per* Theor. II.

The Log. of the Diameter . . AB=20.15=	1.304275
Multiply by	2
The Product is the Log. of . . . . . . DD=	2.608550
To which add the Log. of the conſtant Numb.	0.7854=9.895091
The Log. of the Area of the Circle	ACBD=318,89, &c.=2.503641

11,

11. Or thus ſuppoſing the Periphery given, =63,303, *&c. per* Theor. IV.

To twice the Log. of	P=63.303=	1.801426 1.801426
Add the Log. of the conſtant Number	0.07957=	8.900749
The Sum is the Log. of the Area	A=318,89, *&c.*=	2.503601

12. To meaſure the SECTOR of a CIRCLE ACB, *Fig.* XVIII.

The Sum of the Logarithms of the *Radius*, and $\frac{1}{2}$ the *Arch* (or of the Arch and half the *Radius*) is the Logarithm of the *Area*.

Examp. Suppoſe the Radius AC=12.36, and the Arch AB 10,12; then $\frac{1}{2}$AB=5,11.

Then to the Log. of Radius	AC=12,36=	1.092018
Add the Log. of	$\frac{1}{2}$AB=5,11=	0.707570
The Log. of the Area of the Sector	ACB=63,2596=	1.799588

13. To meaſure the SEGMENT of a Circle, as AFBG, *Fig.* XIX.

The beſt way is to find the Centre C; as by this Theorem, *viz.* $\overline{FB}^2 - FG = N$, then $\frac{N+FG}{2} = CG$; whence C is given; and finding the Area of the whole Sector ACBG, (as *per* laſt Article,) and the Area of the Triangle ABC, (*per* Art. 6.) if the latter Area be ſubducted from the former, it will leave the Area of the Segment required, AFBG.

14. To meaſure a SPHERICAL TRIANGLE, ABC. *Fig.* XX.

From the Sum of the three Angles A, B, C, take 180 Degrees; then from the Logarithm of the Remainder ſubduct the Logarithm of the conſtant Number 720; to that Remainder add the Logarithm of the Superficies of the whole Sphere; the Sum ſhall be the Logarithm of the Area required.

Exam. Let the Angles be $\begin{cases} A=37^\circ & 30' \\ B=92 & 30 \\ C=60 & 45 \end{cases}$

Their Sum is . . . . 190° $45'$
Subſtract . . . . 180 00 Log.
There remains . . . . 10° $45' = 10{,}75 = 1.031408$
Subduct the Logarithm of . . . . . . $720 = 2.857332$

8.174076

Suppoſe the Superficies of the Sphere . . . . $\}$ $=1257{,}3=3.099439$

The Sum is the Log. of the Area of the Triangle . . $\}$ $ABC=23{,}623=1.273515$

15. To meaſure an ELLIPSIS, as ACBD, *Fig.* XXI.

The Sum of the Logarithms of the Tranſverſe Diameter AB, the Conjugate Diameter CD, and the conſtant Number 0.7854; is the Logarithm of the Area.

Exam. Let AB=61,6, and CD=44,4.

Then add the Logarithms of $\begin{cases} AB=61{,}6=1.789581 \\ CD=44{,}4=1.647383 \\ N.0.7854=9.895091 \end{cases}$

The Log. of the Area of Ellipſis . . . . . $\}$ =2148,1004,&c.=3.332055

16.

16. To meaſure a PARABOLA, as ACD, *Fig.* 22.

From the Sum of the Logarithms of the Baſe AB, the perpendicular Height CD, and the Number 2, ſubſtract the Logarithm of 3; the Remainder is the Logarithm of the Area.

Exam. Let AB=61,6, and CD=44,4; as in the Ellipſe.

Then add the Logarithms of	AB=61,6=1.789581
	CD=44,4=1.647383
	and N. 2.=0.301030
	3.737994
Subſtract the Logarithm of	3=0.477121
The Log. of the Area required=1823,36=	3.260873

17. To meaſure any REGULAR POLYGON, *Fig.* XXIII.

In order to this the following Table will be very expedient.

A Table for the more ready finding the Area of any Regular Polygon.

Sides.	Names.	Numbers.
5	Pentagon.	1.72048
6	Hexagon.	2.59808
7	Heptagon.	3.63896
8	Octagon.	4.82843
9	Enneagon.	6.18183
10	Decagon.	7.69421
11	Endecagon.	8.51425
12	Dodecagon.	9.33012

Then the Sum of double the Logarithm of the Side of the Polygon, and of the Number in the Table proper to it, is the Logarithm of the Area.

Exam. Let the *Pentagon* ABCDE be propoſed, and let its Side be AB=14,6.

Then

Then twice the Logarithm of AB=14,6 $\begin{cases} 1.164353 \\ 1.164353 \end{cases}$
Add the Log. of the *tabular Numb.* 1.72048=0.235649

The Log. of the Area required=366,74=2.564355

In the ſame manner you find the Area of any other Polygon mention'd in the Table.

18. *Of the* MENSURATION *of* SOLIDS.

To meaſure a CUBE ABCDFGE, *Fig.* XXIV.

Three times the Logarithm of the Side is the Logarithm of the Solidity.

Exam. Let the Side AB = 31.57 = 1.499275
Multiply by 3

The Log. of the Solidity . . . 31464,81=4.497825

19. To meaſure a PARALLELOPIPEDON AD, *Fig.* XXV.

The Sum of the Logarithms of the *Breadth*, *Depth*, and *Length*, is the Logarithm of the Solidity.

Exam. Let the Width AB=21,56; the Length AG=31,57; and Depth GF=9,03.

Then add the Logarithms of $\begin{cases} AG=31,57=1.499275 \\ AB=21,56=1.333649 \\ GF=\ 9,03=0.955688 \end{cases}$

The Log. of the Solidity required . . . . $\}$=6146,2623=3.788612

20. To meaſure a PRISM ABCDEF, *Fig.* XXVI.

Firſt find the *Area* of the *Baſe*, whether a *Triangle*, *Square*, &c. Then the Sum of the Logarithms of the

the ſaid *Area*, and Length of the *Priſm*, is the Logarithm of its *Solidity*.

Exam. Suppoſe a *Priſm* of a *triangular Baſe*, as in the Figure, then let its Area be ABC=701,4375, and its Length BD=70,15.

Then add the Logarithms of both . . . $\left\{ \begin{array}{l} \text{ABC}=701{,}4375=2.845988 \\ \text{BD}=\ 70,\ 15\ =1.846028 \end{array} \right.$

The Log. of the Solidity=49205,84,&c.=4.692016

21. To meaſure a PYRAMID ABCDE. *Fig.* XXVII.

First find the *Area* of its *Baſe*, whether *triangular*, *quadrangular*, &c. then the Sum of the Logarithms of the Area of the Baſe, and $\frac{1}{3}$ of the *perpendicular Height*, is the Logarithm of the *Solidity*.

Exam. Let the Priſm have a *quadrangular Baſe* ABCD=342,25; and the perpendicular Height GE=180; then $\frac{1}{3}$ GE=60.

Therefore add the Logarithms of . . . . . $\left\{ \begin{array}{r} \text{ABCD}=342{,}25=2.534343 \\ \frac{1}{3}\text{GE}=60=1.778151 \end{array} \right.$

The Log. of the Solidity . . . : . =20535=4.312494

22. To meaſure a CYLINDER ACBDEF, *Fig.* XXVIII.

The Sum of the Logarithms of the *Area* of its Baſe, and of the *Height*, is the Logarithm of the *Solidity*.

Exam. Let the Area of the Baſe be 380,15=AGBH; and the Height BD=50,05.

Then add the Logarithms of . . . . . . $\left\{ \begin{array}{r} \text{AGBH}=380{,}15=2.579955 \\ \text{BD}=50{,}05=1.699404 \end{array} \right.$

Log. of the Solidity . . . =19026,5075=4.279359

22. To meaſure a CONE AEBFD, *Fig.* XXIX.

The Sum of the Logarithms of the *Area* of the *Baſe*, and $\frac{1}{3}$ of the perpendicular Height, is the Logarithm of the *Solidity*.

Exam. Let the Area of the Baſe be 100.75=AEBF; and the perpendicular Height CD=19,95; and therefore $\frac{1}{3}$CD=6,65.

Then add the Logarithms of .... $\begin{cases} AEBF=100,75=2.003245 \\ \frac{1}{3}CD=6,65=0.822822 \end{cases}$

The Sum is the Log. of the Solidity .... } 669,9875=2.826067

23. To meaſure the FRUSTUM of a Pyramid. *Fig.* XXX.

If it be a ſquare Baſe as ABCD, then put ... $\begin{cases} D=\text{Side of the greater Base AB.} \\ d=\text{Side of the lesser Base EF.} \\ D-d=x \text{ and } H=\text{the Height GO.} \end{cases}$

Then we have this Theorem $\overline{Dd+\frac{1}{3}xx} \times H=$ the Solidity.

Exam. Suppoſe D=50, d=21; and H=105,6; then D—d=x=29.

Then add the Logarithms of $\begin{cases} D=50\,..=1.698970 \\ d=21\,..=1.322219 \\ H=105,6=2.023664 \end{cases}$

The Logarithm of .... DdH=110880=5.044853

Again, add the Logarithms of $\begin{cases} \frac{1}{3}xx=280,3=2,447673 \\ H=105,6=2.023664 \end{cases}$

The Logarithm of .... $\frac{1}{3}xx$H=29603,2=4.471337
To which add ..... DdH=110880

The Sum is ... $\overline{Dd+\frac{1}{3}xx} \times H$=140483,2= the Solidity required.

24.

24. But if the Bases of the Frustum be *triangular*; the Theorem is $\overline{Dd+\frac{1}{3}xx}\times 0,433H=$the Solidity. Again, if the Base be any of the regular Polygons; then put N = the Number in Table at Art. 17. proper to the Polygon, and the Theorem will be $\overline{Dd+\frac{1}{3}xx}\times NH=$the Solidity.

25. To measure the Frustum of a RIGHT CONE. *Fig.* XXXI.

Let D=the Diameter of the great Base AB, and d=the Diameter CD of the lesser Base; D—d=x, and H=the perpendicular Height, as before; then the Theorem for the *Solidity* will be $\dfrac{\overline{Dd+\frac{1}{3}xx}\times H}{1,27323}$, or thus $\overline{Dd+\frac{1}{3}xx}\times 0.7854H=$Solidity.

Exam. Let AB=16, CD=12, and GO=9: then D—d=x=4; and $\frac{1}{3}xx=5,\dot{3}$: also Dd=192; and so $Dd+\frac{1}{3}xx=197,\dot{3}$.

Then add the Logarithms of . . . . . $\left\{\begin{array}{r} Dd+\frac{1}{3}xx=197,\dot{3}=2.295199 \\ 0.7854=9.895091 \\ H=9=0.954242 \end{array}\right.$

The Log. of the Solidity . . =1394,8704=3.144532

26. To measure a GLOBE or SPHERE ACBD, *Fig.* XXXII.

The Sum of the triple Logarithm of the Diameter, and the Logarithm of the constant Number 0.5236 is the Logarithm of the Solidity of the Sphere.

Exam. Suppose the Diameter of a Sphere AB= 50,37.

Then add the Logarithms $\left\{ \begin{array}{r} AB=50,37=1.702172 \\ 1.702172 \\ 1.702172 \\ 0,5236=9.718999 \end{array} \right.$

The Log. of the Solidity of the Sphere .... $\}$ =66913,8=4.825515

27. To meaſure the Superficies of a Sphere.

The Sum of Logarithm of double the Diameter, and the conſtant Number 3.1416, is the Logarithm of the Superficies of the Sphere.

Exam. Let the Diameter AB=50.37.

Then add the Logarithms of $\left\{ \begin{array}{r} AB=50.37=1.702172 \\ 1.702172 \\ 3,1416=0.497151 \end{array} \right.$

The Log. of the Superficies .... 7970,7=3.901495

28. To meaſure the SEGMENT of a Sphere, *Fig.* XXXIII.

Let $\left\{ \begin{array}{l} \text{D=The Axis or Diameter of the Sphere CD.} \\ \text{C=Half the Diam. of the Segment's Baſe EB.} \\ \text{H=The Height of the Segment ED.} \end{array} \right.$

Then we have the two following Theorems for finding the Solidity.

viz. $\left\{ \begin{array}{l} \text{Theor. I. } \overline{3CCH+H^3}\times 0.5236=\text{the Solidity.} \\ \text{Theor. II. } \overline{3DH^2-2H^3}\times 0.5236=\text{the Solidity.} \end{array} \right.$

29. To meaſure a SPHEROID ACBDA, *Fig.* XXXIV.

The Sum of the *double Logarithm* of the *Leſſer*, the *Logarithm* of the Greater Diameter, and the *conſtant*

constant Number 0.5236, is the Logarithm of the solid Content of the *Spheroid.*

Exam. Let the lesser Diameter CD=33; and the greater Diameter AB=55.

Then the double Log. of . . . CD=33 $\begin{cases} =1.518514 \\ =1.518514 \end{cases}$
The Logarithm of . . . . AB=55=1.740363
The Logarithm of . . . . 0.5236=9.719000

The Log. of the Solidity . . =31361,022=4.496391

30. To measure the SEGMENT of a SPHEROID.

As the Solidity of the Sphere AFBE is to the Solidity of its Segment AGK; so is the Solidity of the Spheroid ACBD to its like Segment AHI.

31. To measure a PARABOLIC CONOID ACBD, *Fig.* XXXV.

The Sum of the double Logarithm of the *Diameter* of the *Base*, the Logarithm of the *Height*, and *constant Number* 0.3927; is the Logarithm of the Solidity of the *Conoid.*

Exam. Let the Diameter of the Base AB=55; and its Height CD=33.

Then the double Log. of . . .AB=55 $\begin{cases} =1.740363 \\ =1.740363 \end{cases}$
The Logarithm of . . . . CD=33=1.518514
The Logarithm of . . . . .0.3927=9.594061

The Logarithm of the Solidity 39201,4=4.593301

32. To meaſure the FRUSTUM of a PARABOLIC CONOID, *Fig.* XXXVI.

To this end, let D = Diameter of the greater Baſe AB. d = Diameter of the leſſer Baſe CD. H = the Height of the *Fruſtum* FE.

Then we have this Theorem .... $\overline{DD+dd} \times 0.3927H$ = the Solidity.

33. To meaſure a PARABOLIC SPINDLE, *Fig.* XXXVII.

The Sum of the double Logarithm of its Thickneſs, the Logarithm of its Length, and the Logarithm of the conſtant Number 0.41888, is the Logarithm of the Solidity of the Spindle.

Exam. Let the *Diameter* of its *greateſt Circle*, or *Thickneſs* AB=43,45; and the Length CD= 50,075.

Then the double Log. of AB=43,45	=1.637990
	=1.637990
The Logarithm of ..... CD=50,075	=1.699621
The Logarithm of 0.41888	=9.622090
The Log. of the Solidity .. =39599,6	=4.597691

34. To meaſure any of the FIVE REGULAR BODIES, *Fig.* XXXVIII.

For this purpoſe the following Table is neceſſary.

Names.	Superficies.	Solidity.
Tetrahedron	1.73205	0.11785
Octahedron	3.46410	0.47140
Hexahedron	6.00000	1.00000
Icoſahedron	8.66025	2.18169
Dodecahedron	20.64573	7.66312

Exam.

Exam. Suppoſe the Side AB=12, of the *Icoſahedron*, *Fig.* XXXVIII.

The Sum of the Logarithm of the *tabular Number*, and double Logarithm of the Side is the Logarithm of the Superficies; and the Sum of the Logarithm of the *tabular Number*, and triple Logarithm of the Side, is the Logarithm of the Solidity.

Thus the double Log. of . . . AB=12 $\begin{cases} =1.079181 \\ =1.079181 \end{cases}$
The Log. of the *tab. Number* . . 8,66025=0.937530

The Log. of the *Superficies* =1247,0688=3.095892

Again the Logarithm of . . . AB=12=1.079181
3

3.237543
Add the Log. of the *tab. Numb.* 2.18169=0.338793

The Log. of the Solidity . . =3769,9458=3.576336

35. Theſe Propoſitions are ſufficient for meaſuring any *Superficies* or *Solid Body* in common Uſe; and the great Eaſe and Conciſeneſs of performing the ſame by Logarithms, is abundantly manifeſt from theſe Examples.

CHAP.

CHAP. XV.

The DOCTRINE *of* MENSURATION *applied to* GAUGING, MEASURING TIMBER, *and* SURVEYING; *wherein the* PRACTICAL USE *of the* PLAIN SCALE *and* SLIDING RULE, *for these Purposes, is clearly explain'd.*

1. IN the preceeding Chapter you have the *practical Method of absolute* or *general Mensuration* of the *Content* of *Superficies* and *Solids*, laid down in divers Propositions; my Business is here to apply that general Doctrine to particular Uses as those of *Gauging*, *Timber-Measure*, and *Surveying*. For tho' I there shew'd how the *Dimensions* of Bodies might be express'd in *Numbers*; yet since these Numbers represent *divers kinds of Quantities*, as *Inches*, *Feet*, *Yards*, *Poles*, *Chains*, &c. at pleasure, so they are equally subservient to those several Arts above-mention'd, and must be reduced to those *Denominations* or *Measures* which are *peculiar* to each. I shall begin therefore with

2. *Gauging.*

In this Art the Dimensions are taken in *Inches*, and *Decimal Parts* thereof (or must be reduced to such;) and by *Inches* is here to be understood *solid* or *cubic Inches*; and that as well in *superficial* as *solid Measure*. For though it be improper, *geometrically speaking*, to ascribe *Thickness* to a *Superficies*, the *Gaugers* always consider them as *one Inch deep*, and accordingly compute the *superficial Content* in *Gallons*, or *solid Measure*.

3. As *Inches* are the *lineal Dimenſions* of *Gaugers*, ſo *Gallons* are the common Quantities of their *ſolid Meaſure*, of *liquid Subſtances* eſpecially; and *Buſhels* for *ſolid dry Meaſure*, as *Malt*, *Corn*, &c. Now the *ſtandard Gallons* and *Buſhel*, as uſed in *England*, are as follows: the *Ale* or *Beer Gallon* =282 *ſolid Inches*; the *Wine Gallon* = 231; the *Corn Gallon* =268,8; and the *Corn Buſhel* = 2150,42 *Cubic Inches.*

4. From whence it follows, that, ſuppoſing the *Dimenſions* of the *Superficies* in the foregoing Chapter taken in *Inches*, the ſeveral *Area's*, or *ſuperficial Contents*, will by the Gauger be underſtood to be ſo many *ſolid* or *cubic Inches*; which therefore if he divide by 282, 231, 268,8, or 2150,42, the Quotients will be the ſeveral *Contents* in the reſpective *Gallons* or *Buſhels*. For *inſtance*, in Fig. XII. the Side of the Square AB=31,57 Inches, and the Area was therefore found (Art. 2.) to be 996,66 *ſolid Inches.*

Conſequently	282)996,66(=3,53	Ale Gallons.
	231)996,66(=4,31	Wine Gallons.
	268,8)996,66(=3,707	Corn Gallons.
	2150,42)996,66(=0,46	Corn Buſhels.

5. But ſince *Multipliers* are the *Reciprocals* of *Diviſors*, therefore *Unity* divided by theſe *Diviſors* will produce ſo many *Multipliers*. Thus,

282)1.00000(=0.003546	the Mult. for A. Gall.
231)1.00000(=0.004329	the Mult. for W. Gall.
2688)1.00000(=0.003722	the Mult. for C. Gall.
2150,42)1.00000(=0.000465	the Mult. for C. Buſh.

Conſequently, if any Area be

Divided by ..	282	or multiplied by	0.003546	the Area will be expreſſed in	A. Gall.
	231		0.004329		W. Gall.
	268.8		0.003722		C. Gall.
	2150,42		0.000465		C. Buſh.

6.

6. In order to work these Dimensions by the *artificial Line of Numbers*, either on the *Plain Scale*, or *Sliding-Rule*; put B=Breadth, L=Length, D =Depth or Thickness, G=Standard Gallon; and A=Area, and S=Solid Content in those Gallons, *&c.* Then for *Superficies*, $\frac{LB}{G}=A$; and so LB =AG; therefore G : B :: L : A. That is, the *Ratio*, or *Logarithm* of the *Ratio* of the *Standard Gallon* to the *Breadth*, is equal to that of the *Ratio* of the *Length* to the *Area* in Gallons. Wherefore supposing B=31,57, L=41,5 (as in the *Parallelogram* Fig. XIII. Art. 3.) if you set one Foot of the Compasses in 282, and extend the other to 31.57, that Extent will reach from the Length 41,5 to 4 $\frac{6}{10}$ Gallons, Ale Measure. Or, on the *Sliding-Rule*, set 282 on the Rule to 31,57 on the Slider, then against 41,5 on the Rule is 4 $\frac{6}{10}$ on the Slider, the *Area* in *Ale Gallons*, as before; and thus the *Area* is found in *Wine* or *Corn Gallons.*

7. Again, Since AD=S x 1, therefore 1 : A :: D : S. That is, the Logarithm of the *Ratio* of Unity 1 to the Area in Gallons A, is equal to that of the *Ratio* of the Depth (or Height) D to the *Solidity* (or Capacity) S, in Gallons of the same kind. Wherefore if B=21,56, L=31,57, D=9,03, as in Parallelopipedon, Fig. XXV. then G=282 : B=21,56 :: L=31,57 : A=2$\frac{4}{10}$ the *Area* in Ale Gallons; consequently 1 : A=2 $\frac{4}{10}$ :: D=9,03 : S=21 $\frac{7}{10}$, the *solid Content* in Ale Gallons. Where extend the Compasses from Unity to the Area 2 $\frac{4}{10}$, the same Extent will reach from the Depth 9,03 to the *solid Content* 21 $\frac{7}{10}$ Gallons. Or, on the *sliding Rule*, set Unity 1 on the Rule to the Area 2 $\frac{4}{10}$ on the Slider, then against the Depth 9,03 on the Rule you have 21 $\frac{7}{10}$ the *solid Content* in *Ale Gallons* on the *Slider.* Thus the Method of finding the *Content* or *Capacity* of square or rectilinear *Area's* or *Bodies* in *Ale*,

Wine,

Wine, or *Corn* Gallons is exceeding plain and eaſy by the *Single Gunter* and *Sliding-Rule*.

8. In caſe of *circular Area's*, ſince they are all in the Ratio of the Squares of their Diameters; and ſuppoſing the Diameter of a Circle 1 Inch, the Area will be 0,7854 Decimal Parts of a Cubic Inch; therefore having the Diameter of any Circle given in Inches, if the Square thereof be multiplied by 0.7854, the Product will be the Area of that Circle in Cubic Inches. Let D=the Diameter of a Circle, a=0.7854, G=Standard Gall. and A=Area of the Circle in ſuch Gallons, as before. Then 1 : a :: DD : DDa=Area in Inches; therefore DDa=GA; and ſo G : a :: DD : A = Area in Gallons. But a : G :: 1 : $\frac{G}{a}$ = the Square of the Diameter of that Circle, whoſe Area is G. Wherefore ſince DD = $\frac{G}{a}$ A,

therefore if the Square of the Diameter DD be divided by $\frac{G}{a}$ = $\begin{cases} 0.7854)\ 282.0000(=\ 359{,}05 \\ 0.7854)\ 231.0000(=\ 294.12 \\ 0.7854)\ 268.8000(=\ 342.24 \\ 0.7854)2150{,}4200(=2737.92 \end{cases}$

or the ſeveral Diviſors 359,05; 294,12, *&c.* then theſe Quotients will be the Area of the Circle in *Ale*, *Wine*, &c. Gallons.

9. Or thus, ſince DD × $\frac{a}{G}$ = A; therefore if the Square of the Diameter DD

be multiplied by $\frac{a}{G}$ = $\begin{cases} 282\ )0.7854(0.002785 \\ 231\ )0.7854(0.003389 \\ 2688\ )0.7854(0.002922 \\ 2150{,}42)0.7854(0.0000036 \end{cases}$

the ſeveral Products will be the Area in Ale, Wine, *&c.* Gallons, as before. Suppoſe the Diameter of a Circle D=50 Inches, then DD=2500; and put the conſtant Diviſor $\frac{G}{a}$ = dd = 359,05; 294,12, *&c.*

Then since $1 \times DD = ddA$, therefore $dd : DD :: 1 : A =$ the Area in Gallons. Therefore with the Compasses set one Foot in 359,05 extend the other to 2500, the same Extent will reach from 1 to $6\frac{9}{10} =$ the Gallons in the *Area* of the proposed Circle.

10. Otherwise thus; put $\frac{a}{G} = dd = 0.002785$; 0.002389, &c. (see Art. 9.) then because $DDdd = A \times 1$; therefore we have $1 : DD :: dd : A$; or $1 : dd :: DD : A$. If then you set 1 on the Rule to 0,00278 on the Slider, you will see against 2500 on Rule $6\frac{9}{10}$ on the Slider, the Gallons of Ale in the Area of that Circle; and thus you find the Gallons of Wine, Corn, &c. both by the *Sliding-Rule* and *Plain Scale.*

11. Again, putting $H =$ Height of a Cylinder, and $D =$ the Diameter of its circular Base; also $S =$ the solid Content or Capacity in Gallons; then $dd : DD :: (1 : A ::) H : S$. Now let $D = 50$ Inches, and $H = 15$, the Area of such a Cylinder will be thus found by the *Sliding-Rule.* Set 359,05 on the Slider, to 2500 on the Rule; then against 15 on the Slider, is 104 on the Rule; and so many Gallons of Ale would such an hollow Cylinder contain.

12. But since $dd : 1 :: DD : A$, (Art. 9.) therefore $d : 1 :: D : \sqrt{A}$; and for the same Reason $d : \sqrt{H} :: D : \sqrt{S}$. Consequently, if a *single Line of Numbers* be made to slide by a *double one*, if you set d on the *single one* to 1 on the *double one*, then against D on the *former*, you have A on the *latter*. Also, if against d on the *single Line*, you set H on the *double one*; then against D on the former, is S on the latter. By this means therefore, having only the *Diameter* and *Height* of a *Cylinder*, the Area of the Base, and solid Content of the said Cylinder, is immediately known. The Reason of the Method here

here delivered, appears from what has been already ſaid in the Chapter of *Inſtrumental Operations.*

13. Now d =

$$\left\{\begin{array}{l}\sqrt{359,05}=18,95\\ \sqrt{294,12}=17,15\\ \sqrt{342,24}=18.\ 5\\ \sqrt{2737,92}=52,32\end{array}\right\}\begin{array}{c}\text{Which Numbers}\\ \text{are called the}\\ \textit{Gauge Points}\\ \text{for} \ldots\ldots\end{array}\left\{\begin{array}{l}\text{Ale Gall.}\\ \text{Wine Gall.}\\ \text{Corn Gall.}\\ \text{Corn Buſh.}\end{array}\right.$$

And accordingly on the ſingle Line of Numbers againſt thoſe *Gauge Points* there is a ſmall Stroke ('), by which are ſet the Capitals AG, WG, MB, to ſignify they are the *Gauge Points* for *Ale Gallons, Wine Gallons*, and *Malt Buſhels.*

14. Therefore in the forementionꝰd Cylinder, whoſe Height H was = 15 Inches; and the Diameter of its Baſe D = 50; in order to find the Area of the Baſe, and Capacity of the Cylinder in Gallons, by the Sliding-Rule; ſet 1 on the Slider to the *Gauge Point* for Ale 18.95 on the *ſingle Line*, then againſt the Diameter D = 50, on the ſingle Line is $6\frac{9}{10}$ on the *double Line* on the Slider, the Gallons of Ale in the Area of the Baſe.

Secondly, Set the *Gauge Point* 18.95 to the Height 15 on the Slider, then againſt the Diameter 50 on the ſingle Line is 104 = S on the double Line or Slider; therefore the Capacity of the Cylinder is 104 Gallons, *Ale-Meaſure*, as before.

15. Or thus, with the *ſingle Line* and *Compaſſes* only. Set one Foot in the *Gauge Point* 18,95, and extend the other to D = 50, this Extent turn'd twice over from 1 will fall on $6\frac{9}{10}$ = A = the Area of the Baſe in Gallons of Ale. The ſolid Content, in this Caſe, will fall beyond the ſingle Line, or elſe might be performed in the ſame manner.

16. With the double Lines of Numbers, on the Sliding Rule, thus. Set the Gauge-Point 18.95 on the Rule to the Diameter 50 on the Slider, and mark the Number on the Slider againſt 1 on the Rule; then

then againſt that Number on the Rule you have $6\frac{0}{10}$ = A the Area of the Baſe on the Slider. Alſo mark the Number on the Slider againſt the Height 15 on the Rule, then againſt that Number on the Rule is 104 on the Slider, the Capacity of the Cylinder as before; and thus both the Area and Solidity are found at once ſetting the Rule. The ſame is performed by the double Line and Compaſſes, thus; Set one Foot in the *Gauge Point* 18,95, and extend the other to the Diameter 50, the ſame Extent will reach at twice from 1 to $6\frac{0}{10}$ the Area; and from the Height 15 to 104 ſolid Capacity in Ale Gallons.

17. In this manner may Gauge-Points be found for right-lined Areas: for let any ſuch Area given in *Cubic Inches* be called a; then (by Art. 6.) $1 \times a = GA$, and ſo $G : a :: 1 : A$; or thus $G : 1 :: a : A$; therefore $\sqrt{G} : 1 :: \sqrt{a} : \sqrt{A}$.

$$\text{Therefore } \sqrt{G} = \left\{\begin{array}{l} \sqrt{282} = 16.79 = \\ \sqrt{231} = 15.19 = \\ \sqrt{2150.42} = 46.36 = \end{array}\right\} \begin{array}{c}\text{the Gauge}\\ \text{Point for}\end{array} \left\{\begin{array}{l} \text{AleG.} \\ \text{W.G.} \\ \text{Buſh.} \end{array}\right.$$

Call theſe *Gauge Points* d; then $d : 1 : \sqrt{a} : \sqrt{A}$ = the Area in Gallons. Alſo $d : \sqrt{H} :: \sqrt{a} : \sqrt{S}$ = the Solidity in Gallons. Whence you may obſerve, that *right-lined Areas*, as well as *circular ones*, may be found in Gallons, by the *ſingle* and *double Line* of Numbers ſliding by each other; but not the *Solidity* of ſuch Solids, there being *three Terms* of the *four* variable, in the Analogy for that.

18. But notwithſtanding this, there is a Method whereby the Solidity or Capacity of Solids or Veſſels may be found (without knowing the Areas at all) by the *Breadth*, *Length*, and *Depth* only; which call B, L, D; and S = Solidity, and G = the Gallon or Buſhel, as before. Now ſince BDL=GS, therefore $BD = \frac{G}{L} S = G \times \frac{1}{L} \times S$; and ſo $G \times \frac{1}{L} : B$

:: D

:: D : S. But becauſe $\frac{1}{L}$ is the Reciprocal of L, therefore the Line of Numbers on which you ſeek L muſt be *inverted*; and becauſe the Logarithm of G is added, the Number G muſt always begin the *inverted Line*, or be placed equal to 1 on the *direct Line*. Theſe things premiſed, 'tis plain that if B on the *Slider* be ſet to L on the *inverted Line* order'd as before, then againſt D on the *direct Line* you have S on the Slider.

19. For Example; Suppoſe a Ciſtern be 80 Inches long, 50 broad, and 40 deep. Quære the Content in Buſhels?

Here G = 2150,42, and for the Purpoſe of *Malt-Gauging* there is on ſome Rules an *inverted Line of Numbers* fix'd on one Side the Slider; beginning at 2150,42 as before ſaid; then on ſuch a Rule ſet 50 = B on the *Slider* to 80 = L on the inverted Line, and againſt 40 = D on the *direct Line* on the Rule, you have 74=S=the Number of Buſhels the *Ciſtern* will contain. And thus you might proceed for Gallons, had you *inverted Lines* beginning at 282, 231, 268.8.

20. In *gauging Caſks*, the principal Conſideration is the *Curvature* of the *Staves*; as A B D. *Fig.* XXXIX. which *Gaugers* reduce to *four Degrees*, or *Varieties*, *viz.*

Variety I. Thoſe Caſks whoſe Staves are moſt *curved* or *bent*, are conſider'd as the *middle Zone* or *Fruſtum* of a *Spheroid*, ſuch as *Fig.* XXXIV.

Variety II. If the Staves are not quite ſo much *arching* or *bent*, the Caſk is ſuppoſed to be the *middle Zone*, or *Fruſtum* of a *Parabolic Spindle*, as *Fig.* XXXVII.

Variety III. When the Staves of Caſks are but very little *curved*, they are reputed to be in the Form of the *Fruſtums* of two equal *Parabolic Conoids*,

noids, join'd together at the wideſt Baſes. See *Fig.* XXXVI.

Variety IV. When the Staves are ſtrait from the *Bung* to the Head, as the prick'd Lines AB, BD, (or very nearly ſo) then 'tis plain ſuch a Caſk conſiſts of the *Fruſtums* of two equal right *Cones*, ſet together at the greater Baſes. *Fig.* XXXIX.

21. The Caſks being reduced to theſe four Varieties, if you multiply the Difference between the Head and Bung Diameters

$$\text{By} \begin{Bmatrix} 0.\ 7 \\ 0.65 \\ 0.\ 6 \\ 0.55 \end{Bmatrix} \text{for the} \begin{Bmatrix} \text{I.} \\ \text{II.} \\ \text{III.} \\ \text{IV.} \end{Bmatrix} \text{Variety,}$$

and then add the Product to the Head Diameter, then that Sum ſhall be a *mean Diameter*, or that of a *Cylinder*; whoſe *Height* and *Capacity* is equal to that of the *Caſk*, as near as poſſible.

22. For Example; Let there be a Caſk ADEG, whoſe *Bung-Diameter* BF = 31.5 Inches, and *Head-Diameter* AG=24,5; then their Difference is BF —AG=7.

$$\begin{matrix}\text{Conſequently} \\ 24{,}5 + \end{matrix} \begin{Bmatrix} 7\times 0.55=28{,}35 \\ 7\times 0.6\ \ =28{,}7 \\ 7\times 0.65=29{,}05 \\ 7\times 0.7\ \ =29.4 \end{Bmatrix} \begin{matrix}\text{The mean Diameter} \\ \text{of the } \textit{Cylinder} \\ \text{equal to the } \textit{Caſk.}\end{matrix}$$

Having found the Areas (by Art. 8, 9, 10, 12) to be 2,2385; 2,2941, *&c.* Ale Gallons; and ſuppoſing the Length of the Caſk 42 Inches, = HL.

$$\begin{matrix}\text{Then the Contents} \\ \text{according to the} \\ \text{ſeveral Varieties} \\ \text{will be : . . . . .}\end{matrix} \begin{Bmatrix} 2{,}2385\times 42= 94.03 \\ 2{,}2941\times 42= 96.35 \\ 2{,}3504\times 42= 98.71 \\ 2.4073\times 42=101.10 \end{Bmatrix} \begin{matrix}\text{Ale} \\ \text{Gallons.}\end{matrix}$$

And

And theſe being all the principal Articles, wherein the Uſe of *inſtrumental Logarithms*, or the *artificial Line* of *Numbers* is directly concerned, I ſhall ſay no more on the Head of *Gauging*; but proceed to the next Article of

Timber Meaſure.

23. Every Piece of *Timber* is a Solid, like to ſome one or other of thoſe, whoſe *abſolute Menſuration* was ſhewn in the laſt Chapter, *viz.* The *Fruſtum* of a *Cone*, a *Cylinder*, the *Fruſtum* of a *Pyramid*, a *Parallelopipedon*, *Priſm*, &c.

24. In whatever Form the Piece happens to be, find the *Content* or *Solidity* in Inches, as there taught; which divide by 1728, (the *ſolid Inches* in *one Foot* of Timber) the *Quotient* is the *ſolid Content* in *Feet.* But that the Analogies for Operation by Inſtruments may be evident, I ſhall make uſe of the foregoing Method in *Gauging*, by putting $B =$ the Breadth. $D =$ Depth, $L =$ Length, $G = 12$, and $S =$ Solidity, all in Inches. Then ſince $\frac{BDL}{G^3} = S$, therefore $BDL = G^3S$; but $\frac{L}{12} =$ the *Length* in *Feet*, which let be F, then $BD\frac{L}{12} = GGS = BDF$; and conſequently $GG : BD :: F : S$.

25. Now if the Piece of Timber be in form of a *ſquare Priſm*, then the Baſe BD is a *ſquare Number*, which call gg; whence $GG : gg :: F : S$; and therefore $G : g :: \sqrt{F} : \sqrt{S}$. Wherefore, having a *ſingle* and *double Line* of *Numbers*, by the *Sliding-Rule*, ſet $G = 12$ on the *ſingle Line*, to the Length in Feet F on the double one; then againſt g on the *ſingle Line* is $S =$ Solidity on the double Line on the Slider. Example, Suppoſe a Piece of Timber, the Side of whoſe ſquare Baſe is $g = 15$ Inches,

Inches, the Length 18 Feet $=$ F; Quere the ſolid Content S?

Set the conſtant Point G $=$ 12 on the *ſingle Line* to the Length in Feet F $=$ 18; then againſt the Side of the given *ſquare Baſe* $g = 15$ Inches, on the former, is 28 $=$ S $=$ the Number of the ſolid Feet in the Piece, on the latter. Or with the Compaſſes, extend from 12 to 15 on the *ſingle Line*, the ſame will reach from 18 to 28 $=$ S in the double Line, the Anſwer as before.

26. But if you have no *ſingle Line*, proceed with the double Line on the *Sliding-Rule* thus: Set the conſtant Number GG $=$ 144 on the Rule to the given Square $gg = 225$ on the Slider; then againſt the Length in Feet F $=$ 18, on the Rule is 28 $=$ S $=$ the ſolid Feet on the Slider. Or, with the Compaſſes, extend from 144 to 225, the ſame Extent will reach from 18 to 28 the ſolid Feet, as before. Otherwiſe by the Analogy $G : g :: \sqrt{F} : \sqrt{S}$; thus, ſet the conſtant Point G $=$ 12, on the Rule to the given Side $g = 15$, on the Slider, and mark the Point on the Slider againſt 15 on the Rule, bring that Point to 12 on the Rule; then againſt F $=$ 18 on the Rule, is 28 $=$ S on the *Slider*, the *Solidity* as before. But much better with the Compaſſes, thus; Extend from 12 to 15, that Extent turn'd twice over will reach from 18 to 28 $=$ S, as before.

27. If the Piece be in the Form of a *Parallelopipedon*, that is, hath its Breadth and Depth unequal; then the common way is to add the *Depth* and *Breadth* together; and to take half that Sum for the *Side* of a *mean Square*, *viz.* $\frac{B+D}{2} = g$, and then they imagine $gg = BD$, and ſo meaſure the Piece as before. But this is a very erroneous way; and the more ſo, as the Difference between the Breadth and Depth is greater. For ſince, (in order to reduce this Piece to a *ſquare Priſm*) BD $= gg$; 'tis evident

$g =$

$g = \sqrt{BD}$; that is, a *mean Proportional* between B and D, (for B : $\sqrt{BD}$ :: $\sqrt{BD}$: D) and not their *half Sum* $\frac{B+D}{2}$; as they ignorantly ſuppoſe.

28. For Example; Suppoſe a piece of *hewn Timber* in form as aforeſaid, whoſe Breadth is 22,5 Inches = B, the *Depth* or *Thickneſs* 10 Inches = D, and Length 18 Feet = F; what is the ſolid Content?

29. Find a *mean Proportional* between B=22,5 and D = 10, (as heretofore taught) it ſhall be $g = 15 = \sqrt{BD}$; and therefore $G^2 = 144 : g^2\ 225 :: F=18 : S = 28$, as before. But according to the common falſe way, $g = \frac{B+D}{2} = 16,25$, and ſo $gg = 264,0625$, which is greater than the *true Area* of the Baſe BD = 225 by 39,0625 *ſquare Inches*. Alſo ſet G = 12 on the *ſingle Line* of Numbers to the Length F = 18 Feet on the double, then againſt (the falſe) $g = 16,25$ on the *former*, is $32\frac{3}{4}$ = S on the latter. But $32\frac{3}{4} - 28 = 4\frac{3}{4}$ Feet more than is really in the Piece.

29. Becauſe $BDF = G^2S$; therefore $BD = \frac{G^2}{F}S$; and ſo we have $G^2 \times \frac{1}{F} : B :: D : S$. If then you have an *inverted Line* of Numbers beginning from $G^2 = 144$ placed on one ſide the Slider, with a *direct Line* on the other; then may the *ſolid Content* in Feet be found by the B, D, and F, as taught in Art. 18. hereof, thus; Set B = 22,5 on the Slider, to F=18, on the *inverted Line*; then againſt D = 10, on the *direct Line* is 28 = S, on the Slider, the ſolid Feet as before.

30. If the Timber be in Form of the *Fruſtum* of a *ſquare Pyramid*, as Fig. XXX. the Theorem for its Solidity in Inches is $HDd + \frac{1}{3}Hxx$, as per laſt Chap. Art. 23. Therefore $HDd + \frac{1}{3}Hxx = G^3S$ (where S = Solidity in *cubic Feet*) and ſince $\frac{1}{12}H=F$ = the Length in Feet, therefore $FDd + \frac{1}{3}Fxx =$

G^2S, consequently $G^2 : F :: Dd + \frac{1}{3}xx : S$. Example; Suppose a Piece of Timber 25 Inches square at the greatest end, 9 Inches square at the lesser End, and 20 Feet long, how many solid Feet is there in such a Tree?

Here $F = 20$, $Dd = 225$, and $\frac{1}{3}xx = 85,3$, therefore $Dd + \frac{1}{3}xx = 310,3$; wherefore $G^2 = 144 : F = 20 :: 310,3 : 43,1 = S$, the Number of solid Feet as required. Note, the common way, by supposing the Square of $\frac{D+d}{2} = Dd + \frac{1}{3}xx$ is very false; and in this Instance would not give the Solidity above 40,1 Feet, which is three Feet less than the Truth.

31. If the Bases of the *Frustum* be *Parallelograms*, as is the Case of most Pieces of *hew'd Timber*; then they may be reduced to Frustums of *square Pyramids*, thus; Let $A =$ the Area of the greater Base, $a =$ Area of the lesser. Then $\sqrt{A} = D$, and $\sqrt{a} = d$; and $\sqrt{A \times a} = Dd$; also since $D^2 + 2Dd + dd = xx$, therefore $A - 2\sqrt{A \times a} + a = xx$; and so $\frac{3Dd + xx}{3} = \frac{A + \sqrt{A \times a} + a}{3}$; consequently, $G^2 : F :: \frac{1}{3} A + \sqrt{A \times a} + a : S =$ the Number of *solid Feet* in such a Tree.

32. For Example; Suppose a Piece of squared Timber be 32 Inches broad, and 20 deep at the largest End; and 10 broad and 6 deep at the lesser End; the Length 18 Feet; Quære the solid Content?

Here $A = 32 \times 20 = 640$, and $a = 10 \times 6 = 60$; and $\sqrt{A \times a} = \sqrt{38400} = 195,959$; therefore $\frac{1}{3} A + \sqrt{A \times a} + a = 298,653$. Then $G^2 = 144 : F = 18 :: 298,653 : S = 37,33$ the solid Feet required. Note, the Product of $\frac{1}{4}$ of the Girt of each End taken for a *mean Area* is very *false*; and cannot be practised without considerable Error, tho' it be the common or customary way.

33. If the Timber be in form of a Cylinder, *Fig.* XXVIII. then putting $a = 0.7854$, and $G = 12$ Inches, we have (as per Art. 8.) this Analogy; $1 : a :: DD : DDa$ = the Area of the Cylinder's Baſe in Inches (ſuppoſing D = Diameter in Inches;) let H = *Height* in Inches; then $HDDa$ = the Inches ſolid, whereof $1728 = G^3$ = one *ſolid Foot.* Conſequently, if S = ſolid *Content in Feet*, $HDDa = G^3S$; and $\frac{H}{12} = F$ = the Length in Feet, therefore $FDDa = G^2S$; and $FDD = \frac{G^2}{a}S$; and putting $\frac{G^2}{a} = dd = 183{,}34$; then $FDD = ddS$; wherefore $dd : DD :: F : S$, or $d : D :: \sqrt{F} : \sqrt{S}$.

34. Or with the Length in Inches, to find the Solidity in Feet, thus; ſince $HDDa = G^3S$, therefore putting $\frac{G^3}{a} = dd$, we ſhall have $HDD = ddS$; conſequently $dd : DD :: H : S$ = Solidity in Feet; or $d : D :: \sqrt{H} : \sqrt{S}$. Wherefore, if there be a *ſingle Line* of Numbers on your Sliding Rule; ſet the conſtant Number $d = 46.9$ on the ſingle Line to H = the Height in Inches, on the double Line on the Slider; then againſt D = the Diameter of the Cylinder's Baſe in Inches on the *ſingle Line*, is S = the Number of ſolid Feet on the Slider as required. And if the Length be given in Feet; then the Solidity is found, as above, Art. 13.

35. If you have no *ſingle Line* on your Rule, you muſt work with the Analogies $dd : DD :: F : S$; or $dd : DD :: H : S$. In the firſt $dd = 183{,}34$, and $dd = 2200{,}152$; therefore $d = 13.54$ = Diameter of a Circle, when the Area is 144; and $d = 46{,}9$ = the Diameter of a Circle, whoſe Area is 1728 = the Inches in a ſolid Foot. And theſe Numbers being conſtant in all Operations, and the Method of operating ſuch kind of Analogies every way on the *ar-*

tificial Lines of Logarithms already ſufficiently exemplified in the preceeding Articles; I ſhall not here again repeat it.

36. The common way of meaſuring *round Timber*, is by girting them about the *Middle* with a *String*, and taking $\frac{1}{4}$ of the Girt for the *Side* of a *Square* equal to a *mean circular Area*, ſuch as would reduce the propoſed Piece to a *Cylinder*. But this is alſo very falſe and ungeometrical. For, ſince the Area of that Circle, whoſe Circumference is 1, is 0.07958; and the Area of that Square, whoſe Side is ,25 ($= \frac{1}{4}$ of the ſaid Circumference) is 0.0625, and the Solidities being in proportion to theſe Numbers, *viz.* as 0,07958 : 0.0625; that is, as 23 to 18; 'tis evident, the Content by this falſe way is above $\frac{1}{5}$ leſs than what it really is; which *Error*, if it be not conſiderable enough to be regarded and corrected, is great pity indeed.

37. Therefore to meaſure a Piece of *round tapering Timber* truly, it muſt be conſidered as the *Fruſtum* of a *right Cone*; whoſe ſolid Content in Inches is found by the Theorem $\overline{Dd + \frac{1}{3}xx} \times 0.7854H$, as *per* Art. 25. of the laſt Chapter. Now putting $0.7854 = a$, and $G^3 = 1728$, the cubic Inches in a ſolid foot; the Theorem will become $\overline{Dd + \frac{1}{3}xx} \times aH = G^3S$; and again ſince $\frac{1}{12}H = F$, the Length in Feet; therefore $\overline{Dd + \frac{1}{3}xx} \times aF = G^2S$; alſo put $\frac{G^2}{a} = dd$; and then $\overline{Dd + \frac{1}{3}xx} \times F = ddS$; wherefore we have $dd : \overline{Da + \frac{1}{3}xx} :: F : S =$ the *ſolid Content* in Feet.

38. For Example; Suppoſe a Piece of *round Timber* be 36 Inches Diameter at one End, and 9 Inches Diameter at the other; and 24 Feet long; quære the Solidity in Feet?

Here $D = 36$, $d = 9$, $D - d = x = 27$, $Dd = 324$, $\frac{1}{3}xx = 243$, $Dd + \frac{1}{3}xx = 567$, and $F = 24$.

There-

Therefore the Analogy is as $dd = 183,34 : 567 :: F = 24 : S = 74,22$ the ſolid Feet required; as may be wrought by the *Lines of Numbers* in any of the before-mention'd ways.

Of MEASURING LAND.

39. What I principally deſign here, is to ſhew how the true Area or Content of a Plot of Land is to be found by the *artificial Line of Numbers*, in *Acres* and *Decimal Parts*. The Dimenſions of a Field are commonly taken in Rods or *Poles*; each containing $16\frac{1}{2}$ Feet. Of theſe Poles, 40 in Length and 4 in Breadth make an Acre; or an Acre is = 160 ſquare Poles. Some (and indeed moſt) uſe a Chain, called *Gunter's Chain*, in taking Dimenſions; which conſiſteth of 100 Links, and the whole in Length = 4 Poles or Rods. And ſo 10 of theſe *ſquare Chains* make an *Acre*.

40. The Field being meaſur'd with the Pole, if it be in Form of a *Parallelogram*, put $L =$ Length, and $B =$ Breadth; and then it will be $\frac{LB}{160} = A =$ the Number of Acres; therefore $LB = 160\,A$; and ſo we have $160 : L :: B : A =$ the Acres. For Example, ſuppoſe a Field be 35 Pole broad, and 185 Pole in length; how many Acres doth it contain? Set 160 on the Rule to 185 on the Slider, then againſt 35 on the Rule is $40\frac{1}{2}$ nearly, the Number of Acres required.

41. If the Field be in Form of a Triangle, and meaſur'd with a Rod or Pole; then, having plotted it and meaſur'd the *Baſe* and perpendicular Height, which call B and H; then $B \times \frac{1}{2}H = 160A$; and ſo $160 : B :: \frac{1}{2}H : A =$ the Acres as before.

42. If the Field be in Form of a *Trapezium*, as *Fig.* XL. then becauſe, by drawing the Diagonal AB, it is reduced to two Triangles ACB, ADB, and

and calling the two Perpendiculars C*c* and D*d*, H and *h*; and the common Base AB, B; we have $\frac{BH}{2} + \frac{Bh}{2} = \overline{H + h} \times \frac{1}{2}B = 160A$. Therefore 160 : $\frac{1}{2}$B :: H + *h* : A = the Acres contain'd in the Trapezium ACBD.

43. If the Field be of a *multangular Form*, it must, when plotted, be reduced to several *Triangles* and *Trapezia*; and then measur'd, as *per* Art. 41, 42. In case you take the Measures with a Chain of 4 Rods or 100 Links, then the Analogies for Operation will be the same as above, only instead of 160 you must use 10; thus 10 : L :: B : A. Art. 40. and 10 : B :: $\frac{1}{2}$H : A. Art. 41, *&c.* All which is so easy as to need no Example, nor any thing more to be said concerning it.

CHAP.

CHAP. XVI.

The Practical Use *of the* Logistical Logarithms.

1. The *Nature* and *Conſtruction* of *Logiſtical Logarithms*, having been ſufficiently inculcated in Chap. IV. it remains now that I ſhew their Uſe in ſome Caſes of practical Aſtronomy, for in other reſpects it is very little. But in finding the Places, Diſtances, &c. of the *heavenly Bodies*, the Calculation of Eclipſes, &c. they are very neceſſary for finding the proportional Parts, as will, in ſome Degree, appear by the following Examples.

2. Example 1. Admit the *mean Anomaly* of the *Sun* be $4^s\ 7^\circ\ 48'\ 14''$; what is the *true Equation*, and Logarithm of his Diſtance from the Earth?

Mean Anomalies	$4^s\ 7^\circ$	Equations	$1^\circ\ 33'\ 49''$	Logar.	4.995607
	4 8		1 32 36		4.995501
Differences	1=60′		1 13		106

Then for the *Proportional Parts* of the Equation, ſay by the Logiſtical Logarithms,

If one Degree, or	60′ 00″=	0
give the Difference	1 13 =	16930
what gives the Anomaly	48 14 =	948
Anſwer, the prop. Part	0 59 =	17878

This ſubſtracted, (becauſe the Equation is decreaſing) from the Equation $1^\circ\ 32'\ 49''$ (anſwering

the Anomaly $4^s\ 7^\circ$) leaves the *true Equation* = $1^\circ\ 32'\ 50''$, as was requir'd.

3. Then for the proportional Part of the Logarithm ſay

One Degree, or	$60'\ 00''$=	0
gives the Difference of Logarithms . .	106=	15310
what gives the Anomaly	$48'\ 14''$=	948
Anſwer, the Proportional Part.	85=	16258

Which ſubduct from the Logarithm 4,995607, anſwering to the Anom. $4^s\ 7^\circ$ there remains 4,995522, the *true Logarithm* of the *Sun's Diſtance* from the Earth, as required.

4. Example II. Suppoſe the Moon's *annual Argument* be $29^\circ\ 51'\ 37''$, what is the Equation of the *Apogee* and *Eccentricity* of her Orb?

Annual Argum.	29°	Equation	$8^\circ\ 53'\ 8''$	Eccentricity	60392
	30		9 07 14		61045
Differences .	1=60′		14 6		653

Then for the Equation of the Apogee, ſay

If one Degree, or . . . . . . : . .	$60'\ 00''$=	0
give the Difference	$14'\ 6''$=	6289
what gives	$51'\ 37''$=	654
Anſwer, the Prop. Part	$12'\ 8''$	6943

To which add the Equation $8^\circ\ 53'\ 8''$, agreeing to the an. Arg. 29°; the Sum is . . . . $9^\circ\ 05'\ 16''$ the *true Equation* of the Apogee as required.

5. Then for the true Eccentricity, ſay

If

If one Degree, or	60′ 00″=	0
give the Difference of Eccentricity . .	653	=7414
what will the Part	51′ 37 =	654
Anſwer, the Proportional Part	562	=8068
To which add the Eccentricity . . .	60392	

The Sum is the *true Eccentricity* . . 60954, as required.

6. In like manner the Equation of the Moon's Center, Latitude, Inclination of Limit, and Logarithmic Diſtance from the Earth may be found. Alſo the ſame things are in like manner found for any of the Planets, of which there needs no more Examples.

7. Example III. In an Eclipſe of the Moon, admit her horary Motion be 30′ 31″, and the Sun's 2′ 27″; then the horary Motion of the Moon from the Sun will be 28′ 4″: and ſuppoſe the Moon hath paſs'd the Sun by the Diſtance 1° 19′ 4″=4744″; What is the Time requiſite for that Motion? Say thus,

As the horary Motion of the Moon from the Sun	28′	4=3300
is to one Hour	60	0= 0
So is the Diſtance paſs'd above 56′ 8″, *viz.*	22′	56=4177
To the Time above 2 Hours, *viz.* . .	49′	00= 877

Therefore 2h 49′ were paſs'd ſince the *true Oppoſition*, or Moment of the Eclipſe.

Note, becauſe in this Caſe the Motion of the Moon from the Sun, performed in 1 Hour, is 28′ 4″; therefore 56′ 8″ will be paſſed in 2 Hours: But the preſent Diſtance of the Moon from the Sun is

79′ 4″, which becauſe it exceeds the Table of *Logiſtical Logarithms*, therefore ſubduct the Motion of two Hours, *viz.* 79′ 4″—56′ 8″=22′ 56″; and this Exceſs of Motion will give the Exceſs in Time above 2 Hours; as in the Example. And thus you proceed when the Diſtance of the Moon from the Sun exceeds one Degree, or the Table.

8. Example IV. Suppoſe, in a *Lunar Eclipſe*, the *Semidiameter* of the Moon be 15′ 15″, and the *Difference* between the *Moon's Latitude*, and *Sum* of the *Semidiameters* of the Moon and the Earth's Shadow be 9′ 01″; Quære the Digits eclipſed?

Note, the *Semidiameter* of the *Moon* is always ſuppoſed to be 6 *Digits*, or equal to 6 Degrees. Therefore ſay,

As the Semidiameter of the Moon	15′ 15″=	5949
Is to ſix Digits	6° 00′ 00″=	10000
So is the ſaid Difference	9′ 1″=	8231
To the Digits eclips'd	3° 32′ 51″=	12282

9. Example V. In a *Lunar Eclipſe*, let the *Scruples* of *Incidence* be 30′ 17″, and the *horary Motion* of the Moon from the Sun be 28′ 47″; to find the Time of *half* Duration?

Here becauſe 28′ 47″ : 60′ :: 30′ 17″ : a fourth Number greater than 60′, and ſo conſequently beyond the Extent of the Table; therefore (as in Art. 7.) from 30′ 17″ ſubduct the Motion for an Hour, *viz.* 28′ 47″, and to the *Remainder* 1′ 30″ find the Time thus;

As the Motion	28′ 47″=	3190
Is to an Hour	60′ 00″=	0
So the remaining Scruples	1′ 30″=	16021
To the Time	3′ 8″=	12831

The

The Time ſought therefore $1^h\ 3'\ 8''$; the double of which is $2^h\ 6'\ 16''$, the Time of the whole Duration of the Eclipſe.

10. Example VI. If the diurnal Motion of the Sun be $59'\ 8''$, what is the Motion for 7 Hours $15'$?

Say, as one Day	24^h	$00'$	$00''$	=	0
Is to its Motion		$59'$	$8''$	=	63
So is the Time	7^h	$15'$	$0''$	=	5197
To the Motion therein		$17'$	$52''$	=	5260

11. Example VII. If the mean Motion of the Moon in one Hour be $32'\ 56''$, how far doth ſhe move in $17^h\ 45'$?

Say, as one Hour . .	1^h	$00'$	$00''$	=	13802	
Is to its Motion		$32'$	$56''$	=	20392	add
So is	17^h	$45'$	00	=	1309	add
					21701	
To the Motion required . .	9°	$44'$	$00''$	=	7899	

Thus the Motion in 24 Hours, or one natural Day, will be found $13^\circ\ 10'\ 35''$.

12. The *Logiſtical Logarithms* may alſo be uſed with *common Logarithms*. Thus ſuppoſe you would find the Logarithmic Sine of $18^\circ\ 16'\ 47''$, proceed thus;

The Logarithmic Sine of . .	18°	$16'$ =	9.496154
	18	$17'$ =	9.496537
The Differences		$1' = 60''$	383

 Now

Now the Proportion is, 60″ : 383 :: 47 : the Proportional Part. But ſince the *Logiſtical Logarithms* of 47″ and 60″ are reciprocally as thoſe Numbers, and that of 60 is nothing ; therefore (the Differences of the common Logarithms being proportional to the Numbers 60″ and 47″) ſay,

As the Logiſtical Logarithm of	43″=	1447
is to the *Logiſtical* Logarithm of . . .	60 =	0
So is the common Logarithm of . . .	383 =	2.5998
To the common Logarithm of the proportional Part	285 =	2.4551
Which add to the common Logarithm	9.496154	

The Sum is the Logarithm . . . 9.496439 of 18° 16′ 47″ as required. And thus proceed in any other Caſe of like nature.

13. The *Logiſtical Logarithms* may be uſed likewiſe with the *Logarithmic Sines* and *Tangents*, as in finding the Parallaxes of the Planets, *&c.* Thus ſuppoſe the Horizontal Parallax of the Moon 55′ 12″ ; the *Angle* of her *Orb* with the *Horizon* 22° 4′; and her *Longitude* in her Orb from the Horizon 81° 27′, to find her *Parallax* in *Longitude.*

From the Logiſtical Logarithm of the Hor. Paral.		00°	55′	12″=	362
Subduct the Sum of the	Sine of the Angle . . .	22	4	0 =	9.5748
	Co-Sine of Longitude. .	81	27	0 =	9.1722
				viz.	18.7470
There remains the Logiſ. Logar. of the Par. in Long. . .		00°	3′	5″=	12892

14. Becauſe when the Angle of the Moon's Orb with the Horizon is greateſt 67° 14′ 20″, and ſhe in the Horizon, her greateſt Parallax in Longitude will be then; and will be found as in the laſt Example, thus;

Logiſt. Log. of the greateſt Hor. Par.	00°	1′	24″=	17680
The Sine of	67°	14′	20″=	9.9648
Radius	90°	0	0 =	10.0000
Logiſt. Log. of the gr. Par. in Long.	00°	56′	37″=	18032

15. To find the Moon's *Parallax* in *Latitude.*

From the Logiſt. Log. of Horizontal Parallax . . .	00°	55′	12″=	362
Subduct the Co-Sine of the Angle of her Orb with the Horizon	22°	4′	0″=	9.9669
there remains the Logiſtical Logarithm of	00°	51′	9″=	693

which is the *Parallax of Latitude* ſought.

16. To find the greateſt and leaſt *Parallaxes* in *Latitude.*

From the Logiſtical Log. of the gr. Hor. Par.	1°	1′	24″=	17680
Subſtract the Co-Sine of the leaſt Ang. of her Orb with the Hor.	9°	41′	40″=	9.9937
Logiſtical Logarithm of the *greateſt Parallax*	1°	00′	31″=	17783

Again,

Again, from the L. Log. of the least *Horiz. Parallax* . .	00° 54′ 59″=	379
Substract the Co-Sine of the greatest Angle of Orb with the Horizon	67° 14′ 20″=	9.5876
Logistical Logarithm of the *least Parallax*	00° 21′ 16″	4503

17. By these Instances the Reader will be apprized of the great Usefulness and Expediency of *Logistical Logarithms* in his *Astronomical Calculations*; and as the common Logarithms are laid on a Rule, so likewise are these; and such a *Sliding-Rule* of Logistical Logarithms may be very useful to those who desire to be more *expeditious* than *exact* in their Calculations.

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N°	0	1	2	3	4
0		000000	301030	477121	602060
1	000000	041393	079181	113943	146128
2	301030	322219	342423	361728	380211
3	477121	491362	505150	518514	531479
4	602060	612784	623249	633468	643453
5	698970	707570	716003	724276	732394
6	778151	785330	792392	799340	806180
7	845098	851258	857332	863323	869232
8	903090	908485	913814	919078	924279
9	954242	959041	963788	968483	973128
10	000000	004321	008600	012837	017033
11	041393	045323	049218	053078	056905
12	079181	082785	086360	089905	093422
13	113943	117271	120574	123852	127105
14	146128	149219	152288	155336	158362
15	176091	178977	181844	184691	187521
16	204120	206826	209515	212188	214844
17	230449	232996	235528	238046	240549
18	255272	257679	260071	262451	264818
19	278754	281033	283301	285557	287802
20	301030	303196	305351	307496	309630
21	322219	324282	326336	328380	330414
22	342427	344392	346353	348305	350248
23	361728	363612	365488	367356	369216
24	380211	382017	383815	385606	387390
25	397940	399674	401400	403120	404834
26	414973	416640	418301	419956	421204
27	431364	432969	434569	436163	437751
28	447158	448706	450249	451786	453318
29	462398	463893	465383	466868	468347
30	477121	478566	480007	481443	482874
31	491362	492761	494155	495544	496930
32	505150	506505	507856	509202	510545
33	518514	519828	521138	522444	523746
34	531479	532754	534026	535294	536558
35	544068	545307	546543	547775	549003
36	556302	557507	558709	559907	561101
37	568202	569374	570543	571709	572872
38	579784	580925	582063	583199	584331
39	591065	592177	593286	594393	595496

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0	698970	778151	845098	903090	954242
1	176091	204120	230449	255272	278754
2	397940	414973	431364	447158	462398
3	544068	556302	568202	579784	591065
4	653212	662758	672098	681241	770852
5	740363	748188	755875	763428	690196
6	812913	819544	826075	832509	838849
7	875061	880814	886491	892095	897627
8	929419	934498	939519	944483	949390
9	977724	982271	986772	991226	995635
10	021189	025306	029384	033424	037426
11	060698	064458	068186	071882	075547
12	096910	100370	103834	107210	110590
13	130334	133539	136721	139879	143015
14	161368	164353	167317	170262	173186
15	190332	193125	195900	198657	201397
16	217484	220108	222716	225309	227887
17	243038	245513	247973	250420	252853
18	267172	269513	271842	274158	276462
19	290035	292256	294466	296665	298853
20	311754	313867	315970	318063	320146
21	332438	334454	336460	338456	340444
22	352182	354108	356026	357935	359835
23	371068	372912	374748	376577	378398
24	389166	390935	392697	394452	396199
25	406540	408240	409933	411620	413300
26	423246	424882	426511	428135	429752
27	439333	440909	442480	444045	445604
28	454845	456366	457882	459392	460898
29	469822	471292	472756	474216	475671
30	484300	485721	487138	488551	489958
31	498311	499687	501059	502427	503791
32	511883	513218	514548	515874	517196
33	525045	526339	527630	528917	530200
34	537819	539076	540329	541579	542825
35	550228	551450	552668	553885	555095
36	562293	563481	564666	565848	567026
37	574031	575188	576341	577492	578639
38	585461	586587	587711	588832	589950
39	596597	597695	598790	599883	600973

	0	1	2	3	4	5	6	7	8	9
49	60.2060	3144	4226	5305	6381	7455	8526	9594	0660	1723
41	61.2784	3842	4897	5950	7000	8048	9093	0136	1176	2214
42	62.3249	4282	5312	6341	7366	8389	9410	0428	1444	2457
43	63.3468	4477	5484	6488	7490	8489	9486	9481	1474	2464
44	64.3453	4439	5422	6404	7383	8350	9335	0307	1278	2246
45	65 3212	4176	5138	6098	7056	8011	8965	9916	0865	1813
46	66.2758	3701	4642	5581	6518	7453	8386	9317	0246	1173
47	67.2098	3021	3942	4861	5778	6694	7607	8518	9428	0335
48	68.1241	2145	3047	3947	4845	5742	6636	7529	8420	9309
49	69.0196	1081	9659	2847	3727	4605	5482	6356	7229	8101
50	8970	9838	0704	1568	2430	3291	4150	5008	5864	6718
51	70.7570	8421	9270	0117	0963	1807	2648	3491	4330	5167
52	71.6003	6838	7670	8502	9331	0159	0985	1811	2634	3456
53	72.4276	5094	5912	6727	7541	8354	9165	9974	0782	1589
54	73.2394	3197	3999	4800	5599	6396	7193	7987	8781	9572
55	74.0363	1152	1939	2725	3510	4293	5075	5855	6634	7412
56	8188	8963	9736	0508	1279	2048	2816	3583	4348	5112
57	75.5875	6636	7396	8155	8912	9668	0422	1176	1928	2679
58	76.3428	4176	4923	5669	6413	7156	7898	8638	9377	0115
59	77.0852	1587	2322	3055	3786	4517	5246	5974	6701	7427
60	8151	8874	9596	0317	1037	1755	2473	3189	3904	4617
61	78.5330	6041	6751	7460	8168	8875	9581	0285	0988	1691
62	79.2392	3092	3790	4488	5185	5880	6574	7267	7960	8651
63	9340	0029	0717	1404	2089	2774	3457	4139	4821	5501
64	80.6180	6858	7535	8211	8886	9560	0232	0904	1575	2245
65	81.2913	3581	4248	4913	5578	6241	6904	7565	8226	8885
66	9544	0201	0858	1513	2168	2822	3474	4126	4776	5426
67	82.6075	6075	6722	7369	8015	8660	9304	9947	0589	1230
68	83.2509	3147	3784	4421	5056	5691	6324	6957	7588	8219
69	8849	2479	0106	0733	1359	1985	2609	3233	3855	4477
70	84.5098	5718	6337	6955	7573	8189	8805	9419	0033	0646
71	85.1258	1870	2480	3089	3698	4306	4913	5519	6124	6729
72	7332	7935	8537	9138	9739	0338	8937	1534	2131	2727
73	86.3323	3917	4511	5104	5696	6287	6878	7467	8056	8644
74	9232	9818	0404	0989	1573	2156	2739	3321	3902	4482
75	87.5061	5640	6218	6795	7371	7947	8522	9096	9669	0242
76	88.0814	1385	1955	2524	3093	2661	4229	4795	5361	5926
77	6491	7054	7617	8179	8741	9302	9862	0421	0980	1537
78	89.2095	2651	3207	3767	4316	4870	5422	5975	6526	7077
79	7627	8176	8725	9273	9820	0367	0913	1458	2003	2547
80	90.3090	3632	4174	4715	5256	5796	6335	6873	7411	7948
81	8485	9021	9556	0090	0624	1158	1690	2222	2753	3284
82	91.3814	4343	4872	5400	5927	6454	6980	7505	8030	8554
83	9078	9601	0123	0645	1166	1686	2206	2725	3244	3762
84	92.4279	4796	5312	5828	6342	6856	7370	7883	8396	8908
85	9419	9930	0440	0949	1458	1966	2474	2981	3487	3993
86	93.4498	5003	5507	6011	6514	7016	7518	8019	8520	9020
87	9519	0018	0516	1014	1511	2008	2504	3000	3494	3989
88	94.4483	4976	5469	5961	6452	6943	7434	7924	8413	8902
89	9390	9878	0365	0851	1337	1823	2308	2792	3276	3760

N°.	0	1	2	3	4	5	6	7	8	9
90	95.4242	4725	5206	5688	6168	6649	7128	7607	8086	8564
91	9041	9518	9995	0471	0946	1421	1895	2369	2843	3315
92	96.3788	4260	4731	5202	5672	6142	6611	7080	7548	8016
93	8483	8950	9416	9882	0347	0812	1276	1740	2203	2666
94	97.3128	3590	4051	4512	4972	5432	5891	6350	6808	7266
95	7724	8180	8637	9093	9548	0003	0457	0912	1365	1819
96	98.2271	2723	3175	3626	4077	4527	4977	5426	5875	6324
97	6772	7219	7666	8113	8559	9005	9450	9895	0339	0783
98	99.1226	1669	2111	2553	2995	3436	3877	4317	4757	5196
99	5635	6074	6512	6946	7386	7823	8259	8695	9130	9565
100	00.0000	0434	0868	1301	1734	2166	2598	3029	3460	3891
101	4321	4751	5180	5609	6038	6466	6894	7321	7748	8174
102	8600	9026	9451	9876	0300	0724	1147	1570	1993	2415
103	01.2837	3259	3680	4100	4520	4940	5360	5779	6197	6615
104	7033	7451	7868	8284	8700	9116	9532	9947	0361	0775
105	02.1189	1603	9016	2428	2841	3252	3664	4075	4486	4896
106	5306	5715	6124	6533	6942	7350	7757	8164	8571	8978
107	9384	9789	0195	0600	1004	1408	1812	2216	2619	3021
108	03.3424	3826	4227	4628	5029	5430	5830	6229	6630	7028
109	7426	7825	8223	8620	9017	9414	9811	0207	0602	0998
110	04.1393	1787	2182	2575	2969	3362	3755	4148	4540	4931
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830
112	9218	9606	9993	0380	0766	1152	1538	1924	2309	2694
113	05.3078	3463	3846	4230	4613	4996	5378	5760	6142	6524
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	0320
115	06.0698	1075	1452	1829	2206	2582	2958	3333	3709	4083
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7814
117	8186	8557	8928	9298	9668	0038	0407	0776	1145	1514
118	07.1882	2250	2617	2985	3352	3718	4085	4451	4816	5182
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819
120	9181	9543	9904	0266	0626	0987	1347	1707	2067	2426
121	08.2785	3144	3503	3861	4219	4576	4934	5291	5647	6004
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552
123	9905	0258	0611	0963	1315	1667	2018	2370	2721	3071
124	09.3422	3772	4122	4471	4820	5169	5518	5866	6215	6562
125	6910	7257	7604	7951	8297	8644	8990	9335	9681	0026
126	10.0371	0715	1059	1403	1747	2090	2434	2777	3119	3462
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6870
128	7210	7549	7888	8227	8565	8903	9241	9578	9916	0253
129	11.0590	0926	1262	1598	1934	2270	2605	2940	3275	3609
130	3943	4277	4611	4944	5278	5610	5943	6276	6608	6940
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	0245
132	12.0574	0903	1231	1560	1888	2216	2543	2871	3198	3525
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781
134	7105	7429	7752	8076	8399	8722	9045	9367	9690	0012
135	13.0334	0655	0977	1298	1619	1939	2260	2580	2900	3219
136	3539	3858	4177	4496	4814	5133	5451	5768	6086	6403
137	6721	7037	7354	7670	7987	8303	8618	8934	9249	9564
138	9879	0194	0508	0822	1136	1450	1763	2076	2384	2702
139	14.3015	3327	3639	3951	4263	4574	4885	5196	5507	5818

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141	9219	9527	9835	0142	0449	0756	1063	1370	1676	1982
142	15.2288	2594	2900	3205	3510	3815	4119	4424	4728	5032
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061
144	8362	8664	8965	9266	9567	9868	0168	0458	0769	1068
145	16.1368	1667	1967	2266	2564	2863	3161	3460	3757	4055
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022
147	7317	7613	7908	8203	8497	8792	9085	9380	9674	9968
148	17.0261	0555	0848	1141	1434	1726	2019	2311	2603	2895
149	3186	3478	3769	4060	4351	4641	4931	5222	5512	5802
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689
151	8977	9264	9552	9839	0126	0413	0699	0986	1272	1558
152	18.1844	2129	2415	2700	2985	3270	3554	3839	4123	4407
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	0051
155	19.0332	0612	0892	1171	1451	1730	2010	2289	2567	2846
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623
157	5900	6176	6452	6729	7005	7281	7556	7832	8107	8382
158	8657	8932	9206	9481	9755	0029	0303	0577	0850	1124
159	20.1397	1670	1943	2216	2488	2761	3033	3305	3577	3848
160	4120	4391	4662	4933	5204	5475	5745	6016	6286	6556
161	6826	7095	7365	7634	7903	8172	8441	8710	8978	9247
162	9515	9783	0051	0318	0586	0853	1120	1388	1654	1921
163	21.2188	2454	2720	2986	3252	3518	3783	4048	4314	4579
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221
165	7484	7747	8010	8273	8535	8798	9060	9322	9584	9846
166	22.0108	0370	0631	0892	1153	1414	1675	1936	2196	2456
167	2716	2976	3236	3494	3755	4015	4274	4533	4792	5051
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630
169	7881	8144	8400	8657	8913	9170	9426	9682	9938	0193
170	23.0449	0704	0960	1215	1470	1724	1979	2233	2488	2742
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795
173	8046	8297	8548	8799	9049	9299	9550	9799	0050	0300
174	24.0549	0799	1048	1297	1546	1795	2044	2293	2541	2790
175	3038	3286	3534	3782	4030	4277	4524	4772	5019	5266
176	5513	5759	6006	6252	6499	6745	6991	7236	7482	7728
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	0176
178	25.0420	0664	0908	1151	1395	1638	1881	2125	2367	2610
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031
180	5272	5514	5755	5996	6236	6477	6718	6958	7198	7439
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833
182	26.0071	0310	0548	0787	1025	1263	1501	1738	1976	2214
183	2451	2688	2592	3162	3399	3636	3873	4109	4345	4582
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279
186	9513	9746	9980	0213	0446	0679	0912	1144	1377	1609
187	27.1842	2074	2306	2538	2770	3001	3233	3464	3696	3927
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232
189	6462	6691	6921	7151	7380	7609	7838	8067	8296	8525

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191	28.1033	1261	1488	1715	1942	2169	2395	2622	2848	3075
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578
194	7802	8025	8249	8473	8696	8920	9143	9366	9589	9812
195	29.0035	0257	0480	0702	0925	1147	1369	1591	1813	2034
196	2256	2478	2699	2920	3141	3363	3583	3804	4025	4246
197	4466	4687	4907	5127	5347	5567	5787	0007	6226	6446
198	6665	6884	7104	7323	7542	7760	7979	8198	8416	8635
199	8853	9071	9289	9507	9725	9942	0160	0378	0595	0813
200	30.1030	1247	1464	1681	1898	2114	2331	2547	2764	2980
201	3196	3412	3628	3844	4059	4275	4490	4706	4921	5136
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417
204	9630	9843	0056	0268	0481	0693	0906	1118	1330	1542
205	31 1754	1966	2177	2389	2600	2812	3023	3234	3445	3656
206	3867	4078	4289	4499	4710	4920	5130	5340	5550	5760
207	5970	6180	6390	6599	6809	7018	7227	7436	7645	7854
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938
209	32.0146	0354	0562	0769	0977	1184	1391	1598	1805	2012
210	2219	2426	2633	2839	3046	3252	3458	3664	3871	4077
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131
212	6336	6541	6745	6950	7154	7359	7563	7767	7972	8176
213	8380	8583	8787	8991	9194	9398	9601	9804	0008	0211
214	33.0414	0617	0819	1022	1225	1427	1630	1832	2034	2236
215	2438	2640	2842	044	3246	3447	3649	3850	4051	4253
216	4454	4655	4856	5056	5257	5458	5658	5859	6059	6260
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257
218	8456	8656	8855	9054	9253	9451	9650	9849	0047	0246
219	34.0444	0642	0840	1039	1237	1434	1632	1830	2028	2225
220	2423	2620	2817	3014	3212	3409	3605	3802	3999	4196
221	4392	4589	4785	4981	5178	5374	5570	5766	5961	6157
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110
223	8305	8500	8694	8889	9083	9277	9472	9666	9860	0054
224	35.0248	0442	0636	0829	1023	1216	1410	1603	1796	1989
225	2182	2375	2568	2761	2954	3146	3339	3532	3724	3916
226	4108	4301	4493	4684	4876	5068	5260	5451	5643	5834
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646
229	9835	0025	0215	0404	0593	0783	0972	1161	1350	1539
230	36.1728	1917	2105	2294	2482	2671	2859	3048	3236	3424
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169
233	7356	7542	7728	7915	8101	8287	8473	8659	8844	9030
234	9216	9401	9587	9772	9958	0143	0328	0513	0698	0883
235	37.1068	1253	1437	1622	1806	1991	2175	2360	2544	2728
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216
239	8398	8580	8761	8943	9124	9305	9487	9668	9849	0030

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240	38.0211	0392	0573	0754	0934	1115	1296	1476	1656	1837
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5427
243	5606	5785	5964	6142	6321	6499	6677	6855	7034	7212
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989
245	9166	9343	9520	9697	9875	0051	0228	0405	0582	0758
246	39.0935	1112	1288	1464	1641	1817	1993	2169	2345	2521
247	2679	2873	3048	3224	3400	3575	3751	3926	4101	4276
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025
249	6199	6374	6548	6722	6896	7070	7245	7418	7592	7766
250	7940	8114	0287	8461	8634	8808	8981	9154	9327	9501
251	9674	9847	0020	0192	0365	0538	0711	0883	1056	1228
252	40.1400	1573	1745	1917	2089	2261	2433	2605	2777	2949
253	3120	3292	3464	3635	3807	3978	4149	4320	4492	4663
254	4834	5005	5175	5346	5517	5688	5858	6029	6199	6370
255	6540	6710	6881	7051	7221	7391	7561	7731	7900	8070
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764
257	9933	0102	0271	0440	0608	0777	0946	1114	1283	1451
258	41.1620	1788	1956	2124	2292	2460	2628	2796	2964	3132
259	3300	3467	3635	3802	3970	4137	4305	4472	4639	4806
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474
261	6640	6807	6973	7139	7306	7472	7638	7804	7970	8135
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791
263	9956	0121	0286	0451	0616	0781	0945	1110	1275	1439
264	42.1604	1768	1933	2097	2261	2426	2590	2754	2918	3082
265	3246	3410	3573	3737	3901	4064	4228	4392	4555	4718
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973
268	8135	8297	8459	8621	8782	8944	9106	9268	9429	9591
269	9752	9914	0075	0236	0398	0559	0720	0881	1042	1203
270	43.1364	1525	1685	1846	2007	2167	2328	2488	2649	2809
271	2969	3129	3290	3450	3610	3770	3930	4090	4249	4409
272	4569	4728	4888	5048	5207	5366	5526	5685	5844	6003
273	6163	6322	6481	6640	6798	6957	7116	7275	7433	7592
274	7751	7909	8067	8226	8384	8542	8700	8859	9017	9175
275	9333	9491	9648	9806	9964	0122	0279	0437	0594	0752
276	44.0909	1066	1224	1381	1538	1695	1852	2009	2166	2323
277	2480	2636	2793	2950	3106	3263	3419	3576	3732	3888
278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5448
279	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552
281	8706	8861	9015	9170	9324	9478	9633	9787	9941	0095
282	45.0249	0403	0557	0711	0865	1018	1172	1326	1479	1633
283	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165
284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692
285	4845	4997	5149	5302	5454	5606	5758	5910	6062	6214
286	6366	6518	6670	6821	6973	7125	7276	7428	7579	7730
287	7882	8033	8184	8336	8487	8638	8789	8940	9091	9241
288	9392	9543	9694	9845	9995	0146	0296	0447	0597	0747
289	46.0898	1048	1198	1348	1498	1649	1799	1948	2098	2248

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290	46.2398	2548	2697	2847	2997	3146	3296	3445	3594	3744
291	3893	4042	4191	4340	4489	4639	4787	4936	5085	5234
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200
294	8347	8495	8643	8790	8938	9085	9233	9831	9527	9675
295	9822	9969	0116	0263	0410	0557	0704	0851	0998	1145
296	47.1292	1438	1585	1732	1878	2025	2171	2317	2464	2610
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4070
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976
300	7121	7266	7411	7555	7700	7844	7989	8133	8278	8422
301	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863
302	48.0007	0151	0294	0438	0582	0725	0869	1012	1156	1299
303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157
305	4300	4442	4584	4727	4868	5011	5153	5295	5437	5579
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410
308	8551	8692	8833	8973	9114	9255	9396	9537	9677	9818
309	9958	0099	0239	0380	0520	0661	0801	0941	1081	1222
310	49.1362	1502	1642	1782	1922	2062	2201	2341	2481	2621
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406
313	5544	5683	5822	5960	6099	6237	6376	6514	6653	6791
314	6930	7068	7206	7344	7482	7621	7759	7897	8035	8173
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550
316	9687	9824	9962	0099	0236	0374	0511	0648	0785	0922
317	50.1059	1196	1333	1470	1607	1744	1880	2017	2154	2290
318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3654
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014
320	5150	5286	5421	5557	5692	5828	5963	6099	6234	6370
321	6505	6640	6775	6911	7046	7181	7316	7451	7586	7721
322	7856	7991	8125	8260	8395	8530	8664	8799	8933	9068
323	9202	9337	9471	9606	9740	9874	0008	0143	0277	0411
324	51.0545	0679	0813	0947	1081	1215	1348	1482	1616	1750
325	1883	2017	2150	2284	2417	2551	2684	2818	2951	3084
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415
327	4548	4680	4813	4946	5079	5211	5343	5476	5609	5741
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382
330	8514	8645	8777	8909	9040	9171	9303	9434	9565	9697
331	9828	9959	0090	0221	0352	0483	0614	0745	0876	1007
332	52.1138	1269	1400	1530	1661	1792	1922	2053	2183	2314
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915
335	5045	5174	5304	5434	5563	5692	5822	5951	6081	6210
336	6339	6468	6598	6727	6856	6985	7114	7243	7372	7501
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	878[illegible]
338	8917	9045	9174	9302	9434	9559	9687	9815	9943	00[illegible]2
339	53.0200	0328	0456	0584	0712	0840	0968	1095	1223	1351

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340	53.1479	1607	1734	1862	1990	2117	2245	2372	2500	2627
341	2754	2882	3009	3136	3263	3391	3518	3645	3772	3899
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167
343	5294	5421	5547	5674	5800	5927	6053	6179	6306	6432
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951
346	9076	9202	9327	9452	9578	9703	9829	9954	0079	0204
347	54.0329	0455	0580	0705	0830	0955	1080	1205	1330	1454
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183
351	5307	5431	5554	5678	5802	5925	6049	6172	6296	6419
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652
353	7775	7898	8021	8144	8266	8389	8512	8635	8758	8881
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	0106
355	55.0228	0351	0473	0595	0717	0840	0962	1084	1206	1328
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2546
357	2668	2790	2911	3033	3154	3276	3397	3519	3640	3762
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973
359	5094	5211	5336	5457	5578	5699	5820	5940	6061	6182
360	6302	6423	6544	6664	6785	6905	7026	7146	7266	7387
361	7507	7627	7748	7868	7988	8108	8228	8348	8469	8589
362	8709	8828	8948	9068	9188	9308	9428	9548	9667	9787
363	9907	0026	0146	0265	0385	0504	0624	0743	0863	0982
364	56.1101	1221	1340	1459	1578	1697	1817	1936	2055	2174
365	2293	2412	2531	2650	2768	2887	3006	3125	3244	3362
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548
367	4666	4784	4903	5021	5139	5257	5375	5494	5612	5730
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084
370	8202	8319	8436	8554	8671	8778	8905	9023	9140	9257
371	9374	9491	9608	9725	9842	9959	0076	0193	0309	0426
372	57.0543	0660	0776	0893	1010	1126	1243	1359	1476	1592
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915
375	4031	4147	4263	4378	4494	4610	4726	4841	4957	5072
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226
377	6341	6456	6572	6687	6802	6917	7032	7147	7262	7377
378	7492	7607	7721	7836	7951	8066	8181	8295	8410	8525
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669
380	9784	9898	0012	0126	0240	0355	0469	0583	0697	0811
381	58.0925	1039	1153	1267	1381	1494	1608	1722	1836	1950
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085
383	3199	3312	3425	3539	3652	3765	3879	3992	4105	4218
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348
385	5461	7573	5686	5799	5912	6024	6137	6250	6362	6475
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599
387	7711	7823	7925	8047	8160	8272	8384	8496	8608	8720
388	8831	8944	9055	9167	9279	9391	9503	9614	9726	9838
389	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953

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390	59.1065	1176	1287	1397	1510	1621	1732	1843	1955	2066
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282
393	4393	4503	4613	4724	4834	4945	5055	5165	5276	5386
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7585
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681
397	8790	8900	9009	9119	9228	9337	9446	9556	9665	9774
398	9883	9992	0101	0210	0319	0428	0537	0646	0755	0864
399	60.0973	1082	1190	1299	1408	1517	1625	1734	1843	1951
400	2060	2169	2277	2386	2494	2602	2711	2819	2928	3036
401	3144	3252	3361	3469	3577	3685	3794	3902	4010	4118
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197
403	5305	5413	5521	5628	5736	5843	5951	6059	6166	6274
404	6381	6489	6586	6704	6811	6918	7026	7133	7240	7348
405	7455	7562	7677	7777	7884	7991	8098	8205	8312	8419
406	8526	8633	8740	8847	8954	9060	9167	9274	9381	9488
407	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554
408	61.0660	0767	0873	0979	1086	1192	1298	1405	1511	1617
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678
410	2784	2890	2996	3101	3207	3313	3419	3525	3630	3736
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845
413	5950	6055	6160	6265	6370	6475	6580	6685	6790	6895
414	7000	7105	7210	7313	7420	7524	7629	7734	7839	7943
415	8048	8153	8257	8362	8466	8571	8675	8780	8884	8989
416	9093	9198	9302	9406	9511	9615	9719	9823	9928	0032
417	62.0136	0240	0344	0448	0552	0656	0760	0864	0968	1072
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146
420	3249	3353	3456	3559	3663	3766	3869	3972	4076	4179
421	4282	4385	4488	4591	4694	4798	4901	5004	5107	5209
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263
424	7366	7468	7571	7673	7775	7878	7980	8082	8184	8287
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308
426	9410	9511	9613	9715	9817	9919	0021	0123	0224	0326
427	63.0428	0530	0631	0733	0834	0936	1038	1139	1241	1342
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356
429	5427	2558	2660	2761	2862	2963	3064	3165	3266	3367
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376
431	4477	4578	4679	4779	4881	4981	5681	5182	5283	5383
432	5484	5584	5685	5785	5886	5986	6086	6187	6287	6388
433	6488	6588	6688	6789	6889	6989	7089	7189	7289	7390
434	7490	7590	7690	7790	7890	7990	8090	8190	8289	8389
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387
436	9486	9586	9685	9785	9885	9984	0084	0183	0283	0382
437	64.0841	0581	0680	0779	0879	0978	1077	1176	1276	1375
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366
439	2464	2563	2662	2761	2860	2959	3058	3156	3255	3354

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441	644.439	537	635	734	832	931	029	127	226	324
442	645.422	520	619	717	815	913	011	109	208	306
443	646.404	502	600	698	796	894	991	089	187	285
444	647.383	481	579	676	774	872	969	067	165	262
445	648.360	458	555	653	750	848	945	043	140	237
446	649.335	432	530	627	724	821	919	016	113	210
447	650.307	405	502	599	696	793	890	987	084	181
448	651.278	375	472	569	666	762	859	956	053	150
449	652.246	343	440	536	633	730	826	923	019	116
450	653.212	309	405	502	598	695	791	888	984	080
451	654.176	273	369	465	562	658	754	850	946	042
452	655.138	234	331	427	523	619	714	810	906	002
453	656.098	194	290	386	481	577	673	769	864	960
454	657.056	151	247	343	438	534	629	725	820	916
455	658.051	107	202	298	393	488	584	679	774	870
456	965	060	155	250	346	441	536	631	726	821
457	659.916	011	106	201	296	391	486	581	676	771
458	660.865	960	055	150	245	340	434	529	623	718
459	661.813	907	002	096	191	285	380	474	569	663
460	662.758	852	947	041	135	230	324	418	512	607
461	663.701	795	889	983	078	172	266	360	454	548
462	664.642	736	830	924	018	112	205	299	393	487
463	665.581	675	769	862	956	050	143	237	331	424
464	666.518	612	705	799	892	986	079	173	266	359
465	667.453	546	640	733	826	920	013	106	199	293
466	668.386	479	572	665	758	852	945	038	131	224
467	669.317	410	503	596	689	782	874	967	060	153
468	670.246	339	431	524	617	710	802	895	988	080
469	671.173	265	358	451	543	636	728	821	913	005
470	672.098	190	283	375	467	560	652	744	836	929
471	673.021	113	205	297	390	482	574	666	758	850
472	942	034	126	218	310	402	494	586	677	769
473	674.861	953	045	136	228	320	412	503	595	687
474	675.778	870	961	053	145	236	328	419	511	602
475	676.694	785	876	968	059	150	242	333	424	516
476	677.607	698	789	881	972	063	154	245	336	427
477	678.518	609	700	791	882	973	064	155	246	337
478	679.428	519	610	700	791	882	973	063	154	245
479	680.335	426	517	607	698	789	879	970	060	151
480	681.241	332	422	513	603	693	784	874	964	055
481	682.145	235	326	416	506	596	686	777	867	957
482	683.047	137	227	317	407	497	587	667	767	857
483	947	037	127	217	307	396	486	576	666	756
484	684.845	935	025	114	204	294	383	473	563	652
485	685.742	831	921	010	100	189	279	368	457	547
486	686.636	726	815	904	994	083	172	261	351	440
487	687.529	618	707	796	885	975	064	153	242	331
488	688.420	509	598	687	776	865	953	042	131	220
489	689.309	398	486	575	664	753	841	930	019	107

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490	690.196	285	373	462	550	639	727	816	905	993
491	691.081	170	258	347	435	523	612	700	788	877
492	965	053	142	230	318	406	494	583	671	759
493	692.847	935	023	111	199	287	375	463	551	639
494	693.727	815	403	991	078	166	254	342	430	517
495	694.605	693	781	868	956	044	131	219	306	394
496	695.482	569	657	744	832	919	007	094	182	269
497	696.356	444	531	618	706	793	880	968	055	142
498	697.229	316	404	491	578	665	752	839	926	013
499	698.100	188	275	362	448	535	622	709	796	883
500	970	057	144	230	317	404	491	578	664	751
501	699.838	924	011	098	184	271	357	444	531	617
502	700.703	790	877	963	050	136	222	309	395	482
503	701.568	654	741	827	913	999	086	172	258	344
504	702.430	517	603	689	775	861	947	033	119	205
505	703.291	377	463	549	635	721	807	893	979	065
506	704.150	236	322	408	494	579	665	751	837	922
507	705.008	094	179	265	350	436	522	607	643	778
508	864	949	034	120	205	291	376	462	547	632
509	706.718	803	888	974	059	144	229	315	400	485
510	707.570	655	740	826	911	996	081	166	251	336
511	708.421	506	591	676	761	846	930	015	100	185
512	709.270	355	440	524	609	694	779	863	948	033
513	710.117	202	287	371	456	540	625	710	194	879
514	963	048	132	216	301	385	470	554	638	723
515	711.807	891	976	060	144	229	313	397	481	565
516	712.650	734	818	902	986	070	154	238	322	406
517	713.490	574	658	742	826	910	994	078	162	246
518	714.330	414	497	581	665	749	832	916	000	084
519	715.167	251	335	418	502	586	669	753	836	920
520	716.003	087	170	254	337	421	504	588	671	754
521	838	921	004	088	171	254	338	421	504	588
522	717.670	754	837	920	003	086	169	252	336	419
523	718.502	585	668	751	834	917	000	083	165	248
524	719.331	414	497	580	663	745	828	911	994	077
525	720.159	242	325	407	490	573	655	738	821	903
526	986	068	151	233	316	398	481	563	646	728
527	721.811	893	975	058	140	222	305	387	469	552
528	722.634	716	798	881	963	045	127	209	291	374
529	723.456	538	620	702	784	866	948	030	112	194
530	724.276	358	440	522	603	685	767	849	931	013
531	725.094	176	258	340	422	503	585	667	748	830
532	912	993	075	156	238	320	401	483	564	646
533	726.727	809	890	972	053	134	216	297	379	460
534	727.541	623	704	785	866	948	029	110	191	273
535	728.354	435	516	597	678	759	841	922	003	084
536	729.165	246	327	408	489	570	651	732	812	893
537	974	055	136	217	298	378	459	540	621	701
538	730.782	863	944	024	105	186	266	347	428	508
539	731.589	669	750	830	991	911	072	152	233	313

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540	732.394	474	555	635	715	790	876	956	037	117
541	733.197	277	358	438	518	598	679	759	839	919
542	999	079	159	240	320	400	480	560	640	720
543	734.800	880	960	040	120	200	279	359	439	519
544	735.599	679	758	838	918	998	078	157	237	317
545	736.396	476	556	635	715	795	874	954	033	113
546	737.192	272	352	431	511	590	670	749	828	908
547	987	067	146	225	305	384	463	542	622	701
548	738.781	860	939	018	097	177	256	355	414	493
549	739.572	651	730	810	889	968	047	126	205	281
550	740.363	442	521	599	678	757	836	915	994	073
551	741.152	230	309	388	467	545	624	703	782	860
552	939	018	096	175	254	332	411	489	568	647
553	742.725	804	882	961	039	118	196	274	353	431
554	743.510	588	666	745	823	902	980	058	136	215
555	744.293	371	449	528	606	684	762	840	918	997
556	745.075	153	231	309	387	465	543	621	699	777
557	855	933	011	089	167	345	323	401	478	556
558	746.634	712	790	868	945	023	101	179	256	334
559	747.412	489	567	645	722	800	878	955	033	110
560	748.188	256	343	421	498	576	653	731	808	885
561	963	040	118	195	272	350	427	504	582	659
562	749.736	814	891	968	045	122	200	277	354	431
563	750.508	585	663	740	817	894	971	048	125	202
564	751.279	356	433	510	587	664	741	818	895	972
565	752.048	125	202	279	356	433	509	586	663	740
566	816	893	970	047	123	200	277	353	430	506
567	753.583	660	736	813	889	966	042	119	195	272
568	754.348	425	501	578	654	730	807	883	960	036
569	755.112	189	265	341	417	494	570	646	722	799
570	875	951	027	103	179	256	332	408	484	560
571	756.636	712	788	864	940	016	092	168	244	320
572	757.396	472	548	624	700	775	851	927	003	079
573	758.155	230	306	382	458	533	609	685	760	836
574	912	987	063	139	214	290	366	441	517	592
575	759.668	743	819	894	970	045	121	196	272	347
576	760.422	498	573	649	724	799	875	950	025	100
577	761.176	251	326	402	477	552	627	702	777	853
578	928	003	078	153	228	303	378	453	528	603
579	762.679	754	829	903	978	053	128	203	278	35[illegible]
580	763.428	503	568	653	727	802	877	952	027	101
581	764.176	251	326	400	475	550	624	699	774	848
582	923	998	072	147	221	296	370	445	519	594
583	765.669	743	817	892	966	041	115	190	264	338
584	766.413	487	562	636	710	784	859	933	007	082
585	767.156	230	304	378	453	527	601	675	749	823
586	898	972	046	120	194	268	342	416	490	564
587	768.638	712	786	860	934	008	082	156	230	303
588	769.377	451	525	599	673	746	820	894	968	041
589	770.115	189	263	336	410	484	557	631	705	778

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590	770.852	926	999	073	146	220	293	367	440	514
591	771.587	661	734	808	881	955	028	102	175	248
592	772.322	395	468	542	615	688	762	835	908	981
593	773.055	128	201	274	347	420	494	567	640	713
594	786	859	933	006	079	152	225	298	371	444
595	774.517	590	663	736	809	882	955	028	100	173
596	775.246	319	392	465	538	610	683	756	829	902
597	974	047	120	192	265	338	411	483	556	628
598	776.701	774	846	919	992	064	137	209	282	354
599	777.427	499	572	644	717	789	862	934	006	079
600	778.151	224	296	368	441	513	585	658	730	802
601	874	947	019	091	163	236	308	380	452	524
602	779.596	669	141	813	885	957	029	101	173	245
603	780.317	389	461	533	605	677	749	821	893	965
604	781.037	109	181	253	324	396	468	540	612	684
605	755	827	899	971	042	114	186	258	329	401
606	782.473	544	616	688	759	831	902	974	046	117
607	783.189	260	332	403	475	546	618	689	761	832
608	904	975	046	118	189	261	332	403	475	546
609	784.617	689	760	831	902	974	045	116	187	259
610	785.330	401	472	543	614	686	757	828	899	970
611	786.041	112	183	254	325	396	467	538	609	680
612	751	822	893	964	035	106	177	248	319	390
613	787.460	531	602	673	744	815	885	956	027	098
614	788.168	239	310	380	451	522	593	663	734	804
615	875	946	016	087	157	228	299	369	440	510
616	789.581	651	722	792	863	933	003	074	144	215
617	790.285	355	426	496	567	637	707	778	848	918
618	988	059	129	199	269	340	410	480	550	620
619	791.691	761	831	901	971	041	111	181	252	322
620	792.392	462	532	602	672	742	812	882	952	022
621	793.092	161	231	301	371	441	511	581	651	721
622	790	860	930	000	070	139	209	279	349	418
623	794.488	558	627	697	767	836	906	976	045	115
624	795.185	254	324	393	463	532	602	671	741	810
625	880	949	019	088	158	227	297	366	436	505
626	796.574	644	713	782	852	921	990	060	129	198
627	797.267	337	406	475	544	614	683	752	821	890
628	960	029	098	167	236	305	374	443	512	582
629	798.651	720	789	858	927	996	065	134	203	272
630	799.340	409	478	547	616	685	754	823	892	960
631	800.029	098	167	236	305	373	442	511	580	648
632	717	786	854	923	992	060	129	198	266	335
633	801.404	472	541	609	678	747	815	884	952	021
634	802.089	158	226	295	363	432	500	568	637	705
635	774	842	910	979	047	115	184	252	320	389
636	803.457	525	594	662	730	798	867	935	003	071
637	804.139	208	276	344	412	480	448	616	684	753
638	821	889	957	025	093	161	229	297	365	433
639	805.501	569	537	705	773	840	908	976	044	112

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640	806.180	248	316	383	451	519	587	655	722	790
641	858	926	993	061	129	197	264	332	400	467
642	807.535	603	670	738	805	873	941	008	076	143
643	808.211	278	346	413	481	548	616	683	751	818
644	886	953	021	088	155	223	290	358	425	492
645	809.560	627	694	762	829	896	963	031	098	165
746	810.232	300	367	434	501	568	636	703	770	837
647	904	971	038	106	173	240	307	374	440	508
648	811.575	642	709	776	843	910	977	044	111	178
649	812.245	312	378	445	512	579	646	713	780	846
650	913	980	047	114	180	247	314	381	447	514
651	813.581	648	714	781	848	914	981	048	114	181
652	814.248	314	381	447	514	580	647	714	780	847
653	913	980	046	113	179	246	312	378	445	511
654	815.578	644	710	777	843	910	976	042	109	175
655	816.241	308	374	440	506	573	639	705	771	838
656	904	970	036	102	169	235	301	367	433	499
657	817.565	632	697	764	830	896	962	028	094	160
658	818.226	292	358	424	490	556	622	688	754	819
659	885	951	017	083	149	215	281	346	412	478
660	819.544	610	675	741	807	873	939	004	070	136
661	820.201	267	333	398	464	530	595	661	727	792
662	858	924	989	055	120	186	251	317	382	448
663	821.513	579	644	710	775	841	906	972	037	103
664	822.168	233	299	364	430	495	560	626	691	756
665	822	887	952	017	083	148	213	279	344	409
666	823.474	539	605	670	735	800	865	930	996	061
667	824.126	191	256	321	386	451	516	581	646	711
668	776	841	906	971	036	101	166	231	296	361
669	825.426	491	556	621	686	751	815	880	945	010
670	826.075	140	204	269	334	399	463	528	593	658
671	722	787	852	917	981	046	111	175	240	305
672	827.369	434	498	563	628	692	757	821	886	950
673	828.015	080	144	209	273	338	402	466	531	595
674	660	724	789	853	918	982	046	111	175	239
675	829.304	368	432	497	561	625	690	754	818	882
676	947	011	075	139	204	268	332	396	460	524
677	830.589	653	717	781	845	909	973	037	102	166
678	831.230	294	358	422	486	550	614	678	742	806
679	870	934	998	062	125	189	253	317	381	445
680	832.509	573	637	700	764	828	892	956	019	083
681	833.147	211	275	338	402	466	530	593	657	721
682	784	848	912	975	039	103	166	230	293	357
683	834.421	484	548	611	675	738	802	866	929	993
684	835.056	120	183	246	310	373	437	500	564	627
685	691	754	817	881	944	007	071	134	197	261
686	836.324	387	451	514	577	640	704	767	830	893
687	957	020	083	146	209	273	336	399	462	525
688	837.588	652	715	778	841	904	967	030	093	156
689	838.219	282	345	408	471	534	597	660	723	786

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691	839.478	541	604	667	729	792	855	918	981	043
692	840.106	169	232	294	357	420	482	545	608	671
693	733	796	859	921	984	046	109	172	234	297
694	841.359	422	485	547	610	672	735	797	860	922
695	984	047	110	172	235	297	360	422	484	547
696	842.609	672	734	796	859	921	983	046	108	170
697	843.233	295	357	420	482	544	606	669	731	793
698	855	918	980	042	104	166	229	291	353	415
699	844.477	539	601	663	726	788	850	912	974	036
700	845.098	160	222	284	346	408	470	532	594	656
701	718	780	842	904	966	028	090	151	213	275
702	846.337	399	461	523	584	646	708	770	832	893
703	955	017	079	141	202	264	326	388	449	511
704	847.573	634	695	758	819	881	943	004	067	127
705	848.189	251	312	374	435	497	559	620	682	743
706	805	866	928	989	051	112	174	236	296	358
707	849.419	481	542	604	665	726	788	849	911	972
708	850.033	095	156	217	279	340	401	462	523	585
709	646	707	769	830	891	952	014	075	136	197
710	851.258	319	381	442	503	564	625	686	747	808
711	870	931	992	053	114	175	236	297	358	419
712	852.480	541	602	663	724	785	846	907	968	029
713	853.089	150	211	272	333	394	455	516	576	637
714	698	759	820	881	942	002	063	124	184	245
715	854.306	367	427	488	549	610	670	731	792	852
716	913	974	034	095	156	216	277	337	398	459
717	855.519	580	640	701	761	822	882	943	003	064
718	856.124	185	245	306	366	427	487	548	608	668
719	729	789	850	910	970	031	091	151	212	272
720	857.332	393	453	513	574	634	694	754	815	875
721	935	995	056	116	176	236	296	357	417	477
722	858.537	597	657	718	778	838	898	958	018	079
723	859.138	198	258	318	378	438	499	559	619	679
724	739	798	858	918	978	038	098	158	218	278
725	860.338	398	458	518	578	637	697	757	817	877
726	937	996	056	116	176	236	295	355	415	475
727	861.534	594	654	714	773	833	893	952	012	072
728	862.131	191	251	310	370	430	489	549	608	668
729	727	787	846	906	966	025	085	144	204	263
730	863.323	382	442	501	561	620	680	739	798	858
731	917	977	036	096	155	214	274	333	392	452
732	864.511	570	630	689	748	808	867	926	985	045
733	865.104	163	222	282	341	400	459	518	578	637
734	696	755	814	873	933	992	051	110	169	228
735	866.287	346	405	465	524	583	642	701	760	819
736	878	937	996	055	114	173	232	291	350	409
737	867.467	526	585	644	703	762	821	880	939	997
738	868.056	115	174	233	292	350	409	468	527	586
739	644	703	762	821	879	928	997	056	114	173

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740	869.232	290	349	408	466	525	584	642	701	760
741	818	877	935	994	053	111	170	228	287	345
742	870.404	462	521	579	638	696	755	813	872	930
743	989	047	106	164	223	281	339	398	456	515
744	871.573	631	690	748	806	865	923	981	040	098
745	872.156	215	273	331	389	449	506	564	622	681
746	739	797	855	913	972	030	088	146	204	262
747	873.321	378	437	495	553	611	669	727	785	843
748	902	960	017	076	134	192	250	308	366	424
749	874.482	540	598	656	714	772	830	887	945	003
750	875.061	119	177	235	293	351	409	466	524	582
751	640	698	756	813	871	929	987	045	102	160
752	876.218	276	333	391	449	506	564	622	680	737
753	795	853	910	968	026	083	141	198	256	314
754	877.371	429	486	544	602	659	717	774	832	889
755	947	004	062	119	177	234	292	349	407	464
756	878.522	579	637	694	751	809	866	924	981	038
757	879.096	153	211	268	325	383	440	497	555	612
758	669	726	784	841	898	956	013	070	127	185
759	880.242	299	356	413	471	528	585	642	699	756
760	814	871	928	985	042	099	156	213	270	328
761	881.385	442	499	556	613	670	727	784	841	898
762	955	012	069	126	183	240	297	354	411	468
763	882.524	581	638	695	752	809	866	923	980	036
764	883.093	150	207	264	321	377	434	491	548	605
765	661	718	775	832	888	945	002	059	115	172
766	884.229	285	342	399	455	512	569	625	682	739
767	795	852	909	965	022	078	135	191	248	305
768	885.361	418	474	531	587	644	700	757	813	870
769	926	983	039	096	152	209	265	321	378	434
770	886.491	547	603	660	716	773	829	885	942	998
771	887.054	111	167	223	280	336	392	448	505	561
772	617	674	730	786	842	898	955	011	067	123
773	888.179	236	292	348	404	460	516	573	629	685
774	741	797	853	909	965	021	077	134	190	246
775	889.302	358	414	470	526	582	638	694	750	806
776	862	918	974	030	085	141	197	253	309	365
777	890.421	477	533	589	644	700	756	812	868	924
778	980	035	091	147	203	259	314	370	426	482
779	891.537	593	649	705	760	816	872	927	983	039
780	892.095	150	206	262	317	373	428	484	540	595
781	651	707	762	818	873	929	985	040	096	151
782	893.207	262	318	373	429	484	540	595	651	706
783	762	817	873	928	984	039	094	150	205	261
784	894.316	371	427	482	538	593	648	704	759	814
785	870	925	980	036	091	146	201	257	312	367
786	895.422	478	533	588	643	699	754	809	864	919
787	975	030	085	140	195	251	306	361	416	471
788	896.526	581	636	691	747	802	857	912	967	022
789	897.077	132	187	242	297	352	407	462	517	572

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790	897.627	682	737	792	847	902	957	012	067	122
791	898.176	231	286	341	396	451	506	561	615	670
792	725	780	835	890	944	999	054	109	164	218
793	899.273	328	383	437	492	547	602	656	711	766
794	820	875	930	985	039	094	149	203	258	312
795	900.367	422	476	531	586	640	695	749	804	858
796	913	968	022	077	131	186	240	295	349	404
797	901.458	513	567	622	676	731	785	840	894	948
798	902.003	057	112	166	220	275	329	384	438	492
799	547	601	655	710	764	818	873	927	981	036
800	903.090	144	198	253	307	361	416	470	524	578
801	632	687	741	795	849	903	958	012	066	120
802	904.174	228	283	337	391	445	499	553	607	661
803	715	770	824	878	932	986	040	094	148	202
804	905.256	310	364	418	472	526	580	634	688	742
805	796	850	904	958	012	065	119	173	227	281
806	906.335	389	443	497	550	604	658	712	766	820
807	873	927	981	035	089	142	196	250	304	358
808	907.411	465	519	573	626	680	734	787	841	895
809	948	002	056	109	163	217	270	324	378	431
810	908.485	539	592	646	699	753	807	860	914	967
811	909.021	074	128	181	235	288	342	395	449	502
812	556	609	663	716	770	823	877	930	984	037
813	910.090	144	197	251	304	358	411	464	518	571
814	624	678	731	784	838	891	944	998	051	104
815	911.158	211	264	317	371	424	477	530	584	637
816	690	743	797	850	903	956	009	063	116	169
817	912.222	275	328	381	435	488	541	594	647	700
818	753	806	859	913	966	019	072	125	178	231
819	913.284	337	390	443	496	549	602	655	708	761
820	814	867	920	973	026	079	131	184	237	290
821	914.343	396	449	502	555	608	660	713	766	819
822	872	925	977	030	083	136	189	241	294	347
823	915.400	453	505	558	611	664	716	769	822	874
824	927	980	033	085	138	191	243	296	349	401
825	916.454	507	559	612	664	717	770	822	875	927
826	980	033	085	138	190	243	295	348	400	453
827	917.505	558	610	663	715	768	820	873	925	978
828	918.030	083	135	188	240	292	345	397	450	502
829	554	607	660	712	764	816	869	921	973	026
830	919.078	130	183	235	287	340	392	444	496	549
831	601	653	705	758	810	862	914	967	019	071
832	920.123	175	228	280	332	384	436	849	541	593
833	645	697	749	801	853	906	958	010	062	114
834	921.166	218	270	322	374	426	478	530	582	634
835	686	738	790	842	894	946	998	050	102	154
836	922.206	258	310	362	414	466	518	570	622	674
837	725	777	829	881	933	985	037	088	140	192
838	923.244	296	348	399	451	503	555	607	658	710
839	762	814	865	917	969	021	072	124	176	288

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840	924.279	331	383	434	486	538	589	641	693	744
841	796	848	899	951	002	054	106	157	209	260
842	925.312	364	415	467	518	570	621	673	724	776
843	827	879	931	932	034	085	137	183	239	291
844	926.342	394	445	497	548	600	651	702	754	805
845	857	908	959	011	062	114	165	216	268	319
846	927.370	422	473	524	576	627	678	730	781	832
847	883	935	986	037	088	140	191	242	293	345
848	928.396	447	498	549	601	652	703	754	805	856
849	908	959	010	051	112	163	214	269	317	368
850	929.419	470	521	572	623	674	725	776	827	878
851	930	981	032	083	134	185	236	287	338	389
852	930.440	491	541	592	643	694	745	796	847	898
853	949	000	051	102	153	203	254	305	356	407
854	931.458	509	560	610	661	712	763	814	864	915
855	966	017	068	118	169	220	271	321	372	423
856	932.474	524	575	626	677	727	778	829	879	930
857	981	031	082	133	183	234	285	335	386	437
858	933.487	538	588	639	690	740	791	841	892	943
859	993	044	094	145	195	246	296	347	397	448
860	934.498	549	599	650	700	751	801	852	902	953
861	935.003	054	104	154	205	255	306	356	406	457
862	507	558	608	658	709	759	809	860	910	960
863	936.011	061	111	162	212	262	313	363	413	463
864	514	564	614	664	715	765	815	865	916	966
865	937.016	066	116	167	217	267	317	367	418	468
866	518	568	618	668	718	769	819	869	919	969
867	938.019	069	119	169	219	269	319	370	420	470
868	520	570	620	670	720	770	820	870	920	970
869	939.020	070	120	170	220	270	319	369	419	469
870	519	569	619	669	719	769	819	868	918	968
871	940.018	068	118	168	218	267	317	367	417	467
872	516	566	616	666	716	765	815	865	915	964
873	941.014	064	114	163	213	263	313	362	412	462
874	511	561	611	660	710	760	809	859	909	958
875	942.008	058	107	157	206	256	306	355	405	454
876	504	554	603	653	702	752	801	851	900	950
877	943.000	049	099	148	198	247	297	346	396	445
878	494	544	593	643	692	742	791	841	890	939
879	989	038	088	137	186	236	285	335	384	433
880	944.483	532	581	631	680	729	779	828	877	927
881	976	025	074	124	173	222	272	321	370	419
882	945.469	518	567	616	665	715	764	813	862	911
883	961	010	059	108	157	207	256	305	354	403
884	946.452	501	550	600	649	698	747	796	845	894
885	943	992	041	090	139	189	238	287	336	385
886	947.434	483	532	581	630	679	728	777	826	875
887	924	973	021	070	119	168	217	266	315	364
888	948.413	462	511	560	608	657	706	755	804	853
889	902	951	000	048	097	146	195	244	292	341

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890	949.390	439	488	536	585	634	683	731	780	829
891	878	926	975	024	073	121	170	219	267	316
892	950.365	413	462	511	560	608	657	705	754	803
893	851	900	949	997	046	095	143	192	240	289
894	951.337	386	435	483	532	580	629	677	726	774
895	283	872	920	969	017	066	114	163	211	259
896	952.308	356	405	453	502	550	599	647	696	744
897	792	841	889	938	986	034	083	131	180	228
898	953.276	325	373	421	470	518	566	615	663	711
899	760	808	856	905	953	001	049	098	146	194
900	954.242	291	339	387	435	484	532	580	628	677
901	725	773	821	869	918	966	014	062	110	158
902	955.206	255	303	351	399	447	495	543	592	640
903	688	736	784	832	880	928	976	024	072	120
904	956.168	216	264	312	361	409	457	505	553	601
905	649	697	744	792	840	888	936	984	032	080
906	957.128	176	224	272	320	368	416	464	511	559
907	607	655	703	751	799	847	894	942	990	038
908	958.086	134	181	229	277	325	373	420	468	516
909	564	612	659	707	755	803	850	898	946	994
910	959.041	084	137	184	232	280	328	375	423	471
911	518	566	614	661	709	757	804	852	900	947
912	995	042	090	138	185	233	280	328	376	423
913	960.471	518	566	613	661	709	736	804	851	899
914	946	994	041	089	136	184	231	279	326	374
915	961.421	469	516	563	611	658	706	753	801	848
916	896	943	990	038	085	132	180	227	275	322
917	962.369	417	464	511	559	606	653	701	748	795
918	843	890	937	985	032	079	126	174	221	268
919	963.315	363	410	457	504	552	599	646	693	741
920	788	835	882	929	977	024	071	118	165	212
921	964.260	307	354	401	448	495	542	590	637	684
922	731	778	825	872	919	966	013	060	108	155
923	965.202	249	296	343	390	437	484	531	578	625
924	672	719	766	813	860	907	954	001	048	095
925	966.142	189	236	283	329	376	423	470	517	564
926	611	658	705	752	798	845	892	939	986	033
927	967.080	127	173	220	267	314	361	408	454	501
928	548	595	642	688	735	782	829	875	922	969
929	968.016	062	109	156	203	249	296	343	389	436
930	483	530	576	623	670	716	763	810	856	903
931	950	996	043	090	136	183	229	276	323	369
932	969.416	462	509	556	602	649	695	742	788	835
933	882	929	975	021	068	114	161	207	254	300
934	970.347	393	440	486	533	579	626	672	719	765
935	812	858	904	951	997	044	090	137	183	229
936	971.276	322	369	415	461	508	554	600	647	693
937	740	786	832	879	925	971	018	064	110	156
938	972.203	249	295	342	388	434	480	527	573	619
939	665	712	758	804	851	897	943	989	035	082

N°	0	1	2	3	4	5	6	7	8	9
940	973.128	174	220	266	313	359	405	451	497	543
941	590	636	682	728	774	820	866	913	959	005
942	974.051	097	143	189	235	281	327	373	420	466
943	512	558	604	650	696	742	788	834	880	926
944	972	018	064	110	156	202	248	294	340	386
945	975.432	478	524	570	616	661	707	753	799	845
946	891	937	983	029	075	121	166	212	258	304
947	976.350	396	442	487	533	579	625	671	717	762
948	808	854	900	946	991	037	083	129	175	220
949	977.266	312	358	403	449	495	541	586	632	678
950	724	769	815	861	906	952	997	043	089	135
951	978.180	226	272	317	363	409	454	500	546	591
952	637	683	728	774	819	865	911	956	002	047
953	979.093	138	184	230	275	321	366	412	457	503
954	548	594	639	685	730	776	821	867	912	958
955	980.003	049	094	140	185	231	276	322	367	412
956	458	503	549	594	640	685	730	776	821	867
957	912	957	003	048	093	139	184	229	275	320
958	981.365	411	456	501	547	592	637	683	728	773
959	819	864	909	954	000	045	090	135	181	226
960	982.271	316	362	407	452	497	543	588	633	678
961	723	769	814	859	904	949	994	040	085	130
962	983.175	220	265	310	356	401	446	491	536	581
963	626	671	716	762	807	852	897	942	987	032
964	984.077	122	167	212	257	302	347	392	437	482
965	527	572	617	662	707	752	797	842	887	932
966	977	022	067	112	157	202	247	292	337	382
967	985.426	471	516	561	606	651	696	741	786	830
968	875	920	965	010	055	100	144	189	234	279
969	986.324	369	413	458	503	548	593	637	682	727
970	772	816	861	906	951	995	040	085	130	174
971	987.219	264	309	353	398	443	487	532	577	622
972	666	711	756	800	845	890	934	979	024	068
973	988.113	157	202	247	291	336	381	425	470	514
974	559	603	648	693	737	782	826	871	915	960
975	989.005	049	094	138	183	227	272	316	361	405
976	450	494	539	583	628	672	717	761	806	850
977	895	939	983	028	072	117	161	206	250	294
978	990.339	383	428	472	516	561	605	650	694	738
979	783	827	871	916	960	004	049	093	137	182
980	991.226	270	315	359	403	448	492	536	580	625
981	669	713	757	802	846	890	934	979	023	067
982	992.111	156	200	244	288	333	377	421	465	509
983	553	598	642	686	730	774	818	863	907	951
984	995	039	083	127	172	216	260	304	348	392
985	993.436	480	524	568	612	657	701	745	789	833
986	877	921	965	009	053	097	141	185	229	273
987	994.317	361	405	449	493	537	581	625	669	713
988	757	801	845	889	933	967	021	064	108	152
989	995.196	240	284	328	372	416	460	504	547	591

N°.	0	1	2	3	4	5	6	7	8	9
990	995.635	679	723	767	811	854	898	942	986	030
991	996.074	117	161	205	249	293	336	380	424	468
992	512	555	599	643	687	730	774	818	862	905
993	949	993	037	080	124	168	212	255	299	343
994	997.386	430	474	517	561	605	648	692	736	779
995	823	867	910	954	998	041	085	128	172	216
996	998.259	303	346	390	434	477	521	564	608	652
997	695	739	782	826	869	913	956	000	043	087
998	999.130	174	218	261	305	348	392	435	478	522
999	565	609	652	696	739	783	826	870	913	957

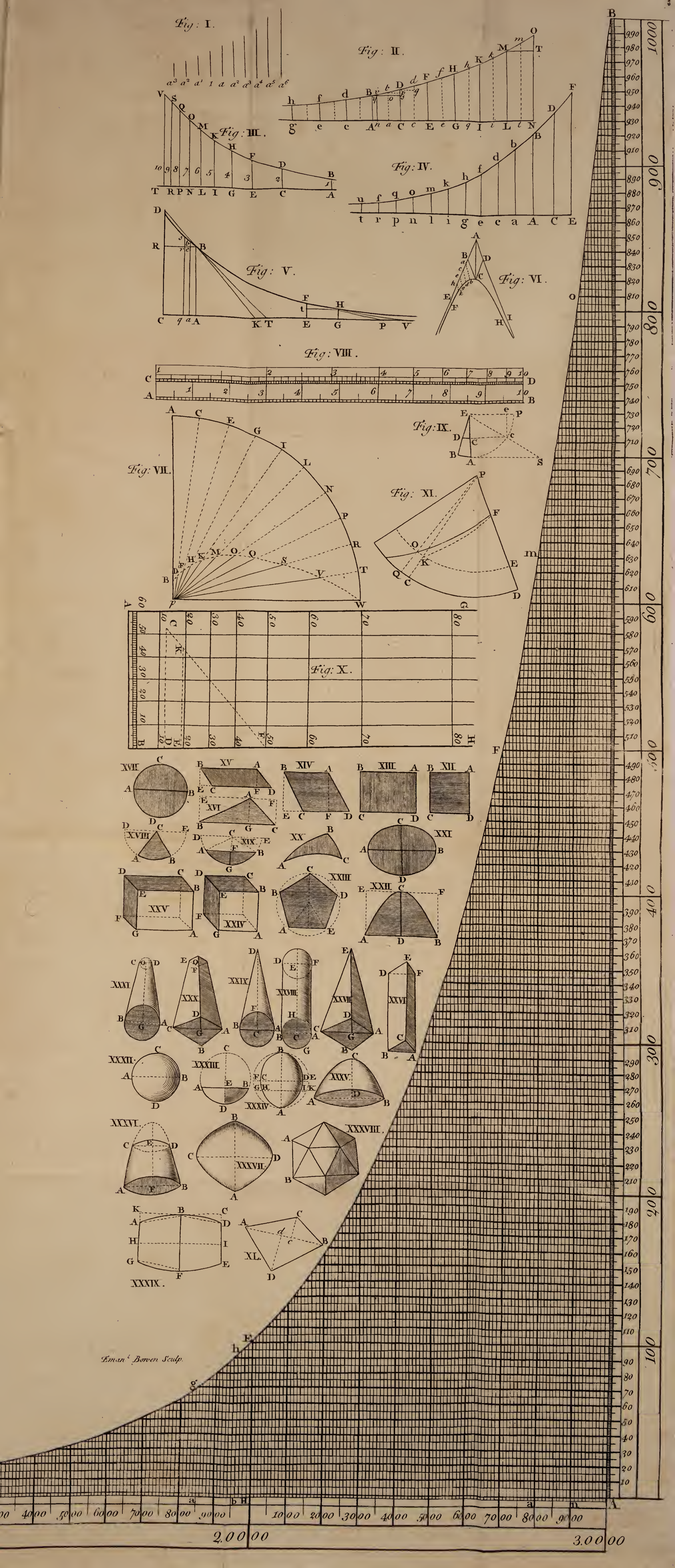
Fig: I.
Fig: II.
Fig: III.
Fig: IV.
Fig: V.
Fig: VI.
Fig: VII.
Fig: VIII.
Fig: IX.
Fig: X.
Fig: XI.
Eman. Bowen Sculp.
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A

T A B L E

O F

Artificial or Logarithmic

SINES and TANGENTS,

To every Degree and Minute of the Quadrant.

A TABLE OF *Artificial or Logarithmic* SINES, *To every Degree and Minute of the Quadrant.*

A Table of Logarithmic Sines.

Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min.
0	0	8.241855	8.542819	8.718800	8.843584	0
1	6.463726	49033	46422	21204	5387	1
2	6.764756	56094	49995	23595	7183	2
3	6.940847	63042	53539	25972	8971	3
4	7.065786	69881	57054	28337	8.850751	4
5	7.162696	76614	60540	8.730688	2524	5
6	7.241877	83243	63999	3027	4290	6
7	7.308824	89773	67431	5353	6049	7
8	7.366816	96207	70836	7667	7801	8
9	7.417963	8.302546	74214	9969	9546	9
10	7.463725	08794	77566	8.742259	8.861283	10
11	7.505118	14954	80892	4536	3014	11
12	7.542906	21027	84193	6801	4738	12
13	7.577668	27016	87469	9055	6454	13
14	7.609853	32924	90721	8.751297	8165	14
15	7.639816	38753	93948	3528	9868	15
16	7.667844	44504	97152	5747	8.871565	16
17	7.694173	50180	8.600332	7955	3255	17
18	7.718997	55783	03489	8.760151	4938	18
19	7.742477	61315	06623	2337	6615	19
20	7.764754	66777	09734	4511	8285	20

A

T A B L E

OF

Artificial or Logarithmic

TANGENTS,

To every Degree and Minute of the Quadrant.

A Table of Logarithmic Tangents.

Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min
0	0	8.241921	8.543083	8.719396	8.844644	0
1	6.463726	49101	46691	21806	6455	1
2	6.764756	56165	50268	24203	8260	2
3	6.940847	63115	53817	26588	8.850057	3
4	7.065786	69956	57336	28959	1846	4
5	7.162696	76691	60828	8.731317	3628	5
6	7.241878	83323	64291	3665	5403	6
7	7.308825	89856	67727	5996	7171	7
8	7.366817	96292	71137	8317	8932	8
9	7.417970	8.302633	74520	8.740626	8.860686	9
10	7.463727	08884	77877	2922	2433	10
11	7.505120	15046	81208	5207	4172	11
12	7.542909	21122	84514	7479	5905	12
13	7.577671	27114	87794	9740	7632	13
14	7.609857	33025	91051	8.751989	9351	14
15	7.639820	38856	94283	4227	8.871064	15
16	7.667849	44610	97492	6453	2770	16
17	7.694179	50289	8.600677	8668	4469	17
18	7.719003	55895	03839	8.760872	6162	18
19	7.742484	61430	06978	3055	7849	19
20	7.764761	66894	10094	5246	9529	20

Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min.
21	7.785943	8.372171	8.612823	8.766675	8.879949	21
22	7.806146	8. 77499	8. 5891	8. 8827	8.881607	22
23	7. 25451	8. 82762	8. 8937	8.770970	8. 3258	23
24	7. 43934	8. 87962	8.621962	8. 3101	8. 4903	24
25	7. 61662	8. 93101	8. 4965	8. 5223	8. 6542	25
26	7. 78695	8. 98179	8. 7948	8. 7333	8. 8174	26
27	7. 95085	8.403199	8.630911	8. 9434	8. 9801	27
28	7.910880	8. 08161	8. 3854	8.781524	8.891421	28
29	7. 26119	8. 13068	8. 6776	8. 3605	8. 3035	29
30	7. 40842	8. 17919	8. 9680	8. 5675	8. 4643	30
31	7. 55082	8. 22717	8.642563	8. 7736	8. 6245	31
32	7. 68870	8. 27462	8. 5428	8. 9787	8. 7842	32
33	7. 82233	8. 32156	8. 8274	8.791828	8. 9432	33
34	7. 95198	8. 36800	8.651102	8. 3859	8.901017	34
35	8.007787	8. 41394	8. 3911	8. 5881	8. 2595	35
36	8. 20021	8. 45941	8. 6702	8. 7894	8. 4168	36
37	8. 31919	8. 50440	8. 9475	8. 9897	8. 5736	37
38	8. 43501	8. 54893	8.662230	8.801891	8. 7297	38
39	8. 54781	8. 59301	8. 4968	8. 3876	8. 8853	39
40	8. 65776	8. 63665	8. 7689	8. 5852	8.910404	40
41	8. 76500	8. 67985	8.670393	8. 7819	8. 1949	41
42	8. 86965	8. 72263	8. 3080	8. 9777	8. 3488	42
43	8. 97183	8. 76498	8. 5751	8.811726	8. 5022	43
44	8.107167	8. 80693	8. 8405	8. 3667	8. 6550	44
55	8. 16926	8. 84848	8.681043	8. 5598	8. 8073	45
46	8. 26471	8. 88963	8. 3665	8. 7522	8. 9591	46
47	8. 35810	8. 93040	8. 6272	8. 9436	8.921103	47
48	8. 44953	8. 97078	8. 8862	8.821342	8. 2610	48
49	8. 53907	8.501080	8.691438	8. 3240	8. 4112	49
50	8. 62681	8. 05045	8. 3998	8. 5130	8. 5609	50
51	8. 71280	8. 08974	8. 6543	8. 7011	8. 7100	51
52	8. 79713	8. 12867	8. 9073	8. 8884	8. 8587	52
53	8. 87985	8. 16726	8.701589	8.830749	8.930068	53
54	8. 96102	8. 20551	8. 4090	8. 2607	8. 1544	54
55	8.204070	8. 24343	8. 6577	8. 4456	8. 3015	55
56	8. 11895	8. 28102	8. 9049	8. 6297	8. 4481	56
57	8. 19581	8. 31828	8.711507	8. 8130	8. 5942	57
58	8. 27133	8. 35523	8. 3952	8. 9956	8. 7398	58
59	8. 34557	8. 39186	8 6383	8.841714	8. 8850	59
60	8. 41855	8. 42819	8. 8800	8. 3584	8.940296	60

Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min.
21	7.785951	8.372291	8.613189	8.767417	8.881202	21
22	7.806154	8. 77622	8. 6261	8. 9578	8. 2869	22
23	7. 25460	8. 82889	8. 9313	8.771727	8. 4530	23
24	7. 43944	8. 88092	8.622343	8. 3866	8. 6185	24
25	7. 61674	8. 93234	8. 5352	8. 5995	8. 7833	25
26	7. 78708	8. 98215	8. 8340	8. 8114	8. 9476	26
27	7. 95099	8.403338	8.631308	8.780222	8.891112	27
28	7.910894	8. 08304	8. 4256	8. 2320	8. 2742	28
29	7. 26134	8. 13213	8. 7185	8. 4408	8. 4366	29
30	7. 40858	8. 18068	8.640093	8. 6486	8. 5984	30
31	7. 55100	8. 22869	8. 2982	8. 8554	8. 7596	31
32	7. 68889	8. 27618	8. 5853	8.790613	8. 9203	32
33	7. 82253	8. 32315	8. 8704	8. 2662	8.900803	33
34	7. 95219	8. 36962	8.651537	8. 4701	8. 2398	34
35	8.007809	8. 41560	8. 4352	8. 6731	8. 3987	35
36	8. 20044	8. 46110	8. 7149	8. 8752	8. 5570	36
37	8. 31945	8. 50613	8. 9928	8.800763	8. 7147	37
38	8. 43527	8. 55070	8.662689	8. 2765	8. 8719	38
39	8. 54809	8. 59481	8. 5433	8. 4758	8.910285	39
40	8. 65806	8. 63849	8. 8160	8. 6742	8. 1846	40
41	8. 76531	8. 68172	8.670870	8. 8717	8. 3401	41
42	8. 86997	8. 72453	8. 3563	8. 0683	8. 4951	42
43	8. 97217	8. 76693	8. 6239	8. 2641	8. 6495	43
44	8.107202	8. 80892	8. 8900	8. 4589	8. 8034	44
45	8. 16963	8. 85050	8.681544	8. 6529	8. 9567	45
46	8. 26510	8. 89170	8. 4172	8. 8461	8.921096	46
47	8. 35851	8. 93250	8. 6784	8.820384	8. 2619	47
48	8. 44995	8. 97293	8. 9381	8. 2298	8. 4136	48
49	8. 53952	8.501298	8.691963	8. 4205	8. 5649	49
50	8. 62727	8. 05267	8. 4529	8. 6103	8. 7156	50
51	8. 71328	8. 09200	8. 7081	8. 7992	8. 8658	51
52	8. 79762	8. 13098	8. 9617	8. 9874	8.930155	52
53	8. 88036	8. 16961	8.702139	8.831748	8. 1647	53
54	8. 96156	8. 20790	8. 4646	8. 3613	8. 3134	54
55	8.204126	8. 24585	8. 7139	8. 5471	8. 4616	55
56	8. 11953	8. 28349	8. 9618	8. 7321	8. 6093	56
57	8. 19641	8. 32080	8.712083	8. 9163	8. 7565	57
58	8. 27195	8. 35779	8. 4534	8.840998	8. 9032	58
59	8. 34621	8. 39447	8. 6972	8. 2824	8.940494	59
60	8. 41921	8. 43084	8. 9396	8. 4644	8. 1952	60

D	'	0	1	2	3	4	5	6	7	8	9
5.	0	8.94.0296	1738	3174	4605	6034	7436	8874	0287	1696	3099
	10	95 4499	5894	7284	8670	0052	1429	2801	4170	5534	6893
	20	96 8249	9600	0947	2289	3628	4962	6293	7619	8941	0259
	30	98.1573	2883	4189	5491	6789	8083	9374	0660	1943	3222
	40	99.4497	5768	7036	8299	9560	0816	2069	3318	4563	5805
	50	9.00.7040	8278	9510	0737	1926	3182	4400	5613	6824	8031
6.	0	01 9235	0435	1632	2825	4016	5203	6386	7567	8744	9918
	10	03.1039	2256	3421	4582	5741	6895	8048	9197	0342	1485
	20	04.2615	3762	4895	6026	7154	8279	9400	0519	1635	2748
	30	05.3859	4966	6071	7172	8271	9367	0460	1551	2639	3723
	40	06.4806	5885	6962	8036	9107	0176	1242	2305	3366	4424
	50	07.5480	6533	7583	8631	9676	0719	1759	2797	3832	4864
7.	0	08 5894	6922	7947	8970	9990	1008	2024	3037	4047	5056
	10	09.6061	7065	8066	9065	0061	1056	2047	3037	4025	5010
	20	10.5992	6973	7951	8921	9901	0873	1842	2809	3774	4737
	30	11.5698	6656	7612	8566	9519	0469	1417	2362	3306	4248
	40	12.5187	6125	7060	7993	8925	9854	0781	1706	2630	3551
	50	13 4470	5387	6303	7216	8127	9037	9944	0850	1754	2655
8.	0	14.3555	4453	5350	6243	7136	8026	8915	9801	0686	1569
	10	15.2451	3330	4208	5083	5957	6830	7700	8569	9435	0300
	20	16.1664	2025	2885	3743	4600	5454	6307	7159	8008	8855
	30	16.9702	0546	1389	2230	3070	3908	4744	5578	6418	7242
	40	17.8072	8900	9726	0551	1374	2196	3016	3834	4651	5466
	50	18.6280	7092	7903	8712	9519	0325	1130	1933	2734	3534
9.	0	19 4332	5129	5925	6719	7511	8302	9091	9879	0666	1451
	10	20 2234	3017	3797	4577	5354	6131	6906	7679	8452	9222
	20	9992	0759	1526	2291	3055	3818	4579	5338	6097	6854
	30	21.7609	8363	9116	9868	0618	1367	2115	2861	3606	4349
	40	22.5092	5833	6572	7311	8448	8784	9518	0252	0984	1714
	50	23 2444	3172	3899	4625	5349	6073	6795	7515	8235	8953
10.	0	9670	0386	1101	1814	2526	3237	3947	4656	5363	6069
	10	24.6775	7478	8181	8883	9583	0282	0980	1677	2375	3067
	20	25.3761	4453	5144	5834	6523	7211	7898	8583	9268	9951
	30	26.0633	1314	1994	2673	3351	4027	4703	5377	6051	6723
	40	7394	8065	8734	9402	0069	0735	1399	2063	2726	3388
	50	27.4094	4708	5367	6024	6681	7337	7991	8644	9297	9948
11.	0	28.0599	1248	1897	2544	3190	3836	4480	5124	5766	6408
	10	7048	7687	8326	8964	9600	0235	0870	1504	2137	2768
	20	29.3399	4029	4658	5286	5913	6539	7164	7788	8411	9034
	30	9655	0276	0895	1514	2132	2748	3364	3979	4593	5207
	40	30.5819	6430	7041	7650	8259	8867	9474	0080	0685	1289
	50	31.1893	24[illegible]5	3097	3698	4297	48[illegible]6	5495	6092	6658	7284
12.	0	7879	8473	9066	9658	0249	0840	1430	2019	2607	3194
	10	32.3780	4356	4950	5534	6117	6700	7281	7862	8442	9021
	20	9599	0176	0753	1328	1903	2478	3051	3624	4195	4766
	30	33.5337	5906	6475	7043	7610	8176	8742	9306	9871	0434
	40	34.0996	1558	2119	2679	3239	3797	4355	4912	5469	6024
	50	6579	7134	7687	8240	8792	9343	9893	0443	0992	1540

D '	0	1	2	3	4	5	6	7	8	9
5. 0	8.94.1952	3404	4852	6295	7734	9168	0597	2021	3441	4856
10	95 6267	7673	9075	0473	1866	3254	4639	6019	7394	8766
20	97.0133	1496	2855	4209	5560	6906	8248	9586	0920	2251
30	98.3577	4893	6217	7532	8842	0149	1451	2750	4045	5337
40	99.6624	7909	9188	0465	1737	3007	4272	5524	6792	8047
50	9.00.9298	0546	1790	3031	4268	5502	6732	7959	9183	0403
6. 0	02.1620	2834	4044	5251	6455	7655	8852	0046	1237	2425
10	03.3606	4791	5969	7144	8316	9485	0651	1813	2973	4130
20	04.5284	6434	7582	8727	9869	1008	2144	3277	4407	5535
30	05.6659	7781	8900	0016	1130	2240	3348	4453	5556	6655
40	06 7752	8846	9938	1027	2113	3197	4278	5356	6432	7505
50	07.8576	9644	0710	1773	2833	3891	4947	6000	7050	8098
7. 0	08.9144	0187	1228	2266	3302	4335	5367	6395	7422	8446
10	09.9468	0487	1504	2519	3532	4542	5550	6556	7559	8560
20	10.9559	0556	1551	2543	3533	4521	5507	6491	7472	8452
30	11 9429	0404	1377	2348	3317	4284	5249	6211	7172	8130
40	12 9086	0041	0993	1944	2893	3839	4783	5726	6666	76 5
50	13.8542	9476	0409	1340	2269	3196	4121	5044	5965	6885
8. 0	14.7802	8718	9632	0544	1454	2363	3269	4171	5077	5978
10	15.6877	7775	8671	9565	0457	1347	2236	3123	4008	4892
20	16 5774	6654	7532	8409	9284	0157	1029	1899	2767	3634
30	17.4499	5362	6224	7084	7942	8799	9655	0508	1360	2211
40	18.3059	3907	4752	5596	6439	7280	8119	8957	9794	0629
50	19.1462	2294	3124	3953	4780	5606	6430	7253	8074	8891
9. 0	9712	0529	1345	2159	2971	3782	4592	5400	6207	7012
10	20.7816	8619	9420	0220	1018	1815	2611	3405	4198	4989
20	21.5779	6568	7356	8141	8926	9710	0492	1272	2052	2830
30	22.3606	4382	5156	5929	6700	7471	8239	9007	9773	0·39
40	23.1302	2065	2826	3586	4345	5103	5859	6614	7368	8120
50	8871	9622	0371	1118	1865	2610	3354	4097	4839	5579
10. 0	24 6319	7057	7794	8530	9264	9998	0730	1461	2191	2920
10	25.3648	4374	5100	5824	6547	7269	7990	8710	9428	0146
20	26.0862	1578	2292	3005	3717	4428	5138	5847	6555	7261
30	7967	8671	9375	0077	0779	1479	2178	2876	3573	4269
40	27.4964	5658	6351	7043	7734	8424	9113	9801	0489	1174
50	28.1858	2542	3225	3907	4588	5268	5947	6624	7301	7977
11. 0	8652	9326	9999	0671	1342	2013	2682	3350	4017	4684
10	29.5349	6013	6677	7339	8001	8662	9322	9980	0638	1295
20	30.1951	2607	3261	3914	4567	5218	5869	6519	7167	7815
30	8463	9109	9754	0398	1042	1685	2327	2967	3608	4247
40	31.4885	5523	6159	6795	7430	8064	8697	9329	9961	0592
50	32.1222	1851	2479	3106	3733	4358	4983	5607	6230	6853
12. 0	7474	8095	8715	9334	9953	0570	1187	1803	2418	3033
10	33.3646	4259	4871	5482	6093	6702	7311	7919	8527	9133
20	9739	0344	0948	1552	2155	2757	3358	3958	4558	5157
30	34.5755	6353	6949	7545	8141	8735	9329	9922	0514	1106
40	35.1697	2287	2876	3465	4053	4640	5227	5813	6398	6982
50	7566	8149	8731	9313	9893	0474	1053	1632	2210	2787

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13. 0	35.2088	2635	3181	3726	4271	4815	5358	5901	6443	6984
10	7524	8064	8603	9141	9678	0215	0751	1287	1822	2356
20	36.2889	3422	3954	4485	5016	5546	6075	6604	7131	7659
30	8185	8711	9235	9761	0285	0808	1330	1852	2373	2894
40	37.3414	3933	4452	4970	5487	6003	6519	7035	7549	8063
50	8576	9089	9601	0113	0624	1134	1643	2152	2660	3168
14. 0	38.3675	4181	4687	5192	5697	6201	6704	7207	7709	8210
10	8711	9211	9711	0210	0708	1206	1703	2199	2695	3190
20	39.3685	4179	4673	5166	5658	6150	6641	7131	7621	8111
30	8600	9088	9575	0062	0549	1035	1520	2005	2489	2972
40	40.3455	3938	4420	4901	5381	5862	6341	6820	7299	7777
50	8254	8731	9207	9682	0157	0632	1106	1579	2052	2524
15. 0	41.2996	3467	3938	4408	4878	5345	5815	6283	6751	7217
10	7684	8149	8615	9079	9514	0007	0470	0933	1395	1857
20	42.2318	2778	3238	3697	4156	4615	5073	5530	5987	6443
30	6809	7354	7809	8263	8717	9170	9623	0075	0526	0978
40	43.1429	1879	2328	2778	3226	3675	4122	4569	5016	5462
50	5908	6353	6798	7242	7686	8129	8572	9014	9456	9897
16. 0	44.0330	0778	1218	1658	2095	2535	2973	3410	3847	4284
10	4720	5155	5590	6025	6459	6893	7326	7759	8191	3623
20	9054	9485	9915	0345	0775	1204	1632	2060	2488	2915
30	45.3342	3768	4194	4619	5044	5469	5893	6316	6739	7162
40	7584	8066	8427	8848	9268	9688	0108	0526	0945	1364
50	46.1782	2199	2616	3031	3448	3864	4279	4694	5108	5522
17. 0	5935	6348	6761	7173	7585	7997	8407	8817	9227	9637
10	47.0046	0455	0863	1271	1678	2086	2492	2898	3304	3710
20	4115	4519	4923	5327	5730	6133	6536	6938	7340	7741
60	8142	8542	8942	9342	9741	0140	0538	0937	1334	1731
40	48 2128	2525	2921	3316	3712	4107	4501	4895	5289	5682
50	6075	6467	6859	7251	7643	8033	8424	8814	9204	9593
18. 0	9982	0371	0759	1147	1534	1922	2308	2695	3081	3466
10	49.3851	4236	4620	5005	5388	5772	6154	6537	6919	7301
20	7682	8063	8444	8324	9204	9584	9963	0342	0721	1099
30	50.1476	1854	2231	2607	2984	3340	3735	4110	4485	4860
40	5234	5608	5981	6354	6727	7099	7471	7843	8214	8585
50	8956	9326	9696	0065	0434	0803	1172	1540	1907	2275
19. 0	51.2642	3009	3375	3741	4107	4472	4837	5202	5566	5930
10	6294	6657	7020	7382	7745	8107	8468	8829	9190	9551
20	9911	0271	0631	0990	1349	1707	2066	2423	2781	3138
30	52.3495	3852	4208	4564	4920	5275	5630	5984	6339	6693
40	7046	7409	7753	8105	8458	8810	9161	9513	9864	0215
50	53 0565	0915	1265	1614	1963	2312	2661	3009	3357	3704
20 0	4052	4399	4745	5091	5437	5783	6129	6474	6818	7163
10	7507	7851	8194	8537	8880	9223	9565	9907	0249	0590
20	54.0931	1272	1613	1953	2293	2632	2971	3310	3649	3987
30	4325	4663	5001	5338	5674	6011	6347	6683	7019	7354
40	7689	8024	8358	8693	9026	9360	9693	0026	0359	0691
50	55.1024	1356	1687	2018	2349	2680	3010	3341	3670	4000

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13.0	36.3364	3940	4515	5090	5664	6237	6810	7382	7953	8524
10	9094	9663	0231	0799	1367	1933	2499	3964	3629	4193
20	37.4756	5319	5881	6442	7003	7563	8122	8681	9229	9797
30	38.0354	0910	1465	2020	2575	3128	3682	4234	4786	5337
40	5888	6438	6987	7536	8084	8631	9178	9724	0270	0815
50	39.1359	1903	2447	2989	3531	4073	4614	5654	5694	6233
14.0	6771	7309	7846	8383	8919	9455	9990	0524	1058	1591
10	40.2124	2656	3187	3718	4249	4778	5308	5836	6364	6892
20	7419	7945	8471	8996	9521	0045	0569	1092	1615	2137
30	41.2658	3179	3699	4219	4738	5257	5775	6293	6810	7326
40	7842	8358	8873	9378	9901	0415	0927	1440	1951	2463
50	42.2973	3484	3993	4503	5011	5519	6027	6534	7041	7547
15.0	8052	8557	9062	9566	0070	0573	1075	1577	2079	2580
10	43.3080	3580	4080	4579	5078	5576	6073	6570	7067	7563
20	8059	8554	9048	9513	0036	0529	1022	1514	2006	2497
30	44.2988	3479	3968	4458	4947	5435	5923	6411	6898	7384
40	7870	8356	8841	9326	9810	0294	0777	1260	1743	2225
50	45.2706	3187	3668	4148	4628	5107	5586	6064	6542	7019
16.0	7496	7973	8449	8925	9400	9875	0349	0823	1297	1770
10	46.2242	2714	3186	3658	4128	4599	5069	5539	6008	6476
20	6945	7413	788	8347	8814	9280	9746	0211	0676	1141
30	47.1605	2068	2532	2995	3457	3919	4381	4842	5303	5763
40	6223	6683	7142	7601	8059	8517	8975	9432	9889	0345
50	48.0801	1257	1712	2167	2621	3075	3529	3982	4435	4887
17.0	5339	5791	6242	6693	7143	7593	8043	8492	8941	9390
10	9838	0286	0733	1180	1627	2073	2519	2965	3410	3854
20	49.4299	4743	5186	5630	6073	6515	6957	7399	7841	8282
30	8722	9162	9603	0042	0481	0920	1359	1797	2235	2672
40	50.3109	3546	3982	4418	4854	5289	5724	6159	6593	7027
50	7460	7893	8326	8759	9191	9622	0054	0485	0916	1346
18.0	51.1776	2206	2635	3064	3493	3921	4349	4777	5204	5631
10	6057	6484	6910	7335	7761	8185	8610	9034	9458	9882
20	52.0305	0728	1151	1573	1995	2417	2838	3259	3679	4100
30	4520	4939	5359	5778	6197	6615	7033	7451	7868	8285
40	8702	9119	9535	9950	0366	0781	1196	1611	2025	2439
50	53.2853	3266	3679	4092	4504	4916	5328	5739	6150	6561
19.0	6972	7382	7792	8202	8611	9020	9429	9837	0245	0653
10	54.1061	1468	1875	2281	2689	3094	3500	3905	4310	4715
20	5119	5524	5927	6331	6735	7138	7540	7943	8345	8747
30	9149	9550	9951	0352	0752	1152	1552	1952	2351	2750
40	55.3149	3548	3946	4344	4741	5139	5536	5933	6329	6725
50	7121	7517	7912	8308	8702	9097	9491	9885	0279	0673
20.0	56.1066	1459	1851	2244	2636	3028	3419	3811	4202	4592
10	4983	5373	5763	6153	6542	6932	7320	7709	8097	8486
20	8873	9261	9648	0035	0422	0809	1195	1581	1967	2353
30	57.2738	3124	3507	3892	4276	4660	5044	5427	5810	6193
40	6576	6958	7341	7723	8104	8486	8867	9248	9629	0009
50	58.0389	0769	1149	1528	1907	2286	2665	3043	3422	3800

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21.0	4329	4658	4987	5315	5643	5971	6299	6626	6953	7280
10	7606	7932	8258	8583	8909	9234	9558	9883	0207	0531
20	56.0854	1178	1501	1824	2146	2468	2790	3112	3433	3755
30	4075	4396	4716	5036	5356	5676	5995	6314	6632	6951
40	7269	7587	7904	8222	8539	8855	9172	9488	9804	0120
50	57.0435	0751	1066	1380	1695	2009	2323	2636	2949	3263
22.0	3575	3888	4200	4512	4824	5136	5447	5758	6068	6379
10	6689	6999	7309	7618	7927	8236	8545	8853	9162	9469
20	9777	0084	0391	0699	1005	1312	1618	1924	2229	2534
30	58.2840	3144	3449	3753	4058	4361	4665	4968	5272	5574
40	5877	6179	6482	6783	7085	7386	7688	7988	8289	8590
50	8890	9190	9489	9789	0088	0387	0686	0984	1282	1580
23.0	59.1878	2175	2473	2770	3067	3363	3659	3955	4251	4547
10	4842	5137	5432	5727	6021	6315	6609	6903	7196	7490
20	7783	8075	8368	8660	8952	9244	9536	9827	0118	0409
30	60.0700	0990	1280	1570	1860	2150	2439	2728	3017	3305
40	3594	3881	4170	4457	4755	5032	5319	5606	5892	6178
50	6465	6751	7036	7321	7607	7892	8176	8461	8745	9029
24.0	9313	9597	9880	0163	0446	0729	1012	1294	1576	1858
10	61.2140	2421	2702	2983	3264	3545	3825	4105	4385	4665
20	4944	5223	5502	5781	6060	6338	6616	6894	7172	7450
30	7727	8004	8281	8558	8834	9110	9386	9662	9937	0213
40	62.0488	0763	1038	1313	1587	1861	2135	2409	2682	2956
50	3229	3502	3774	4047	4319	4591	4863	5135	5406	5677
25.0	5948	6219	6490	6760	7030	7300	7570	7840	8109	8378
10	8647	8916	9184	9453	9721	9989	0257	0524	0792	1059
20	63.1326	1593	1859	2125	2391	2658	2923	3189	3454	3719
30	3984	4249	4514	4778	5042	5306	5570	5833	6097	6360
40	6623	6886	7148	7411	7673	7935	8197	8458	8720	8981
50	9242	9503	9764	0024	0284	0544	0804	1064	1323	1583
26.0	64.1842	2101	2360	2618	2876	3135	3393	3650	3908	4165
10	4423	4679	4936	5193	5450	5706	5962	6218	6473	6729
20	6984	7239	7494	7749	8004	8258	8512	8766	9020	9274
30	9527	9781	0034	0287	0539	0792	1044	1297	1549	1800
40	65.2052	2303	2555	2806	3057	3307	3558	3808	4058	4308
50	4558	4808	5057	5307	5556	5805	6054	6302	6550	6799
27.0	7047	7295	7542	7790	8037	8284	8531	8778	9025	9271
10	9517	9763	0009	0255	0500	0746	0991	1236	1481	1726
20	66.1970	2214	2459	2703	2946	3190	3433	3677	3920	4163
30	4406	4648	4891	5133	5375	5617	5859	6100	6341	6583
40	6824	7065	7305	7546	7786	8026	8266	8506	8746	8986
50	9225	9464	9703	9942	0181	0419	0658	0896	1131	1372
28.0	67.1609	1847	2084	2321	2558	2795	3032	3268	3505	3741
10	3977	4213	4448	4684	4919	5155	5389	5624	5859	6094
20	6328	6562	6796	7030	7264	7497	7731	7964	8197	8430
30	8663	8895	9278	9360	9592	9824	0056	0287	0519	0750
40	68.0982	1213	1443	1674	1905	2135	2365	2595	2852	3055
50	2284	2514	3743	3072	4201	4430	4658	4887	5115	5343

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21.	0	58.4177	4555	4932	5309	5686	6062	6439	6815	7190	7566
	10	7941	8316	8691	9066	9440	9814	0188	0562	0935	1308
	20	59.1681	2054	2426	2798	3170	3542	3914	4285	4656	5027
	30	5397	5768	6138	6508	6878	7247	7616	7985	8354	8722
	40	9090	9459	9827	0194	0562	0929	1296	1662	2029	2395
	50	60.2761	3127	3493	3858	4223	4588	4953	5317	5682	6046
22.	0	6410	6773	7137	7500	7893	8225	8588	8950	9312	9674
	10	61.0036	0397	0759	1120	1480	1841	2201	2561	2921	3281
	20	3641	4000	4359	4718	5077	5435	5793	6151	6509	6867
	30	7224	7581	7938	8295	8652	9008	9364	9720	0076	0432
	40	62.0787	1142	1497	1852	2207	2561	2915	3269	3623	3976
	50	4330	4683	5036	5388	5701	6093	6445	6797	7149	7501
23.	0	7852	8203	8554	8905	9255	9606	9956	0306	0655	1005
	10	63.1354	1704	2053	2401	2750	3098	3447	3795	4143	4490
	20	4838	5185	5532	5879	6226	6572	6918	7265	7611	7956
	30	8302	8647	8992	9337	9682	0027	0371	0716	1060	1404
	40	64.1747	2091	2434	2777	3120	3463	3806	4148	4490	4832
	50	5174	5516	5857	6199	6540	6881	7222	7562	7903	8243
24.	0	8583	8923	9263	9602	9942	0281	0620	0959	1297	1636
	10	65.1974	2312	2650	2988	3326	3663	4000	4337	4674	5011
	20	5348	5684	6020	6356	6692	7028	7363	7699	8034	8369
	30	8704	9039	9373	9707	0042	0376	0710	1043	1377	1710
	40	66.2043	2376	2709	3042	3374	3707	4039	4371	4703	5035
	50	5366	5697	6029	6360	6691	7021	7352	7682	8013	8343
25.	0	8672	9002	9332	9661	9991	0320	0649	0977	1306	1634
	10	67.1963	2291	2619	2947	3274	3602	3929	4257	4584	4910
	20	5237	5564	5890	6216	6543	6869	7194	7520	7846	8171
	30	8496	8821	9146	9471	9795	0120	0444	0768	1092	1416
	40	68.1740	2063	2386	2710	3033	3356	3678	4001	4323	4646
	50	4968	5290	5612	5934	6255	6577	6898	7219	7540	7861
26.	0	8182	8502	88[illegible]3	9143	9463	9783	0103	0423	0742	1062
	10	69.1381	1700	2019	2338	2656	2975	3293	3612	3930	4248
	20	4566	4883	5201	5518	5835	6153	6470	6786	7103	7420
	30	7736	8053	8369	8685	9001	9316	9632	9947	0263	0578
	40	70.0893	1208	1523	1837	2152	2466	2780	3095	3409	3722
	50	4036	4349	4663	4975	5290	5603	5916	6228	6541	6853
27.	0	7166	7478	7790	8102	8414	8726	9037	9349	9660	9971
	10	71.0282	0593	0904	1215	1525	1836	2146	2456	2766	3076
	20	3386	3596	4005	4314	4624	4933	5242	5551	5859	6168
	30	6477	6785	7093	7401	7709	8017	8325	8633	8940	9248
	40	9555	9862	0169	0476	0782	1089	1396	1702	2008	2315
	50	72.2621	2927	3232	3538	3844	4149	4454	4759	5065	5369
28.	0	5674	5979	6284	6588	6892	7197	7501	7805	8109	8412
	10	8716	9020	9323	9626	9929	0232	0535	0838	1141	1444
	20	73.1746	2048	2351	2653	2955	3257	3558	3861	4162	4463
	30	4764	5065	5367	5658	5958	6269	6570	6871	7171	7471
	40	7771	8070	8371	8671	8971	9271	9570	9870	0169	0468
	50	74.0767	1066	1365	1664	1962	2261	2559	2858	3156	3454

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29.0	68.5571	5799	6027	6254	6482	6709	6936	7163	7389	7616
10	7842	8069	8295	8521	8747	8972	9198	9423	9648	9873
20	69.0098	0323	0548	0772	0996	1220	1444	1668	1892	2115
30	2339	2562	2785	3008	3231	3453	3676	3898	4120	4342
40	4564	4786	5007	5229	5450	5671	5892	6113	6334	6554
50	6774	6995	7215	7435	7654	7874	8094	8313	8532	8751
30.0	8970	9189	9407	9626	9844	0062	0280	0498	0716	0933
10	70.1151	1368	1585	1802	2019	2236	2452	2669	2885	3101
20	3317	3533	3749	3964	4179	4395	4610	4825	5040	5254
30	5469	5683	5897	6112	6326	6539	6753	6967	7180	7393
40	7606	7819	8032	8245	8457	8670	8882	9094	9306	9518
50	9730	9941	0153	0364	0575	0786	0997	1208	1419	1629
31.0	71.1839	2049	2260	2469	2679	2889	3098	3308	3517	3726
10	3935	4144	4352	4561	4769	4978	5186	5394	5601	5809
20	6017	6224	6432	6639	6846	7053	7259	7466	7672	7879
30	8085	8291	8497	8703	8908	9114	9320	9525	9730	9935
40	72.0140	0345	0549	0754	0958	1162	1366	1570	1774	1978
50	2181	2385	2588	2791	2994	3197	3400	3603	3805	4007
32.0	4210	4411	4614	4815	5017	5219	5420	5622	5823	6024
10	6225	6426	6626	6827	7027	7228	7428	7628	7828	8027
20	8227	8427	8626	8825	9024	9223	9422	9621	9819	0018
30	73.0216	0415	0613	0811	1009	1206	1404	1601	1799	1996
40	2193	2390	2587	2784	2980	3177	3373	3569	3765	3961
50	4157	4353	4548	4744	4939	5134	5329	5525	5719	5914
33.0	6109	6303	6498	6692	6886	7080	7274	7467	7661	7854
10	8048	8241	8434	8627	8820	9013	9205	9398	9590	9783
20	9974	0167	0359	0550	0742	0934	1125	1316	1507	1699
30	74.1889	2080	2271	2462	2652	2842	3032	3223	3413	3602
40	3792	3982	4171	4361	4550	4739	4928	5117	5306	5490
50	5683	5871	6059	6248	6436	6624	6811	6999	7187	7374
34.0	7562	7749	7936	8123	8310	8497	8683	8870	9056	9242
10	9429	9615	9801	9987	0172	0358	0543	0728	0914	1099
20	75.1284	1469	1654	1838	2023	2207	2392	2576	2760	2944
30	3128	3312	3495	3679	3862	4046	4229	4412	4595	4778
40	4960	5143	5326	5508	5690	5872	6054	6236	6418	6600
50	6781	6963	7144	7326	7507	7688	7869	8049	8230	8411
35.0	8591	8772	8952	9132	9312	9492	9672	9851	0031	0211
10	76.0390	0569	0748	0927	1106	1285	1464	1642	1821	1999
20	2177	2356	2534	2712	2889	3067	3245	3422	3599	3777
30	3954	4131	4308	4485	4662	4838	5015	5191	5367	5543
40	5720	5896	6071	6247	6423	6598	6774	6949	7124	7299
50	7475	7649	7824	7999	8173	8348	8522	8696	8871	9045
36.0	9219	9392	9566	9740	9913	0087	0260	0433	0606	0779
10	77.0952	1125	1298	1470	1643	1815	1987	2159	2331	2503
20	2675	2847	3018	3190	3361	3532	3704	3875	4046	4216
30	4388	4558	4729	4899	5069	5240	5410	5580	5750	5920
40	6090	6259	6429	6598	6768	6937	7106	7275	7444	7613
0	7781	7950	8119	8287	8455	8623	8791	8959	9127	9295

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29.0	74.3752	4050	4348	4645	4943	5240	5538	5835	6132	6429
10	6726	7023	7319	7616	7912	8209	8505	8801	9097	9393
20	9689	9985	0281	0576	0872	1167	1462	1757	2052	2347
30	75.2642	2937	3231	3526	3820	4115	4409	4703	4997	5291
40	5585	5878	6172	6465	6759	7052	7345	7638	7931	8224
50	8517	8810	9102	9395	9687	9979	0272	0564	0856	1148
30.0	76.1439	1731	2023	2314	2606	2897	3188	3479	3770	4061
10	4352	4642	4933	5224	5514	5805	6095	6385	6675	6965
20	7255	7545	7834	8124	8413	8703	8992	9281	9570	9859
30	77.0148	0437	0726	1015	1303	1592	1880	2168	2457	2745
40	3033	3321	3608	3895	4184	4471	4759	5046	5333	5621
50	5908	6195	6482	6768	7055	7342	7628	7915	8210	8487
31.0	8774	9060	9346	9632	9918	0203	0489	0775	1060	1346
10	78.1631	1916	2201	2486	2771	3056	3341	3626	3910	4195
20	4479	4764	5048	5332	5616	5900	6184	6468	6752	7036
30	7319	7603	7886	8170	8453	8736	9019	9302	9585	9868
40	79.0151	0433	0716	0999	1281	1563	1846	2128	2410	2692
50	2974	3256	3538	3819	4101	4383	4664	4945	5227	5508
32.0	5789	6070	6351	6632	6913	7194	7474	7755	8036	8316
10	8596	8877	9157	9437	9717	9997	0277	0557	0836	1116
20	80.1369	1675	1955	2234	2513	2792	3072	3351	3630	3908
30	4187	4466	4745	5023	5302	5580	5859	6137	6415	6693
40	6978	7249	7527	7805	8083	8361	8638	8916	9193	9471
50	9748	0025	0302	0580	0857	1134	1410	1687	1964	2241
33.0	81.2517	2794	3070	3347	3623	3899	4175	4452	4728	5004
10	5279	5555	5831	6107	6382	6658	6933	7209	7484	7759
20	8035	8310	8585	8860	9135	9410	9684	9959	0234	0508
30	82.0783	1057	1332	1606	1880	2154	2429	2703	2977	3250
40	3524	3798	4072	4345	4619	4893	5166	5439	5713	5986
50	6259	6532	6805	7078	7351	7624	7897	8170	8442	8715
34.0	8987	9260	9532	9805	0077	0349	0621	0893	1165	1437
10	83.1709	1981	2253	2525	2796	3068	3339	3611	3882	4154
20	4425	4696	4967	5238	5509	5780	6051	6322	6593	6864
30	7134	7405	7675	7946	8216	8487	8757	9027	9297	9568
40	9838	0108	0378	0647	0917	1187	1457	1726	1996	2266
50	84.2535	2805	3074	3343	3612	3882	4151	4420	4689	4958
35.0	5227	5496	5764	6033	6302	6571	6839	7107	7376	7644
10	7913	8181	8449	8717	8985	9254	9522	9790	0057	0325
20	85.0593	0861	1128	1396	1664	1931	2199	2466	2733	3001
30	3268	3535	3802	4069	4336	4603	4870	5137	5404	5671
40	5938	6204	6471	6737	7004	7270	7537	7803	8069	8336
50	8602	8868	9134	9400	9666	9932	0198	0464	0730	0995
36.0	86.1261	1527	1792	2058	2323	2589	2854	3119	3385	3650
10	3915	4180	4445	4710	4975	5240	5505	5770	6035	6300
20	6564	6829	7094	7358	7623	7887	8152	8416	8680	8945
30	9209	9473	9737	0001	0265	0529	0793	1057	1321	1585
40	87.1849	2112	2376	2640	2903	3167	3430	3694	3957	4220
50	4484	4747	5010	5273	5536	5800	6063	6326	6589	6851

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37.0	77.9463	9631	9798	9965	0133	0300	0467	0634	0801	0968
10	78.1134	1301	1467	1634	1800	1966	2132	2298	2464	2630
20	2796	2961	3127	3292	3457	3623	3788	3953	4118	4282
30	4447	4612	4776	4941	5107	5269	5433	5597	5761	5925
40	6089	6252	6416	6579	6742	6905	7069	7232	7395	7557
50	7720	7883	8045	8280	8370	8532	8694	8856	9018	9180
38.0	9342	9540	9665	9826	9988	0149	0310	0471	0632	0793
10	79.0954	1115	1275	1436	1596	1757	1917	2077	2237	2397
20	2557	2716	2876	3035	3195	3354	3513	3673	3832	3991
30	4149	4308	4467	4626	4784	4942	5101	5259	5417	5525
40	5733	5891	6048	6206	6364	6571	6679	6836	6993	7150
50	7307	7464	7621	7777	7934	8091	8247	8403	8560	8716
39.0	8872	9028	9184	9339	9495	9651	9806	9962	0117	0272
10	80.0427	0582	0737	0892	1047	1201	1356	1511	1665	1819
20	1973	2128	2282	2435	2589	2743	2897	3050	3204	3357
30	3510	3664	3817	3970	4123	4276	4428	4581	4734	4886
40	5038	5191	5343	5495	5647	5799	5951	6103	6254	6406
50	6557	6709	6860	7011	7163	7314	7465	7615	7766	7917
40.0	8067	8218	8368	8519	8669	8819	8959	9119	9269	9419
10	9569	9718	9867	0017	0167	0316	0465	0614	0763	0912
20	81.1061	1210	1358	1507	1655	1804	1952	2100	2248	2396
30	2544	2692	2840	2987	3135	3283	3430	3578	3725	3872
40	4019	4166	4313	4460	4607	4753	4900	5046	5193	5339
50	5485	5631	5778	5923	6069	6215	6361	6507	6652	6797
41.0	6943	7088	7233	7378	7523	7668	7813	7958	8103	8247
10	8392	8536	8681	8825	8969	9113	9257	9401	9545	9689
20	9832	9976	0120	0263	0406	0550	0693	0836	0979	1122
30	82.1265	1407	1550	1693	1835	1977	2120	2262	2404	2546
40	2688	2830	2972	3114	3255	3397	3538	3680	3821	3963
50	4104	4245	4386	4527	4668	4808	4949	5089	5230	5370
42.0	5511	5651	5791	5931	6071	6211	6351	6491	6631	6770
10	6910	7049	7189	7328	7467	7606	7745	7884	8023	8162
20	8301	8439	8578	8716	8855	8993	9131	9269	9407	9545
30	9683	9821	9959	0097	0234	0372	0509	0646	0784	0921
40	83.1058	1195	1332	1469	1606	1742	1879	2015	2152	2288
50	2425	2561	2697	2833	2969	3105	3241	3377	3512	3648
43.0	3783	3919	4054	4189	4325	4459	4595	4730	4865	4999
10	5134	5269	5403	5538	5672	5806	5941	6075	6209	6343
20	6477	6611	6745	6878	7012	7145	7279	7412	7546	7679
30	7812	7945	8078	8211	8344	8477	8609	8742	8875	9007
40	9139	9272	9404	9536	9668	9800	9932	0064	0196	0327
50	84.0359	0591	0722	0854	0985	1116	1247	1378	1509	1640
44.0	1771	1902	2033	2163	2294	2424	2555	2685	2815	2945
10	3076	3206	3336	3465	3595	3725	3855	3984	4114	4243
20	4372	4502	4631	4760	4889	5018	5147	5276	5404	5533
30	5662	5790	5919	6047	6175	6304	6432	6560	6688	6816
40	6944	7071	7199	7327	7454	7582	7709	7836	7954	8091
50	8218	8345	8472	8599	8726	8852	8879	9106	9232	9359

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37.0	87.7114	7377	7640	7903	8165	8428	8691	8953	9216	9478
10	9741	0003	0265	0528	0790	1052	1314	1576	1839	2101
20	88.2363	2625	2887	3148	3410	3672	3934	4196	4457	4719
30	4980	5242	5503	5765	6026	6288	6549	6810	7072	7333
40	7594	7855	8116	0377	8639	8900	9160	9421	9682	9943
50	89.0204	0465	0725	0986	1247	1507	1768	2028	2289	2549
38.0	2810	3070	3331	3591	3851	4111	4371	4632	4892	5152
10	5412	5672	5932	6192	6452	6712	6971	7231	7491	7751
20	8010	8270	8530	8789	9049	9308	9568	9827	0086	0346
30	90.0605	0864	1124	1383	1642	1901	2160	2419	2679	2938
40	3197	3455	3714	3973	4232	4491	4750	5008	5267	5526
50	5784	6043	6302	6560	6819	7077	7336	7594	7852	8111
39.0	8369	8672	8886	9144	9402	9660	9918	0177	0435	0693
10	91.0951	1209	1467	1724	1982	2240	2498	2756	3014	3271
20	3529	3787	4044	4302	4560	4817	5075	5332	5590	5847
30	6104	6362	6619	6876	7134	7391	7648	7905	8163	8420
40	8677	8934	9191	9446	9705	9962	0219	0476	0733	0990
50	92.1247	1503	1760	2017	2274	2530	2787	3044	3300	3557
40.0	3813	4070	4327	4583	4840	5096	5352	5609	5865	6121
10	6378	6634	6890	7147	7403	7659	7915	8171	8427	8683
20	8940	9196	9452	9708	9964	0219	0475	0731	0987	1243
30	93.1499	1755	2010	2266	2522	2778	3033	3289	3545	3800
40	4056	4311	4567	4822	5078	5333	5589	5844	6100	6355
50	6610	6866	7121	7376	7632	7887	8142	8397	8653	8908
41.0	9163	9418	9673	9928	0183	0438	0694	0949	1204	1458
10	94.1713	1968	2223	2478	2733	2988	3243	3498	3752	4007
20	4262	4517	4771	5026	5281	5535	5790	6045	6299	6554
30	6808	7063	7317	7572	7826	8081	8335	8590	8844	9099
40	9353	9607	9862	0116	0370	0625	0879	1133	1387	1642
50	95.1896	2150	2404	2659	2913	3167	3421	3675	3929	4183
42.0	4437	4691	4945	5199	5453	5707	5961	6215	6469	6723
10	6977	7231	7485	7739	7993	8246	8500	8754	9008	9262
20	9515	9769	0023	0277	0530	0784	1038	1291	1545	1799
30	96.2052	2306	2560	2813	3067	3320	3574	3827	4081	4335
40	4588	4842	5095	5349	5602	5855	6109	6362	6616	6869
50	7122	7376	7629	7883	8136	8389	8643	8896	9149	9403
43.0	9656	9909	0162	0416	0669	0922	1175	1429	1682	1935
10	97.2188	2441	2694	2948	3201	3454	3707	3960	4213	4466
20	4719	4973	5226	5479	5732	5985	6238	6491	6744	6997
30	7250	7503	7756	8009	8262	8515	8768	9021	9274	9527
40	9780	0033	0286	0538	0791	1044	1297	1550	1803	2056
50	98.2309	2562	2814	3067	3320	3573	3826	4079	4331	4584
44.0	4837	5090	5343	5596	5848	6101	6354	6607	6860	7112
10	7365	7618	7871	8123	8376	8629	8882	9134	9387	9640
20	9893	0145	0398	0651	0993	1156	1409	1662	1914	2167
30	99.2420	2672	2925	3178	3430	3683	3936	4189	4441	4694
40	4947	5199	5452	5705	5957	6210	6463	6715	6968	7221
50	7473	7726	7979	8231	8484	8737	8989	9242	9495	9747

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45.0	84.9485	9611	9737	9854	9990	0116	0242	0367	0493	0619
10	85.0745	0870	0996	1121	1246	1372	1497	1622	1747	1872
20	1997	2123	2247	2371	2496	2620	2745	2869	2994	3118
30	3243	3366	3490	3614	3738	3862	3986	4109	4233	4356
40	4480	4603	4727	4850	4973	5096	5219	5342	5465	5588
50	5711	5833	5955	6078	6201	6323	6445	6568	6690	6812
46.0	6934	7056	7178	7300	7421	7513	7665	7786	7908	8029
10	8150	8272	8393	8514	8635	8756	8877	8998	9119	9329
20	9340	9480	9601	9721	9842	9962	0082	0202	0322	0442
30	86.0562	0682	0802	0921	1041	1161	1280	1399	1519	1638
40	1758	1877	1996	2115	2234	2353	2471	2590	2709	2827
50	2946	3064	3183	3301	3419	3537	3656	3774	3892	4010
47.0	4127	4245	4363	4481	4598	4715	4833	4950	5068	5185
10	5302	5419	5536	5653	5770	5887	6004	6120	6237	6353
20	6470	6586	6703	6819	6935	7051	7167	7283	7399	7515
30	7631	7747	7862	7978	8094	8209	8324	8439	8555	8670
40	8785	8900	9015	9130	9245	9360	9474	9589	9704	9818
50	87.9932	0047	0161	0276	0390	0504	0618	0732	0846	0960
48.0	1073	1187	1301	1414	1528	1641	1755	1868	1981	2094
10	2208	2321	2434	2547	2659	2772	2885	2997	3110	3223
20	3335	3448	3560	3672	3784	3896	4008	4120	4232	4344
30	4456	4568	4679	4791	4903	5014	5125	5237	5348	5459
40	5570	5681	5793	5904	6014	6125	6236	6347	6457	6568
50	6678	6789	6900	7010	7120	7230	7340	7450	7560	7670
49.0	7780	7889	7999	8109	8218	8328	8437	8547	8656	8765
10	8875	8984	9093	9202	9311	9420	9529	9637	9746	9855
20	9963	0072	0180	0289	0397	0505	0613	0721	0829	0937
30	88.1045	1153	1261	1369	1477	1584	1692	1799	1907	2014
40	2121	2228	2336	2443	2550	2657	2764	2871	2977	3084
50	3191	3297	3404	3510	3617	3723	3829	3936	4042	4148
50.0	4254	4360	4466	4572	4677	4783	4889	4994	5100	5205
10	5311	5416	5521	5627	5732	5837	5942	6047	6152	6257
20	6362	6466	6571	6675	6780	6885	6989	7093	7198	7302
30	7406	7510	7614	7718	7822	7926	8030	8133	8237	8341
40	8444	8549	8651	8755	8858	8961	9064	8167	9271	9374
50	9476	9579	9682	9785	9888	9990	0093	0195	0297	0400
51.0	890.503	605	707	809	911	013	115	217	319	402
10	891.523	624	726	827	929	030	132	233	334	435
20	892.536	637	738	839	940	041	142	242	343	444
30	893.544	645	745	846	946	046	146	246	346	446
40	894.546	646	746	846	945	045	144	244	343	443
50	895.542	641	741	840	939	038	137	236	335	433
52.0	896.532	631	729	828	926	025	123	222	320	418
10	897.516	614	712	810	908	006	104	201	299	397
20	898 494	592	689	787	884	981	078	176	273	370
30	899.467	564	660	757	854	951	047	144	240	337
40	890.433	529	626	722	818	914	010	106	202	298
50	901.394	489	585	681	776	872	967	063	158	253

D '	0	1	2	3	4	5	6	7	8	9
45.0	00.0000	0253	0505	0758	1011	1263	1516	1769	2021	2274
10	2527	2779	3032	3285	3537	3790	4043	4295	4548	480[illegible]
20	5053	5306	5559	5811	6064	6317	6569	6822	7075	7328
30	7580	7833	8086	8338	8591	8844	9096	9349	9602	9855
40	01.0107	0360	0613	0866	1118	1371	1624	1877	2129	2382
50	2635	2888	3140	3393	3646	3899	4152	4404	4657	4910
45.0	5163	5416	5668	5921	6174	6427	6680	6932	7185	7438
10	7691	7944	8197	8450	8703	8956	9209	9461	9714	9967
20	02.0220	0473	0726	0979	1232	1485	1738	1991	2244	2497
30	2750	3003	3256	3509	3762	4015	4268	4521	4774	5027
40	5280	5534	5787	6040	6293	6546	6799	7052	7305	7559
50	7812	8065	8318	8571	8825	9078	9331	9584	9837	0091
47.0	03.0344	0597	0851	1104	1357	1611	1864	2117	2371	2624
10	2877	3131	3384	3638	3891	4144	4398	4651	4905	5158
20	5412	5665	5919	6172	6426	6680	6933	7187	7440	7694
30	7947	8201	8455	8708	8962	9216	9469	9723	9977	0231
40	04.0484	0738	0992	1246	1500	1753	2007	2261	2515	2769
50	3023	3277	3531	3785	4738	4290	4546	4800	5054	5308
48.0	5563	5817	6071	6325	6579	6833	7087	7341	7595	7850
10	8104	8358	8612	8867	9121	9375	9629	9884	0138	0392
20	05.0647	0901	1156	1410	1664	1919	2173	2428	2682	2937
30	3191	3446	3701	3955	4210	4465	4719	4974	5229	5483
40	5738	5992	6248	6502	6757	7012	7267	7522	7777	8032
50	8286	8541	8796	9051	9306	9561	9816	0072	0327	0582
49.0	06 0837	1092	1347	1602	1858	2113	2368	2623	2879	3134
10	3389	3645	3900	4156	4411	4666	4922	5177	5433	5689
20	5944	6200	6455	6711	6467	7222	7478	7734	7989	8245
30	8501	8757	9013	9269	9524	9780	0036	0292	0548	0804
40	07.1060	1316	1573	1829	2085	2341	2597	2853	3110	3365
50	3622	3878	4135	4391	4648	4904	5160	5417	5673	5930
50.0	6186	6443	6700	6956	7213	7470	7726	7983	8240	8497
10	8753	9010	9267	9524	9780	0038	0295	0552	0809	1066
20	08.1323	1580	1837	2094	2352	2609	2866	3123	3381	3638
30	3895	4153	4410	4668	4925	5183	5440	5698	5956	6213
40	6471	6728	6986	7244	7502	7760	8018	8275	8533	8791
50	9049	9307	9565	9823	0081	0340	0598	0856	1114	1372
51.0	09.1631	1889	2147	2406	2664	2923	3181	3440	3698	3957
10	4215	4474	4733	4991	5250	5509	5768	6026	6286	6544
20	6803	7062	7321	7580	7839	8099	8358	8617	8876	9135
30	9395	9654	9913	0173	0432	0692	0951	1211	1470	1730
40	10.1990	2249	2500	2769	3028	3288	3548	3808	4068	4328
50	4588	4848	5108	5368	5628	5889	6149	6409	6670	6930
52.0	7190	7451	7711	7971	8232	8493	8753	9013	9275	9535
10	9796	0057	0318	0579	0839	1100	1361	1622	1883	2145
20	11.2406	2667	2928	3189	3451	3712	3974	4235	4496	4758
30	5019	5281	5543	5804	6066	6328	6590	6852	7113	7375
40	7637	7899	8161	8423	8686	8948	9210	9472	9735	9997
50	12.0259	0522	0784	1047	1309	1572	1835	2097	2360	2623

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53.0	902.349	444	539	634	729	824	919	014	108	203
10	903.298	392	487	581	676	770	864	959	053	147
20	904.241	335	429	523	617	711	804	898	991	085
30	905.179	272	366	459	552	645	739	832	925	017
40	906.111	204	296	389	482	574	667	760	852	945
50	907.037	129	222	314	406	498	591	682	774	866
54.0	958	049	141	233	324	416	507	599	690	781
10	908.873	964	055	146	237	328	419	510	601	691
20	909.782	873	963	054	144	235	325	415	506	596
30	910.686	776	866	956	046	136	226	315	405	495
40	911.584	674	763	853	942	031	121	210	299	388
50	912.477	566	655	744	833	921	010	099	187	276
55.0	913.364	453	541	630	718	806	894	982	070	158
10	914.246	334	422	510	598	685	773	860	948	035
20	915.123	210	297	385	472	559	646	733	820	907
30	994	080	167	254	341	427	514	600	687	773
40	916.859	945	032	118	204	290	376	462	548	634
50	917.719	805	891	976	062	147	233	318	404	489
56.0	918.574	659	744	830	915	000	084	169	254	339
10	919.424	508	593	677	762	846	931	015	099	184
20	920.268	352	436	520	604	688	772	855	939	023
30	921.107	190	174	357	441	524	607	691	774	857
40	941	023	106	189	272	355	438	520	603	685
50	922.768	851	933	016	098	180	263	345	427	509
57.0	923.591	673	755	837	919	001	083	164	246	328
10	924.409	491	572	653	735	816	897	978	060	141
20	925.222	303	384	465	545	626	707	787	868	949
30	926.029	110	190	270	351	431	511	591	671	751
40	831	911	991	071	151	231	310	390	469	549
50	927.628	708	787	867	946	025	104	183	262	341
58.0	928.420	499	578	657	736	814	893	972	050	129
10	929.207	286	364	442	521	599	677	755	833	911
20	989	067	145	223	300	378	456	533	611	688
30	930.766	843	920	998	075	152	229	306	383	460
40	931.537	611	691	768	845	921	998	074	151	227
50	932.304	380	457	533	609	685	762	838	914	990
59.0	933.066	141	217	293	369	444	520	596	671	747
10	822	897	973	048	123	198	274	349	424	499
20	934.574	649	723	798	873	948	022	097	171	246
30	935.320	395	469	543	618	692	766	840	914	988
40	936.062	136	210	283	357	431	505	578	652	725
50	799	872	945	019	092	165	238	311	385	458
60.0	937.531	603	674	749	822	895	967	040	113	185
10	938.258	330	402	475	547	619	691	763	835	908
20	979	051	123	195	267	339	410	482	554	625
30	939.697	768	839	911	982	053	125	196	267	338
40	940.409	480	515	622	693	763	834	905	975	046
50	941.117	187	257	328	398	468	539	609	679	749

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53.0	12.2886	3148	3411	3674	3937	4200	4463	4727	4990	5253
10	5516	5780	6043	6306	6570	6833	7097	7360	7624	7888
20	8151	8415	8679	8942	9207	9471	9735	9999	0263	0527
30	13.0791	1055	1320	1584	1848	2113	2377	2642	2906	3171
40	3436	3700	3965	4230	4495	4760	5024	5289	5555	5820
50	6085	6350	6615	6880	7146	7411	7677	7942	8208	8473
54.0	8739	9005	9270	9536	9802	0068	0334	0600	0866	1132
10	14.1398	1664	1931	2197	2463	2730	2996	3263	3529	3796
20	4062	4329	4596	4863	5130	5397	5663	5931	6198	6465
30	6732	6999	7266	7534	7801	8069	8336	8604	8871	9139
40	9407	9675	9942	0210	0478	0746	1014	1283	1551	1819
50	15.2087	2356	2624	2892	3161	3429	3698	3967	4236	4504
55.0	4773	5042	5311	5580	5849	6118	6387	6657	6926	7195
10	7465	7734	8004	8273	8543	8813	9083	9352	9622	9892
20	16.0162	0432	0702	0973	1243	1513	1783	2054	2324	2595
30	2866	3136	3407	3678	3949	4220	4491	4762	5033	5304
40	5575	5846	6118	6389	6661	6932	7204	7475	7747	8019
50	8291	8563	8835	9107	9379	9651	9923	0195	0468	0740
56.0	17.1023	1205	1558	1830	2103	2376	2649	2922	3195	3468
10	3741	4014	4287	4561	4834	5107	5381	5651	5928	6202
20	6476	6749	7023	7297	7571	7845	8120	8394	8668	8943
30	9217	9492	9766	0041	0316	0590	0865	1140	1415	1690
40	18.1965	2240	2516	2791	3066	3342	3618	3893	4169	4445
50	4720	4996	5272	5548	5824	6101	6377	6653	6930	7206
57.0	7483	7759	8036	8313	8589	8866	9143	9420	9697	9975
10	19.0252	0529	0807	1084	1362	1639	1917	2195	2473	2751
20	3029	3307	3585	3863	4141	4420	4698	4977	5255	5534
30	5813	6091	6370	6649	6928	7207	7487	7766	8045	8325
40	8604	8884	9163	9443	9723	0003	0283	0563	0843	1123
50	20.1403	1684	1964	2245	2525	2806	3087	3368	3649	3930
58.0	4211	4492	4773	5054	5336	5617	5899	6180	6462	6744
10	7026	7308	7590	7872	8154	8536	8719	9001	9284	9566
20	9849	0132	0415	0698	0981	1264	1547	1836	2114	2397
30	21.2681	2964	3248	3532	3816	4100	4384	4668	4952	5236
40	5521	5805	6090	6374	6659	6944	7229	7514	7799	8084
50	8369	8654	8940	9225	9511	9797	0082	0368	0654	0940
59.0	22.1226	1512	1799	2085	2371	2658	2945	3231	3518	3805
10	4092	4379	4667	4954	5241	5529	5816	6104	6392	6679
20	6967	7255	7543	7832	8120	8408	8697	8985	9274	9563
30	9851	0140	0429	0719	1008	1297	1586	1876	2166	2455
40	23.2745	3035	3325	3615	3905	4195	4486	4776	5067	5357
50	5648	5939	6230	6520	6812	7103	7394	7685	7977	8268
60.0	8561	8852	9144	9436	9728	0021	0313	0605	0898	1190
10	24.1483	1776	2069	2362	2655	2948	3241	3535	3828	4122
20	4415	4709	5003	5297	5592	5885	6180	6474	6769	7063
30	7358	7653	7948	8243	8538	8833	9128	9424	9719	0015
40	25.0311	0607	0903	1199	1495	1791	2087	2384	2681	2977
50	3274	3571	3868	4165	4462	4760	5057	5355	5652	5950

D	'	0	1	2	3	4	5	6	7	8	9
61.	0	941.819	889	959	029	099	169	239	308	378	448
	10	942.517	587	656	725	795	864	933	003	072	141
	20	943.201	279	348	417	486	555	624	692	761	830
	30	898	967	036	104	172	241	309	377	446	514
	40	944.582	650	718	786	854	922	990	058	125	193
	50	945.261	328	396	464	531	598	666	733	800	868
62.	0	935	002	069	136	203	270	337	404	471	538
	10	946.604	671	738	804	870	937	004	070	136	203
	20	947.269	335	401	467	533	599	665	731	797	863
	30	929	995	060	126	192	257	323	388	453	519
	40	948.584	649	715	780	845	910	975	040	105	170
	50	949.235	300	3[illegible]4	429	494	558	623	688	752	816
63.	0	881	945	009	074	138	202	266	330	394	428
	10	950.522	586	650	714	777	841	905	968	032	096
	20	951.159	222	286	349	412	476	539	602	665	728
	30	791	854	917	980	043	105	168	231	294	356
	40	952.419	481	544	606	668	731	793	855	917	979
	50	953.042	104	166	228	290	351	413	475	537	598
64.	0	660	722	783	845	906	967	029	090	152	213
	10	954.274	335	396	457	518	579	640	701	762	823
	20	883	944	005	065	126	186	247	307	368	428
	30	955.488	548	609	669	729	789	849	909	969	029
	40	956.089	148	208	268	328	387	447	506	566	625
	50	684	744	804	862	921	980	040	099	158	217
65.	0	957.276	335	393	452	511	570	628	687	746	804
	10	862	921	979	038	096	154	212	271	329	387
	20	958.445	503	561	619	677	734	792	850	908	965
	30	959.023	080	138	195	253	310	367	425	482	539
	40	596	653	711	768	825	881	938	995	052	109
	50	960.165	222	279	335	392	448	505	561	618	674
66.	0	730	786	845	899	955	011	067	123	179	235
	10	961.290	346	402	458	513	569	624	680	735	791
	20	846	902	957	012	067	123	178	233	288	343
	30	962.398	453	508	562	617	672	727	781	836	891
	40	945	999	054	108	162	217	271	325	379	434
	50	963.488	542	596	650	704	757	811	865	919	972
67.	0	964.0[illegible]6	080	133	187	240	294	347	400	454	507
	10	560	613	666	719	773	826	878	931	984	073
	20	965.090	143	195	248	301	353	406	458	511	563
	30	615	668	720	772	824	879	928	980	032	085
	40	966.136	188	240	292	344	395	447	499	550	602
	50	653	705	756	807	858	910	961	012	064	115
68.	0	967.166	217	268	319	369	420	471	522	573	623
	10	674	725	775	826	876	927	977	027	078	128
	20	968.178	228	278	328	379	429	478	528	578	628
	30	678	728	777	827	877	926	976	025	075	124
	40	969.173	223	272	321	370	420	469	518	567	616
	50	665	714	762	811	860	9[illegible]9	957	006	055	103

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61.0	25.6248	6546	6844	7142	7441	7739	8038	8336	8635	8934
10	9233	9532	9831	0130	0430	0730	1029	1329	1629	1928
20	26.2229	2529	2829	3129	3430	3731	4031	4332	4633	4934
30	5235	5537	5838	6140	6442	6743	7045	7347	7649	7952
40	8254	8556	8859	9162	9465	9767	0070	0374	0677	0980
50	27.1284	1588	1891	2195	2499	2803	3107	3412	3716	4021
62.0	4326	4630	4935	5240	5546	5851	6156	6462	6768	7073
10	7379	7685	7991	8298	8604	8911	9217	9524	9831	0138
20	28.0445	0752	1060	1367	1673	1983	2291	2599	2907	3215
30	3523	3832	4140	4449	4758	5067	5376	5685	5995	6304
40	6614	6924	7234	7544	7854	8164	8475	8785	9096	9407
50	9718	0029	0340	0651	0963	1274	1586	1898	2210	2522
63.0	29.2834	3146	3459	3772	4084	4397	4710	5023	5337	5650
10	5964	6277	6591	6905	7219	7534	7848	8163	8477	8792
20	9167	9422	9737	0053	0368	0684	0999	1315	1631	1947
30	30.2264	2580	2897	3213	3530	3847	4164	4482	4799	5117
40	5434	5752	6070	6388	6707	7025	7323	7662	7981	8300
50	8619	8938	9258	9577	9897	0217	0537	0857	1177	1498
64.0	31.1818	2139	2460	2781	3102	3423	3745	4066	4388	4710
10	5032	5354	5676	5999	6321	6644	6967	7290	7613	7937
20	8260	8584	8908	9232	9556	9880	0205	0529	0854	1179
30	32.1504	1829	2154	2480	2806	3131	3457	3783	4110	4430
40	4763	5089	5416	5743	6071	6398	6725	7053	7381	7709
50	8037	8365	8694	9023	9351	9680	0009	0339	0668	0998
65.0	33.1327	1657	1987	2318	2648	2979	3309	3640	3971	4302
10	4634	4965	5297	5629	5961	6293	6625	6958	7291	7623
20	7957	8290	8623	8957	9290	9624	9958	0292	0627	0961
30	34.1296	1631	1966	2301	2636	2272	3308	3644	3980	4316
40	4652	4989	5326	5662	6000	6337	6674	7012	7350	7688
50	8026	8364	8702	9041	9380	9719	0058	0398	0737	1077
66.0	35.1417	1757	2097	2438	2778	3119	3460	3801	4142	4484
10	4826	5168	5510	5852	6194	6537	6880	7223	7566	7909
20	8253	8596	8940	9284	9629	9973	0318	0662	1008	1353
30	36.1698	2044	2389	[illegible]	3081	3428	3774	4121	4468	4815
40	5162	5510	5857	6205	6553	6901	7250	7598	7947	8296
50	8645	8995	9344	9694	0044	0394	0745	1095	1446	1797
67.0	37.2148	2499	2851	3203	3555	3907	4259	4612	4965	5317
10	5671	6024	6377	6731	7085	7439	7793	8148	8503	8858
20	9213	9568	9924	0279	0635	0992	1348	1705	2061	2418
30	38.2776	3133	3491	3849	4207	4565	4923	5282	5641	6000
40	6359	6719	7079	7438	7799	8159	8519	8880	9241	9603
50	9964	0326	0689	1050	1412	1775	2137	2500	2863	3226
68.0	39.3590	3954	4318	4683	5047	5412	5777	6142	6507	6873
10	7239	7605	7971	8337	8704	9071	9438	9806	0173	0541
20	40 0909	1277	1646	2015	2384	2753	3122	3492	3862	4232
30	4602	4973	5344	5715	6086	6458	6829	7201	7574	7946
40	8319	8692	9065	9438	9812	0186	0560	0934	1309	1684
50	41.2059	2434	2809	3185	3561	3938	4314	4691	5068	5445

D '	0	1	2	3	4	5	6	7	8	9
69.0	970.152	200	248	297	345	394	442	490	538	586
10	635	683	731	779	826	874	922	970	018	065
20	971.113	161	208	256	303	351	398	446	493	540
30	588	635	682	729	776	823	870	917	964	011
40	972.058	105	151	198	245	291	338	384	431	477
50	524	570	617	662	709	755	802	848	894	940
70.0	985	032	078	124	169	215	261	307	352	398
10	973 443	489	535	580	625	671	716	761	807	852
20	897	942	987	032	077	122	167	212	257	302
30	974 347	391	436	481	525	570	614	659	703	747
40	792	836	881	925	969	013	057	101	145	189
50	975 233	277	321	365	408	452	496	539	583	626
71.0	670	713	757	800	844	887	930	974	017	060
10	976.103	146	189	232	275	318	361	404	446	489
20	532	574	617	660	702	745	787	830	872	914
30	957	999	041	083	125	167	209	251	293	335
40	977.377	419	461	503	544	586	628	669	711	752
50	794	835	877	918	959	001	042	083	124	165
72.0	978.206	247	288	329	370	411	452	493	533	574
10	615	655	696	736	777	817	858	898	939	979
20	979 019	059	100	140	180	220	260	300	340	380
30	420	460	500	539	579	618	658	697	737	776
40	816	855	895	934	973	012	052	091	130	169
50	980.208	247	286	325	364	403	441	480	519	558
73.0	596	635	673	712	750	789	827	866	904	942
10	980	019	057	095	133	171	209	247	285	323
20	981.361	399	436	474	512	549	587	624	662	699
30	737	774	812	849	886	924	961	998	035	072
40	982.109	146	183	220	257	294	331	367	404	440
50	477	514	551	587	623	660	696	733	769	805
74.0	842	878	914	950	986	022	058	094	130	166
10	983.202	238	273	309	345	380	416	452	487	523
20	558	594	629	664	700	735	770	805	840	875
30	910	945	980	015	050	085	120	155	189	224
40	984.259	293	328	363	397	432	466	500	535	569
50	603	637	672	706	740	774	808	843	876	910
75 0	944	978	011	045	079	112	146	180	213	247
10	985.280	314	347	380	414	447	480	513	547	580
20	613	646	679	712	745	778	811	843	876	909
30	945	974	007	039	072	104	137	169	202	234
40	986.266	299	331	363	395	437	459	491	523	555
50	587	619	651	683	714	746	778	809	841	873
76.0	904	936	967	998	030	061	092	124	155	186
10	987.217	248	279	310	341	372	403	434	465	495
20	526	557	587	618	649	679	710	740	771	801
30	831	862	892	922	953	983	013	043	073	803
40	988.133	163	193	222	252	282	312	341	371	401
50	430	460	490	519	548	578	607	636	665	695

D ′	0	1	2	3	4	5	6	7	8	9
69.0	41.5823	6200	6578	6956	7335	7714	8093	8472	8851	9231
10	9611	9991	0371	0752	1133	1514	1896	2277	2659	3041
20	42.3424	3807	4190	4573	4956	5340	5724	6108	6493	6877
30	7262	7648	8033	8419	8805	9191	9578	9964	0352	0739
40	43.1126	1514	1902	2291	2679	3068	3458	3847	4237	4627
50	5017	5407	5798	6189	6581	6972	7364	7756	8148	8541
70.0	8934	9327	9721	0115	0509	0903	1297	1692	2087	2483
10	44.2879	3274	3671	4067	4464	4861	5258	5655	6054	6452
20	6851	7250	7649	8048	8448	8847	9248	9648	0049	0450
30	45.0851	1253	1655	2057	2459	2862	3265	3669	4072	4475
40	4881	5285	5690	6095	6501	6906	7312	7719	8125	8532
50	8939	9347	9755	0163	0571	0980	1389	1798	2208	2618
71.0	46.3028	3439	3849	4261	4672	5084	5496	5908	6321	6734
10	7147	7561	7975	8389	8804	9219	9634	0049	0465	0881
20	47.1298	1715	2132	2549	2967	3385	3803	4222	4641	5060
30	5480	5900	6320	6741	7162	7583	8005	8427	8849	9272
40	9695	0118	0542	0966	1390	1814	2239	2665	3090	3516
50	48.3942	4369	4796	5223	5651	6079	6507	6936	7365	7794
72.0	8224	8654	9084	9515	9946	0378	0809	1241	1674	2107
10	49.2540	2973	3407	3841	4276	4711	5146	5582	6018	6454
20	6891	7328	7765	8203	8641	9080	9519	9958	0397	0837
30	50.1278	1718	2159	2601	3043	3485	3927	4370	4813	5257
40	5701	6145	6590	7035	7481	7927	8373	8820	9267	9714
50	51.1062	0610	1059	1508	1957	2407	2857	3307	3758	4209
73.0	4661	5113	5565	6018	6471	6925	7379	7833	8288	8743
10	9199	9655	0111	0568	1025	1483	1941	2399	2858	3317
20	52.3777	4237	4697	5158	5619	6081	6543	7005	7468	7931
30	8395	8859	9324	9789	0254	0720	1186	1653	2120	2587
40	53.3055	3523	3992	4461	4931	5401	5871	6342	6814	7285
50	7758	8230	8703	9177	9651	0125	0600	1075	1551	2027
74.0	54.2504	2981	3458	3936	4414	4893	5372	5852	6332	6813
10	7294	7775	8257	8740	9223	9706	0190	0674	1159	1644
20	55.2130	2616	3102	3589	4077	4565	5053	5542	6031	6521
30	7012	7502	7994	8485	8978	9471	9964	0457	0951	1446
40	56.1941	2437	2933	3430	3927	4424	4922	5421	5920	6419
50	6920	7420	7921	8423	8925	9427	9930	0431	0928	1442
75.0	57.1947	2453	2959	3466	3973	4481	4989	5497	6006	6516
10	7026	7537	8048	8560	9072	9585	0099	0613	1127	1642
20	58.2157	2673	3190	3707	4225	4743	5262	5781	6301	6821
30	7342	7863	8385	8908	9431	9955	0479	1003	1529	2055
40	59.2581	3108	3636	4164	4692	5222	5751	6282	6813	7344
50	7876	8409	8942	9476	0010	0545	1081	1617	2154	2691
76.0	60.3229	3767	4306	4846	5386	5927	6469	7011	7553	8097
10	8640	9185	9730	0276	0822	1369	1916	2464	3013	3562
20	61.4112	4663	5214	5766	6318	6871	7425	7979	8534	9090
30	9646	0203	0761	1319	1877	2434	2997	3558	4119	4681
40	62.5244	5807	6371	6935	7501	8067	8633	9201	9768	0337
50	63.0906	1476	2047	2618	3190	3762	4336	4910	5484	6060

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77.0	724	753	782	811	840	869	898	927	956	985
10	989.014	042	071	100	128	157	185	214	243	271
20	299	328	356	384	413	441	469	497	525	553
30	581	609	637	665	693	721	749	777	804	832
40	859	887	915	942	970	997	025	052	079	107
50	990.134	161	188	215	242	269	296	324	351	377
78.0	404	431	458	485	511	538	565	591	618	644
10	671	697	724	750	777	803	829	855	881	908
20	934	960	986	012	038	064	090	115	141	167
30	991.193	218	244	270	295	321	346	372	397	422
40	448	473	498	524	549	574	599	624	649	674
50	699	724	749	774	799	823	848	873	897	992
79.0	947	971	996	020	044	069	093	117	142	166
10	992.190	214	238	263	287	311	335	358	382	406
20	430	454	478	501	525	549	572	596	619	643
30	666	689	713	736	759	783	806	829	852	875
40	898	921	944	967	990	013	036	058	081	104
50	993.127	149	172	195	217	240	262	284	307	329
80.0	351	374	396	418	440	462	484	506	528	550
10	572	594	616	638	660	681	703	725	746	768
20	789	811	832	853	875	896	918	939	960	981
30	994.003	024	045	066	087	108	129	150	171	191
40	212	233	254	274	295	316	336	357	377	397
50	418	438	459	479	499	519	540	560	580	600
81.0	620	640	660	680	700	719	739	759	779	798
10	818	838	857	877	896	916	935	955	974	993
20	995.013	032	051	070	089	108	127	146	165	184
30	203	222	241	260	278	297	316	334	353	372
40	390	409	427	445	464	482	500	519	537	555
50	573	591	609	628	646	663	681	699	717	735
82.0	753	770	788	806	823	841	859	876	894	911
10	928	946	963	980	998	015	032	049	066	083
20	996.100	117	134	151	168	185	202	218	235	252
30	268	285	302	318	335	351	368	384	400	417
40	434	449	465	482	498	514	530	546	562	578
50	594	610	625	641	657	673	688	704	720	735
83.0	751	766	782	797	812	828	843	858	874	889
10	904	919	934	949	964	679	994	009	024	039
20	997.053	068	083	097	112	127	141	156	170	185
30	199	214	228	242	257	271	285	299	313	327
40	341	355	369	383	397	411	425	439	452	466
50	480	493	507	520	534	547	501	574	588	601
84.0	614	627	641	654	667	680	693	706	719	732
10	745	758	771	784	797	809	822	835	847	860
20	872	885	897	910	922	935	947	959	972	984
30	896	008	020	032	044	056	068	080	092	104
40	116	127	139	151	163	174	186	197	209	220
50	232	243	254	266	277	288	300	311	322	333

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77. 0	63.6636	7213	7790	8368	8947	9526	0106	0687	1269	1851
10	64.2434	3018	3602	4187	4773	5360	5947	6535	7124	7713
20	8303	8894	9486	0078	0671	1265	1859	2455	3051	3647
30	65.4245	4843	5442	6042	6642	7243	7845	8448	9052	9656
40	66.0261	0867	1473	2081	2689	3298	3907	4518	5129	5741
50	6354	6967	7582	8197	8813	9430	0047	0665	1285	1905
78. 0	67.2525	3147	3769	4393	5017	5642	6267	6894	7521	8149
10	8778	9408	0039	0670	1303	1936	2570	3205	3841	4477
20	68.5115	5753	6392	7032	7673	8315	8958	9601	0246	0891
30	69.1537	2184	2832	3481	4131	4782	5433	6086	6739	7393
40	8049	8705	9362	0020	0678	1338	1999	2660	3323	3987
50	70.4651	5316	5983	6650	7318	7987	8658	9329	0001	0674
79. 0	71.1348	2023	2699	3375	4053	4732	5412	6093	6775	7458
10	8141	8826	9512	0199	0887	1576	2266	2957	3649	4342
20	72.5036	5731	6427	7124	7822	8521	9221	9923	0625	1329
30	73.2033	2739	3445	4153	4862	5572	6283	6995	7708	8422
40	9137	9854	0571	1290	2010	2731	3453	4176	4900	5626
50	74.6352	7080	7809	8539	9270	0002	0736	1470	2206	2943
80. 0	75.3681	4421	5161	5903	6646	7390	8135	8881	9629	0378
10	76.1128	1880	2632	3386	4141	4897	5656	6414	7174	7935
20	8698	9461	0226	0993	1761	2529	3300	4071	4844	5618
30	77.6393	7170	7948	8728	9508	0290	1074	1858	2644	3432
40	78.4220	5011	5802	6595	7389	8185	8982	9780	0580	1381
50	79.2183	2987	3793	4600	5408	6217	7029	7841	8655	9471
81. 0	80.0287	1106	1926	2747	3570	4394	5220	6047	6876	7706
10	8538	9371	0206	1042	1880	2720	3561	4403	5247	6093
20	81.6940	7789	8640	9492	0345	1201	2057	2916	3776	4638
30	82.5501	6366	7233	8101	8971	9843	0716	1591	2468	3346
40	83.4226	5108	5992	6877	7764	8653	9543	0435	1329	2225
50	84.3123	4022	4923	5826	6730	7637	8546	9456	0368	1282
82. 0	85.2197	3115	4034	4956	5879	6804	7731	8660	9591	0524
10	86.1458	2395	3333	4274	5216	6161	7107	8056	9006	9959
20	87.0913	1870	2828	3789	4751	5716	6683	7652	8623	9596
30	88.0571	1548	2528	3509	4493	5479	6467	7457	8449	9444
40	89.0441	1440	2441	3444	4450	5458	6468	7481	8495	9513
50	90.0532	1554	2578	3604	4633	5664	6698	7734	8772	9813
83. 0	91.0856	1902	2950	4000	5053	6109	7167	8227	9290	0356
10	92.1424	2495	3567	4644	5722	6803	7887	8973	0062	1153
20	93.2248	3345	4444	5547	6652	7760	8870	9984	1100	2219
30	94.3340	4465	5593	6723	7856	8992	0131	1273	2418	3566
40	95.4716	5870	7027	8187	9349	0515	1684	2856	4031	5209
50	96.6391	7575	8763	9953	1148	2345	3545	4749	5956	7166
84. 0	97.8380	9597	0817	2041	3267	4498	5732	6969	8210	9454
10	99.0702	1953	3208	4466	5728	6993	8262	9535	0812	2192
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30	01.6423	7749	9079	0413	1752	3094	4440	5791	7145	8504
40	02.9867	1234	2606	3981	5361	6745	8134	9527	0925	2326
50	04.3733	5144	6560	7979	9403	0832	2266	3705	5148	6596

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10	9984.53	64	74	85	95	06	6	27	37	47
20	9985.58	68	78	89	99	09	9	29	39	49
30	9986.59	69	79	89	99	08	1	28	37	47
40	9987.57	66	76	85	95	04	13	23	32	41
50	9988.51	60	69	78	87	96	05	14	23	32
86. 0	9989.41	50	58	67	76	84	93	02	10	19
10	9990.27	36	44	52	60	69	77	86	94	02
20	9991.10	18	26	34	42	50	58	66	74	81
30	89	97	05	12	20	27	35	42	50	57
40	9992.65	72	79	86	93	01	08	15	22	29
50	9993.36	43	50	57	64	71	78	84	91	98
87. 0	9994.04	11	18	24	31	37	43	50	56	62
10	69	75	81	87	93	00	06	12	18	24
20	9995.29	35	41	47	53	58	64	70	75	81
30	86	92	97	03	08	14	19	24	29	35
40	9996.40	45	50	55	60	65	70	75	80	85
50	89	94	99	04	08	13	17	22	26	31
88. 0	9997.35	40	44	48	53	57	61	65	69	74
10	78	82	86	90	93	97	01	05	09	12
20	9998.16	20	23	27	31	34	38	41	44	48
30	51	54	58	61	64	67	70	73	76	79
40	82	85	88	91	94	97	99	02	04	07
50	9999.10	12	15	17	20	22	25	27	29	32
89. 0	34	36	38	40	42	44	46	48	50	52
10	54	56	58	59	61	63	64	66	68	69
20	71	72	73	75	76	77	79	80	81	82
30	83	84	86	87	88	88	89	90	91	92
40	93	93	94	95	95	9[illegible]	96	97	97	98
50	98	98	99	99	99	99	99	99	99	99
60	10.000000									

Min.	85 Deg.	86 Deg.	87 Deg.	88 Deg.	89 Deg.	Min.
0	11.058048	11.155356	11.280604	11.456916	11.758078	0
1	9506	11. 7175	11. 3028	11. 60553	11. 65375	1
2	11.060968	11. 9092	11. 5465	11. 64221	11. 72805	2
3	11. 2435	11.160837	11. 7917	11. 67920	11. 80359	3
4	11. 3907	11. 2679	11.290381	11. 71651	11. 88047	4
5	11. 5384	11. 4529	11. 2860	11. 75414	11. 95874	5
6	11. 6866	1. 6387	11. 5353	11. 79210	11.803844	6
7	11. 8353	11. 8252	11. 7861	11. 83039	11. 11964	7
8	11. 9845	11.170126	11.300383	11. 86902	11. 20237	8
9	11.071342	11. 2008	11. 2919	11. 90800	11. 28672	9
10	11. 2844	11. 3897	11. 5471	11. 94733	11. 37273	10
11	11. 4351	11. 5795	11. 8037	11. 98702	11. 46048	11
12	11. 5864	11. 7702	11.310618	11.502707	11. 55004	12
13	11. 7381	11. 9616	11. 3216	11. 06750	11. 64149	13
14	11. 8904	11.181539	11. 5828	11. 10830	11. 73490	14
15	11.080432	11. 3471	11. 8456	11. 14949	11. 83037	15
16	11. 1966	11. 5410	11.321100	11. 19108	11. 92797	16
17	11. 3505	11. 7359	11. 3761	11. 23307	11.902783	17
18	11. 5049	11. 9317	11. 6437	11. 27546	11. 13003	18
19	11. 6599	11.191283	11. 9730	11. 31827	11. 23469	19
20	11. 8154	11. 3258	11.331848	11. 36151	11. 34194	20
21	11. 9715	11. 5242	11. 4566	11. 40519	11. 45191	21
22	11.091281	11. 7235	11. 7311	11. 44930	11. 56473	22
23	11. 2853	11. 9236	11.340072	11. 49387	11. 68055	23
24	11. 4430	11.201248	11. 2851	11. 53890	11. 79955	24
25	11. 6013	11. 3269	11. 5648	11. 58440	11. 92191	25
26	11. 7602	11. 5299	11. 8462	11. 63078	12.004781	26
27	11. 9197	11. 7338	11.351296	11. 67685	12. 17746	27
28	11.100797	11. 9387	11. 4147	11. 72382	12. 31111	28
29	11. 2404	11.211446	11. 7017	11. 77131	12. 44900	29
30	11. 4016	11. 3514	11. 9907	11. 81932	12. 59142	30
31	11. 5634	11. 5592	11.362815	11. 86787	12. 73866	31
32	11. 7258	11. 7680	11. 5744	11. 91696	12. 89106	32
33	11. 8888	11. 9778	11. 8692	11. 96662	12.104901	33
34	11.110524	11.221886	11.371660	11.601685	12. 21292	34
35	11. 2167	11. 4005	11. 4648	11. 06766	12. 38326	35
36	11. 3815	11. 6133	11. 7657	11. 11908	12. 56056	36
37	11. 5470	11. 8272	11.380687	11. 17111	12. 74540	37
38	11. 7131	11.230422	11. 3738	11. 22378	12. 93845	38
39	11. 8798	11. 2582	11. 6811	11. 27708	12.214049	39
40	11.120471	11. 4753	11. 9906	11. 33105	12. 35239	40

Min.	85 Deg.	86 Deg.	87 Deg	88 Deg.	89 Deg.	Min.
41	11.122151	11.236935	11.393022	11.638570	12.257516	41
42	11. 3838	11. 9128	11. 6161	11. 44105	12. 80997	42
43	11. 5531	11 241332	11. 9323	11. 49710	12.305821	43
44	11. 7230	11. 3547	11.402508	11. 55379	12. 32151	44
45	11. 8936	11. 5773	11. 5717	11. 61144	12. 60180	45
46	11.130649	11. 8011	11. 8949	11. 66975	12. 90143	46
47	11. 2368	11.250260	11.412205	11. 72886	12.422328	47
48	11. 4094	11. 2521	11. 5486	11. 78878	12. 57091	48
49	11. 5827	11. 4793	11. 8792	11. 84954	12. 94880	49
50	11. 7567	11. 7078	11.422123	11. 91116	12.536273	50
51	11. 9314	11. 9374	11. 5480	11. 97366	12. 82030	51
52	11.141068	11.261683	11. 8863	11.703708	12.633183	52
53	11. 2829	11. 4004	11·432272	11. 10144	12. 91175	53
54	11. 4597	11. 6337	11. 5709	11. 16677	12.758122	54
55	11. 6372	11. 8683	11. 9172	11. 23309	12.837304	55
56	11. 8154	11.271041	11.442664	11. 30044	12.934214	56
57	11. 9943	11. 3412	11. 6183	11. 36885	13.059153	57
58	11.151740	11. 5796	11. 9732	11. 43835	13.235244	58
59	11. 3545	11. 8194	11.453309	11. 50898	13.536274	59
60	11. 5356	11.280604	11. 6916	11. 58078		60

Mr *STREET*'s

TABLE

OF

Logiſtical Logarithms

To every $\left\{\begin{array}{l}\text{Minute}\\ \text{Second}\end{array}\right\}$ of a $\left\{\begin{array}{l}\text{Degree.}\\ \text{Minute.}\end{array}\right.$

This Day is Publiſhed,

Neatly printed, in 4 Vols. 12mo. Price bound in Calf. 12 s.

Embelliſhed with near 100 Copper-Plates, curiouſly engraved by the beſt Maſters.

Tranſlated from the *French*, by *John Kelly*, Eſq; of the *Inner Temple*; Dr *Bellamy* of St *John's College*, *Oxford*; and *J. Sparrow*, Surgeon and Mathematician.

NATURE DELINEATED: Being Philoſophical Converſations, wherein the wonderful Works of Providence, in the Animal, Vegetable and Mineral Creation are laid open, the Solar and Planetary Syſtem, and whatever is curious in the Methematicks explained. The Whole being a Compleat Courſe of Natural and Experimental Philoſophy, calculated for the Inſtruction of Youth, in order to prepare them for an early Knowledge of Natural Hiſtory, and create in their Minds an exalted Idea of the Wiſdom of the GREAT CREATOR. Written by way of Dialogue to render the Conception more familiar and eaſy. With a particular Table of Contents to each Volume.

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A New Treatiſe of Husbandry, Gardening, and other curious Matters relating to Country Affairs: Containing, A Plain and Practical Method of Improving all Sort of Meadow, Paſture, and Arable Land, *&c.* And making them produce greater Crops of all Kinds, and at much leſs than the preſent Expence. Under the following Heads: I. Of Wheat, Rye, Oats, Barley, Peaſe, Beans, and all other Sorts of Grain. II. Turnips, Carrots, Buckwheat, Clover, Hemp, Rape, Flax and Coleſeed, *&c.* III. Weld or Would, Woad or Wade, Madder Saffron, *&c.* IV. Meadow, Paſture Grounds, and the different Manner of Feeding Cattle and making other Improvements agreeable to the Soil of the ſeveral Counties in *Great Britain.* V. Hops, Foreſt and Fruit Trees, Vine and Garden Plants of all Sorts. VI. All Kinds of Flowers, Shrubs in general, and Greenhouſe Plants. VII. A Curious Scheme of a Farm, the Annual Expence of it, and its Produce. With many New, Uſeful, and Curious Improvements, never before publiſhed. The Whole founded upon many Years Experience. By *Samuel Trowel*, Gent. To which are added, Several Letters to Mr *Thomas Liveings*, concerning his Compound Manure for Land, with ſome Practical Obſervations thereon:

'	"	0	1	2	3	4	5	6	7	8	9	"
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20	14	13	11	09	08	06	04	03	01	00	2660
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40	82	80	78	77	75	74	72	70	69	68	2680
50	66	64	62	61	59	57	56	54	53	51	2690
45.0	49	48	46	45	43	41	40	38	37	35	2700
10	33	32	30	29	27	25	24	22	21	19	2710
20	17	16	14	13	11	09	08	06	05	02	2720
30	01	00	98	97	95	94	92	90	89	87	2730
40	11.86	84	82	81	79	78	76	74	73	71	2740
50	70	68	67	65	63	62	60	59	57	56	2750
46.0	54	52	51	49	48	46	45	43	41	40	2760
10	38	37	35	34	32	30	29	27	26	24	2770
20	23	21	19	18	16	15	13	12	10	09	2780
20	07	05	04	02	01	99	98	96	95	93	2790
40	10.91	90	88	87	85	84	82	81	79	78	2800
50	76	74	73	71	70	68	67	65	64	62	2810
47.0	61	59	57	56	54	53	51	50	48	47	2820
10	45	44	42	41	39	37	36	34	33	31	2830
20	30	28	27	25	24	22	21	19	18	16	2840
30	15	13	12	10	08	07	05	04	02	01	2850
40	9.99	98	96	95	93	92	90	89	87	86	2860
50	84	83	81	80	78	77	75	74	72	71	2870

' "	0	1	2	3	4	5	6	7	8	9	"
48.0	9.69	68	66	65	63	62	60	59	57	56	2880
10	54	53	51	50	48	47	45	44	42	41	2890
20	39	38	36	35	33	32	30	29	27	26	2900
30	24	23	21	20	18	17	15	14	12	11	2910
40	09	08	06	05	03	02	00	99	97	96	2920
50	8.94	93	91	90	88	87	85	84	83	82	2930
49.0	80	78	77	75	74	72	71	69	68	66	2940
10	65	63	62	60	59	57	56	55	54	53	2950
20	50	49	47	46	44	43	41	40	38	37	2960
30	35	34	33	31	30	28	27	25	24	22	2970
40	21	19	18	16	15	14	12	11	09	08	2980
50	06	05	03	02	01	99	98	96	95	93	2990
50.0	7.92	90	89	87	86	85	83	82	80	79	3000
10	77	76	74	73	72	70	69	67	66	64	2010
20	63	62	60	59	57	56	54	53	51	50	3020
30	49	47	46	44	43	40	41	39	37	36	3030
40	34	33	32	30	29	27	26	24	23	21	3040
50	20	19	17	16	14	13	11	10	09	07	3050
51.0	06	04	03	02	00	99	97	96	94	93	3060
10	6.92	90	89	87	86	85	83	82	80	79	3070
20	78	76	73	73	72	70	69	68	66	65	3080
30	63	62	61	59	58	56	55	54	52	51	3090
40	49	48	47	45	44	42	41	40	38	37	3100
50	35	34	33	31	30	28	27	26	24	23	3110
52.0	21	20	19	17	16	15	13	12	11	09	3120
10	08	06	05	03	02	01	99	98	96	95	3130
20	5.94	92	91	90	88	87	85	84	83	81	3140
30	80	79	77	76	74	73	72	70	69	68	3150
40	66	65	63	62	61	59	58	57	55	54	3160
50	52	51	50	48	47	46	44	43	41	40	3170
53.0	39	37	36	35	33	32	31	29	28	26	3180
10	25	24	22	21	20	18	17	16	14	13	3190
20	12	10	09	07	06	05	03	02	01	99	3200
30	4.98	96	95	94	93	91	90	89	87	86	3210
40	84	83	82	80	79	78	76	75	74	72	3220
50	71	70	68	67	66	64	63	62	60	59	3230
54.0	58	56	55	54	52	51	50	48	47	46	3240
10	44	43	42	40	39	38	36	35	34	32	3250
20	31	30	28	26	24	23	22	21	20	19	3260
30	18	16	15	14	12	11	10	08	07	06	3270
40	04	03	02	00	99	98	96	95	94	92	3280
50	3.91	90	88	87	86	84	83	82	81	79	3290
55.0	78	77	75	74	73	71	70	69	67	66	3300
10	65	63	62	61	59	58	57	56	54	53	3310
20	52	50	49	48	46	45	44	42	41	40	3320
30	39	37	36	35	33	32	31	29	28	27	3330
40	26	24	23	22	20	19	18	16	15	14	3340
50	13	11	10	09	07	06	05	04	02	01	3350

' ''	0	1	2	3	4	5	6	7	8	9	''
56.0	3.00	98	97	96	94	93	92	91	89	88	3360
10	2.87	85	84	83	82	80	79	78	76	75	3370
20	74	73	71	70	69	67	66	65	64	62	3380
30	61	60	58	57	56	55	53	52	51	50	3390
40	48	47	46	44	43	42	41	39	38	37	3400
50	35	34	33	32	30	29	28	27	25	24	3410
57.0	23	21	20	19	18	16	15	14	13	11	3420
10	10	09	08	06	05	04	02	01	00	99	3430
20	1.97	96	95	94	92	91	90	89	87	86	3440
30	85	84	82	81	80	79	77	76	75	74	3450
40	72	71	70	69	67	66	65	63	62	61	3460
50	60	58	57	56	55	53	52	51	50	48	3470
58.0	47	46	45	43	42	41	40	39	37	36	3480
10	35	34	32	31	30	29	27	26	25	24	3490
20	22	21	20	19	17	16	15	14	12	11	3500
30	10	09	07	06	05	04	03	01	00	99	3510
40	98	96	95	94	93	91	90	89	88	87	3520
50	85	84	83	82	80	79	78	77	75	74	3530
59.0	73	72	71	69	68	67	66	64	63	62	3540
10	61	60	58	57	56	55	53	52	51	50	3550
20	49	47	46	45	44	42	41	40	39	38	3560
30	36	35	34	33	31	30	29	28	27	25	3570
40	24	23	22	21	19	18	17	16	15	13	3580
50	12	11	10	8	7	6	5	4	2	1	3590
60.0	0										3600

Hours	Deg.	'	M.	Deg.	'	M.	Deg.	'
I	2	30	1	0	3	31	1	18
II	5	00	2	0	5	32	1	20
III	7	30	3	0	8	33	1	23
IV	10	00	4	0	10	34	1	25
V	12	30	5	0	13	35	1	28
VI	15	00	6	0	15	36	1	30
VII	17	30	7	0	18	37	1	33
VIII	20	00	8	0	20	38	1	35
IX	22	30	9	0	23	39	1	38
X	25	00	10	0	25	40	1	40
XI	27	30	11	0	28	41	1	43
XII	30	00	12	0	30	42	1	45
XIII	32	30	13	0	33	43	1	48
XIV	35	00	14	0	35	44	1	50
XV	37	30	15	0	38	45	1	53
XVI	40	00	16	0	40	46	1	55
XVII	42	30	17	0	43	47	1	58
XVIII	45	00	18	0	45	48	2	00
XIX	47	30	19	0	48	49	2	3
XX	50	00	20	0	50	50	2	5
XXI	52	30	21	0	53	51	2	8
XXII	55	00	22	0	55	52	2	10
XXIII	57	30	23	0	58	53	2	13
XXIV	60	00	24	1	00	54	2	15
			25	1	3	55	2	18
			26	1	5	56	2	20
			27	1	8	57	2	23
			28	1	10	58	2	25
			29	1	13	59	2	28
			30	1	15	60	2	30